Fields and Waves
Quiz 2
Fall 2007

Name Solution

Do all problems. Show work on problems to receive partial credit.

(1) ____________________

(2) ____________________

(3) ____________________

(4) ____________________

Total ____________________

1. An air-filled parallel plate capacitor has a layer of dielectric material of width d inserted as shown in the figure. The area of the plates is A, and there is a charge Q and $-Q$ as indicated.

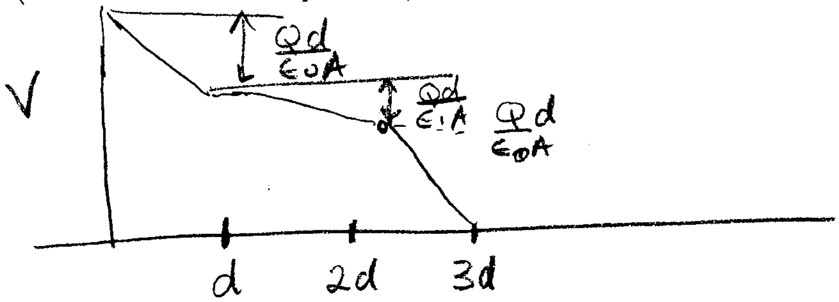
(5) Find the flux density in the 3 regions.

$$D_1 = D_2 = D_3 = Q/A \quad \frac{C}{m^2}$$

(5) Find the electric field in the 3 regions.

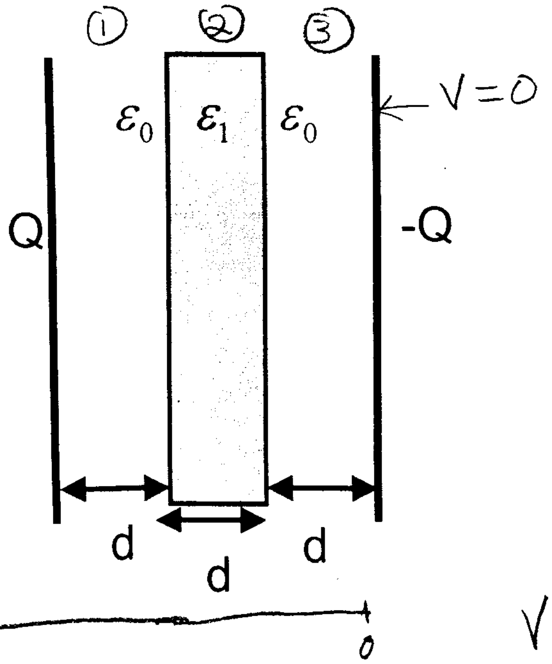
$$E_1 = E_3 = \frac{Q}{\epsilon_0 A} \ \frac{V}{m}, \quad E_2 = \frac{Q}{\epsilon_1 A} \ \frac{V}{m}$$

(10) If we set the potential of the right plate to zero, find the potential in the capacitor (will be a function of position).



(5) Find the capacitance.

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A}\left(\frac{2}{\epsilon_0} + \frac{1}{\epsilon_1}\right)} = \frac{A}{d\left(\frac{2}{\epsilon_0} + \frac{1}{\epsilon_1}\right)}$$



$$V = \int_0^x \vec{E}\cdot d\vec{l} = \begin{cases} \frac{Q}{A\varepsilon_0} x & 0 \le x \le d \\ \frac{Q}{A\varepsilon_0} d + \frac{Q}{A\varepsilon_1}(x-d) & d \le x \le 2d \\ \frac{Q}{A}\left(\frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_1}\right)d + \frac{Q}{A\varepsilon_0}(x-2d) & 2d \le x \le 3d \end{cases}$$

$$V_{3d} = \frac{Q}{A}\left(\frac{2}{\varepsilon_0} + \frac{1}{\varepsilon_1}\right)d$$

2. A uniform charge density of ρ_v C/m^3 exists in the volume between x = -1 m and x = +1 m. This region extends to infinity in the y and z directions.

(15) Use Gauss's law to find the flux density in all of space. $\oint D \cdot ds = Q_{enc}$

$D \cdot A = \rho_v x A$

$D = \rho_v x \; \hat{a}_x \quad 1 > x > 0$ $\qquad D = \rho_v \; \hat{a}_x$ for $x > 1$

$D = \rho_v x \; -\hat{a}_x \quad -1 < x < 0$ $\qquad D = \rho_v \; (-\hat{a}_x)$ for $x < -1$

(10) Use Poisson's equation $\frac{d^2V}{dx^2} = -\frac{\rho_v}{\varepsilon_0}$ to find the flux density and compare your answers.

assume $V(0) = 0$

$\frac{\partial^2 V}{\partial x^2} = -\rho/\varepsilon_0$

$\frac{\partial V}{\partial x} = -\frac{\rho}{\varepsilon_0} x + C_1$ $\qquad \frac{dV}{dx} = E$ so $E = 0$

$V = -\frac{\rho}{\varepsilon_0}\frac{x^2}{2} + C_2$ at $x = 0$; $C_1 = 0$

at $x = 0$, $V = 0$

$V = -\frac{\rho}{2\varepsilon_0} x^2$

Check

$\frac{\partial V}{\partial x} = -E = \frac{\rho}{\varepsilon_0} x$

for $x > 1$

$\frac{\partial^2 V}{\partial x^2} = 0$

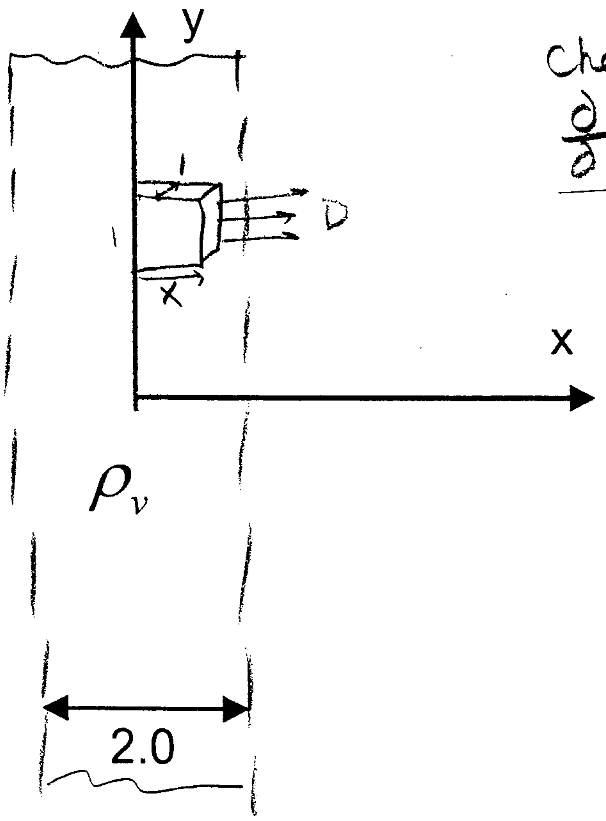
$\frac{\partial V}{\partial x} = C_1$

$V = C_1 x + C_2$

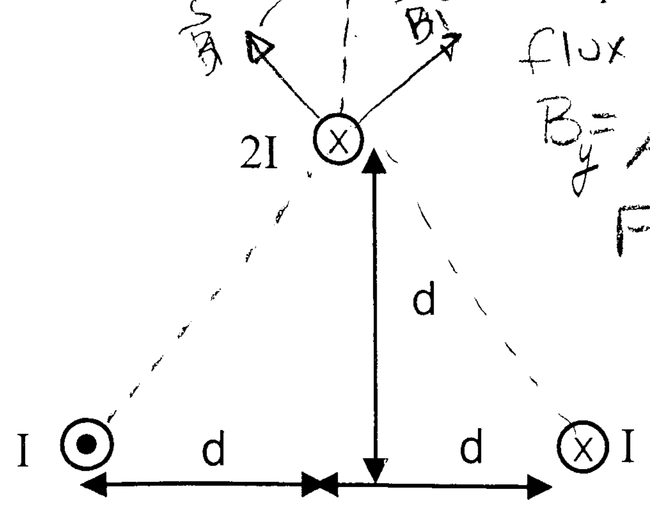
at $x = 1$

$V = -\frac{\rho}{2\varepsilon_0} \Rightarrow C_1 = -\frac{\rho}{2\varepsilon_0}$

$C_2 = 0$



3. a) (10) Three long wires are parallel to each other. The currents are shown on the figure. Find the force per unit length on the top conductor.



flux density at top conductor

$B_y = \frac{\mu_0 I}{2\pi\sqrt{2}d}(\sqrt{2})$

$F = i\, l \times B \qquad (l = 1)$

$= 2I \cdot \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I^2}{\pi d}$

b) (15) A person is standing under a dc transmission line using a compass to find his way. The transmission line is running North-South and the current is going South. The tangential component of the earths field at the observation point is 20 μT. The line current is 2000 Amperes and the line is 10 meters above the compass. What direction is the compass needle pointing? Give and angle from due north.

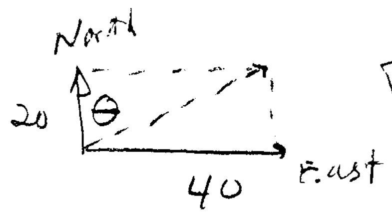
N

I ↓

S

$B_{earth} = 20\,\mu T\ \hat{a}_y$

$B_{wire} = \frac{\mu_0 \times 2000}{2\pi \times 10}\hat{a}_x = \frac{4\pi \times 10^{-7} \times 200}{2\pi} = 40\,\mu T$



$B_{mag} = \sqrt{20^2 + 40^2}\ \mu T$

$\theta = \operatorname{atan}\frac{40}{20} = \operatorname{atan} 2$

4. (10) a) The magnetic vector potential is given by $A = y\cos x\hat{a}_x + (y - e^{-x})\hat{a}_z$. Find the magnetic flux density.

$\nabla \times A = B$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y\cos x & 0 & y - e^{-x} \end{vmatrix}$$

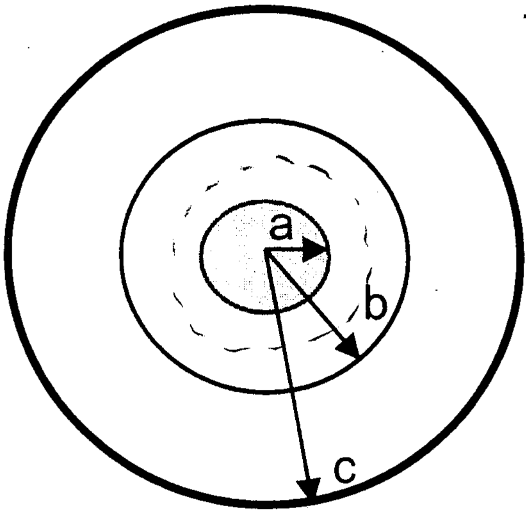
$B = \hat{a}_x(1) - \hat{a}_y(e^{-x}) + \hat{a}_z(\cos x)$

b) A coax has the inner conductor made of 2 different conducting materials. The center material (radius a) has conductivity σ_1 and the outer material (radius b) has conductivity σ_2. The return (radius c) is a very thin outer conductor. We apply a voltage so that the electric field in the inner conductors is E V/m in the axial direction.

(3) Why is the electric field the same in both inner conductors? (one sentence will do)

Because E_t is continuous.

(12) Find the magnetic field everywhere.



$J_1 = \sigma_1 E$

$J_2 = \sigma_2 E$

$\oint H \cdot dl = I_{enc}$

$r < a$

$\oint H \cdot dl = \pi r^2 \sigma_1 E$

$H = \frac{\pi r^2 \sigma_1 E}{2\pi r}$

$= \frac{\sigma_1 E r}{2}$

$a < r < b$

$\oint H \cdot dl = \sigma_1 E \pi a^2 + \sigma_2 E(\pi r^2 - \pi a^2)$

$H = \frac{\sigma_1 E \pi a^2 + \sigma_2 E(\pi r^2 - \pi a^2)}{2\pi r}$

$b < r < c$

$H = \frac{\sigma_1 E \pi a^2 + \sigma_2 E(\pi b^2 - \pi a^2)}{2\pi r}$