

## Notes:

1. Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.
2. For short answer questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians \& not degrees \& use 4 significant digits.

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## Quiz 1

## MULTIPLE CHOICE AND SHORT ANSWER QUESTIONS

## 1. Input Impedance of Transmission Lines ( 15 points)



Assume a sinusoidal source is connected to a transmission line, as shown.
a. (3 pts) The transmission line is of infinite length and it has characteristic impedance $Z_{0}$. What is the input impedance of the transmission line? $Z_{I N}=Z_{o}$ since an infinite line has nothing to reflect off of, there can be no reflected wave.
b. (3 pts) What is the input impedance of a transmission line of length $\lambda / 4$, terminated by an open circuit? $Z_{I N}=Z_{o} \frac{\infty+j Z_{o} \tan \beta d}{Z_{o}+j \infty \tan \beta d}=-j Z_{o} \cot \frac{2 \pi}{\lambda} \frac{\lambda}{4}=-j Z_{o} \cot \frac{\pi}{2}=0$
c. (3 pts) A transmission line has length $\lambda / 2$, characteristic impedance $Z_{o}=50 \Omega$ and load $Z_{L}=75 \Omega+j 50 \Omega$. What is the input impedance? $Z_{I N}=Z_{L}$ whenever the line is $a$ half wavelength long. $Z_{\text {IN }}=Z_{o} \frac{Z_{L}+j Z_{o} \tan \pi}{Z_{o}+j Z_{L} \tan \pi}=Z_{o} \frac{Z_{L}}{Z_{o}}=Z_{L}$
d. (3 pts) A transmission line has characteristic impedance $Z_{o}=50 \Omega$ and the load is short-circuited. What is the minimum length the line should have (in terms of wavelength) so that the input impedance is $Z_{I N}=j 50 \Omega$ ? What minimum line length for $Z_{\text {IN }}=-j 50 \Omega$ ? $Z_{\text {IN }}=Z_{o} \frac{0+j Z_{o} \tan \beta d}{Z_{o}+j 0 \tan \beta d}=j Z_{o} \tan \beta d$ thus we need the lengths were $\tan \beta d$ is +1 or -1 . This requires an argument of $\pi / 4$ or $3 \pi / 4$ or $d \pi / \lambda=\pi / 4$ or $d=\lambda / 8,3 \lambda / 8$
e. ( 3 pts ) What is the input impedance of the network below?

$Z_{\text {IN }}=Z_{o} \frac{Z_{L}+j Z_{o} \tan \beta d}{Z_{o}+j Z_{L} \tan \beta d} \beta d=2 \pi / \lambda 3 \lambda / 4=1.5 \pi \tan 1.5 \pi=\infty j u s t$ like $\tan 0.5 \pi=\infty$ For such cases, $Z_{\text {IN }}=Z_{o}^{2} / Z_{L}$ Thus, $Z_{\text {IN } 2}=10^{2} / 100=1, Z_{\text {IN } 1}=50^{2} / 1=2500$.

## Quiz 1

## 2. Waves on Transmission Lines (15 points) (Includes an extra credit question.)

For all of the following questions, assume that a lossless transmission line has a characteristic impedance of $Z_{o}=50 \Omega$.
a. (4 pts) The load reflection coefficient for a given transmission line is $\Gamma_{L}=0.5$. What is the load impedance? $\Gamma_{L}=0.5=\frac{Z_{L}-Z_{o}}{Z_{L}+Z_{o}}=\frac{Z_{L}-50}{Z_{L}+50}, Z_{L}=150$
b. (3 pts Extra Credit) Same question as a, but for $\Gamma_{L}=j 0.5$.
$\Gamma_{L}=j 0.5=\frac{Z_{L}-Z_{o}}{Z_{L}+Z_{o}}=\frac{Z_{L}-50}{Z_{L}+50} Z_{L}=30+j 40$
c. (4 pts) A transmission line has a matched load ( $Z_{L}=Z_{o}$ ). What does the Standing Wave Pattern look like? Circle the correct diagram below.

$\mathrm{d} .(4 \mathrm{pts})$ The voltage wave propagating on a transmission line is given in real space-time form as $v(z, t)=10 \cos \left(8 \pi 10^{7} t-0.4 \pi z\right)-6 \cos \left(8 \pi 10^{7} t+0.4 \pi z\right)$. Write this voltage expression in phasor form. $V(z)=V^{+} e^{-j \beta z}+V^{-} e^{j \beta z}=10 e^{-j 0.4 \pi z}-6 e^{+j 0.4 \pi z}$
e. (3 pts) Is the wave of part da traveling wave or a tanding wave?

Technically, one could argue that tis iseth, sitite ticonsists of both a standing and a traveling wave. However, the best answer is standing wave since it does include a standing wave and standing wave effects will be observed. Both answers are OK.

## 3 Lossy Transmission Line (5 points)

A sinusoidal voltage wave is propagating on a low loss transmission line. The voltage as a function of position appears as shown below. From this plot, determine the damping coefficient. $\alpha=0.05$ At 10meters, the voltage is 0.6. Thus $-10 \alpha=\ln (0.6)=-.5$ Anything close to this number will be accepted.



What is each known for? Rowan Atkinson - actor, Mr. Bean, Black Adder. Roger Corman - King of the B Movies (Pit and the Pendulum, Little Shop of Horrors, Fall of the House of Ushwer). Herbie Hancock - jazz musician. Allen Dumont - developed practical CRT and TV. Founded Dumont Network. Graduated from RPI.

## Quiz 1

## REGULAR QUESTIONS

## 5. Sinusoidal Voltages on a Lossless Transmission Line (20 points)

A lossless transmission line has a characteristic impedance of $Z_{o}=50$ Ohms. The propagation velocity on the line is $u=1.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$. The load impedance $Z_{L}$ is purely resistive.

a. (3 pts) Consider the voltage Standing Wave Pattern shown on the following page.

What is the Voltage Standing Wave Ratio? $V S W R=\frac{32}{8}=4$
b. (2 pts) Is the load impedance larger or smaller than the characteristic impedance $Z_{o}$ ? It is larger since the standing wave pattern is a maximum at the load.
c. (3 pts) What is the load reflection coefficient? $V S W R=\frac{1+\left|\Gamma_{L}\right|}{1-\left|\Gamma_{L}\right|}=4, \Gamma_{L}=0.6$ Make sure that the sign is correct. The reflection coefficient has to be positive to have a maximum at the load.
d. (3 pts) What is the value of the load impedance? $\Gamma_{L}=0.6=\frac{Z_{L}-Z_{o}}{Z_{L}+Z_{o}}=\frac{Z_{L}-50}{Z_{L}+50}$, $Z_{L}=200$
e. (3 pts) What is the amplitude of the injected voltage wave? $V^{+}=20$ since the standing wave pattern average is 20.
f. (2 pts) What percentage of power is reflected by the load? $36 \%$ because it involves the square of the reflection coefficient.

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g. (2 pts) What is the wavelength? $\lambda=5$ meters. The distance from the load to the next maximum will be a half wavelength.
h. (2 pts) What is the frequency of the source? $f=\frac{u}{\lambda}=\frac{1.5 \times 10^{8}}{5}=3 \times 10^{7}=30 \mathrm{MHz}$

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## 6. Sinusoidal Transmission Line ( 20 pts )

A long lossless transmission line is driven by a standard source with $R_{g}=50 \mathrm{Ohms}$, but has no load (open circuit). The propagation velocity on the line is $u=1.6 \times 10^{8} \mathrm{~m} / \mathrm{s}$ and the characteristic impedance of the line is $Z_{o}=100$ Ohms.

a. What is the reflection coefficient $\Gamma_{L}$ at the load? (2 pts)
$\Gamma_{L}=\frac{Z_{L}-Z_{o}}{Z_{L}+Z_{o}}=\frac{\infty-Z_{o}}{\infty+Z_{o}}=1$
b. What is the standing wave ratio $S W R$ ? (2 pts) $V S W R=\frac{1+\left|\Gamma_{L}\right|}{1-\left|\Gamma_{L}\right|}=\frac{1+1}{1-1}=\infty$
c. For a frequency $f=100 \times 10^{3} \mathrm{~Hz}$, what is the propagation constant $\beta$ ? ( 2 pts )

$$
\beta=\frac{\omega}{u}=\frac{2 \pi f}{u}=\frac{2 \pi 10^{5}}{1.6 \times 10^{8}}=0.00125 \pi=0.003927
$$

d. What is the wavelength $\lambda$ ? (2 pts) $\lambda=\frac{2 \pi}{\beta}=\frac{u}{f}=1600$ meters
e. Will the voltage be a maximum or a minimum at the load? (2 pts) Maximum, because the load impedance is larger than the characteristic impedance.
f. If the voltage at the load is a maximum, what is the distance from the load to the first minimum? If the voltage at the load is a minimum, what is the distance from the load to the first maximum? (Answer only one of these questions.) Express this length both in meters and in wavelengths. Distance $=$ a quarter wavelength. It is a half wavelength between like features and the opposite feature lies exactly half way in between.
g. As luck would have it, this transmission line is made up of 10 lines connected in series. Thus, it is possible to monitor the voltage at the input and output and 9 evenly spaced internal locations. A PSpice simulation of the line and the voltages observed are shown on the following page. From the information given, determine the length of the entire line. Express this length both in meters and in wavelengths.


INPUT


Note: In the first half cycle, the top sinusoid is the input and the bottom is the output. The nine internal points sequence from the top to the bottom. Only 3, 5 , and 7 are shown to avoid clutter in the diagram.
The Length is exactly one half wavelength, as can be seen from the data, which goes through one complete cycle in its entire length. Everything repeats each half wavelength
in a transmission line. The length must by $\frac{\lambda}{2}=\frac{1600}{2}=800$
h. What voltages will be observed at each location if a matched load is connected to the line? The voltage will be constant at each location and equal to exactly half the voltage at the maximum or 5 Volts.

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## 7. Transients on Transmission Lines ( 20 points)



A 9V battery with a 10 Ohm internal impedance is connected to a 50 Ohm transmission line with a 10 Ohm load. The length of the line is 10 meters and the propagation speed is $2.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$.
a. Generate the bounce diagram for this configuration. (10 pts)


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b. Determine and plot the voltage observed at the load as a function of time. Indicate the value the voltage will eventually reach if we wait long enough (time goes to infinity). (10 pts)
Below are shown the voltages observed after several bounces. It is only necessary to determine the first three values shown (through 4.1V) A PSpice model with a different time scale is included for completeness.



