

Quiz 2  
19 March 2007

**VECTOR CALCULUS  
ELECTROSTATICS  
CAPACITANCE**

Name \_\_\_\_\_

Section \_\_\_\_\_



## Notes:

1. In the multiple choice questions, each question may have more than one correct answer; circle all correct answers.
2. For multiple choice questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees.
4. Draw pictures for each problem to be sure that you understand the problem statement.

1. (5 Pts) \_\_\_\_\_

2. (12 Pts) \_\_\_\_\_

3. (28 Pts) \_\_\_\_\_

4. (20 Pts) \_\_\_\_\_

5. (25 Pts) \_\_\_\_\_

6. (10 Pts) \_\_\_\_\_

7. (Ex Cred) \_\_\_\_\_

**Total (100 Pts)** \_\_\_\_\_

## Some Comments and Helpful Info:

In this course, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \quad \hat{a}_y = \hat{y} \quad \hat{a}_z = \hat{z} \quad \hat{a}_r = \hat{r} \quad \hat{a}_\phi = \hat{\phi} \quad \hat{a}_\theta = \hat{\theta}$$

Also, sometimes  $R$  is used for spherical radius instead of  $r$ , so  $R$  is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.

Quiz 2  
19 March 2007

**1. The Electric Field (5 pts)**

While studying for this quiz, you decide that you need the expression for the electric field due to a point charge. Since you are working with five classmates, your group comes up with six possible expressions. Which one of the following is correct and why? Use one of Maxwell's equations to show why your choice is correct.

From  $\oint \vec{E} \cdot d\vec{S} = Q_{encl}/\epsilon$  or  $\nabla \cdot \vec{E} = \rho/\epsilon$  one can show that d is correct.

a.  $\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0 r^2}$

b.  $\vec{E}(\vec{r}) = \frac{q}{2\pi\epsilon_0 r}$

c.  $\vec{E}(\vec{r}) = \frac{q}{2\pi r} \hat{r}$

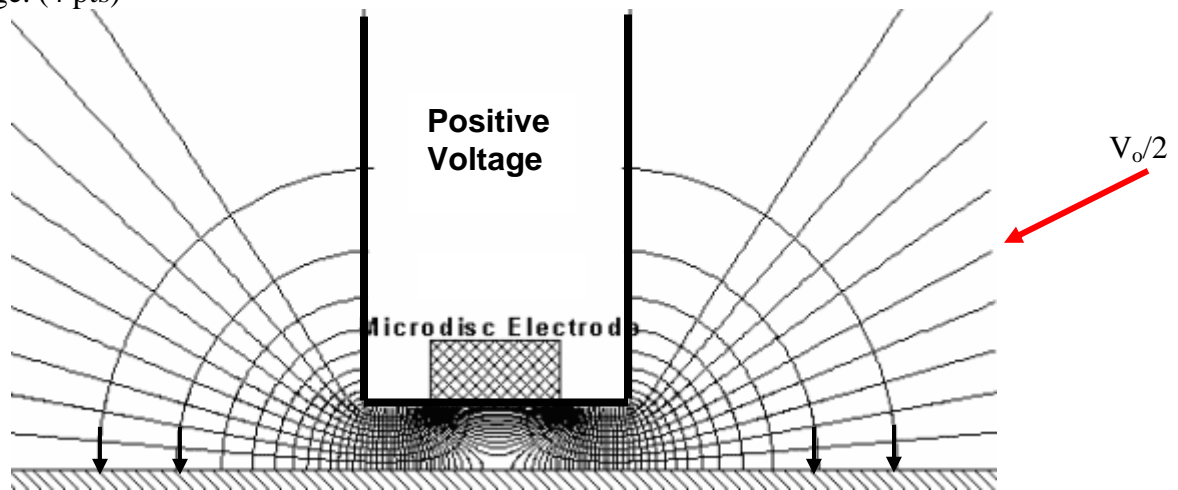
d.  $\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$

e.  $\vec{E}(\vec{r}) = \frac{q}{2\pi\epsilon_0 r^2} \hat{r}$

f.  $\vec{E}(\vec{r}) = \frac{q}{4\pi r^2} \hat{r}$

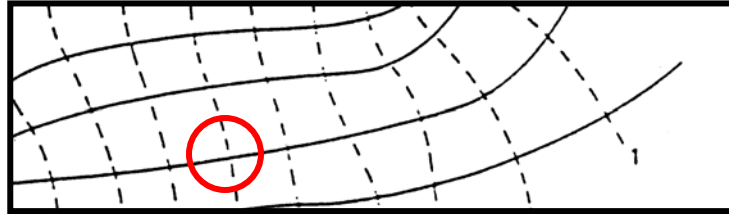
**2. Equipotentials and Electric Field Lines (12 pts)**

Shown below are some drawings of electric field lines and equipotentials. The first drawing shows the E field lines and constant voltage lines around a positive electrode located over a ground plane. The ground plane is at the bottom with the slanted lines. Identify a small number of electric field lines by putting arrows on them showing direction. If the positive electrode voltage is  $V_0$ , label the equipotential at half this voltage. (4 pts)



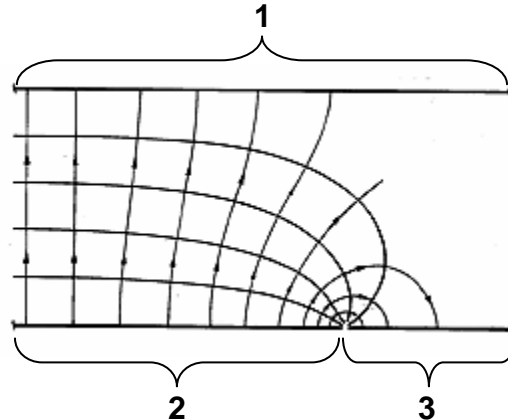
Quiz 2  
19 March 2007

The following figure shows dashed equipotentials and solid field lines. Since they are sketches, they are not perfect. What relationship should exist where the two types of lines cross? Circle at least one point where this is shown correctly. (4 pts)



*The two types of lines should be perpendicular to one another. A good point is shown by the circle.*

The following figure shows the equipotentials and field lines for three electrodes. The top plane is one electrode and the bottom plane is divided into two electrodes. Two of the three electrodes are grounded and one is connected to a positive voltage source. Identify the positive electrode and the two grounded electrodes. (4 pts)

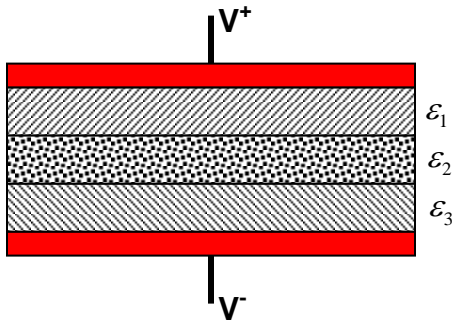


*Electrodes 1 and 3 are grounded and 2 is the positive electrode. The field lines begin on the positive electrode and end on the grounded electrode.*

Quiz 2  
19 March 2007

**3. Numerical Determination of Fields and Capacitance (28 pts)**

On the following pages are found the spreadsheet solutions for a series of parallel plate capacitor configurations. Each of the configurations consists of a pair of conducting plates, one connected to a positive voltage  $V^+$  and the other connected to a negative voltage  $V^-$ . Between the plates are three slabs of insulator of equal thickness, with dielectric constants  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$ , respectively. The width of the plates is  $w$ , the distance between the plates is  $d$  and the plates have unit depth ( $depth = 1$ ).



The cases shown on the following pages include:

- a.  $\epsilon_1 = \epsilon_2 < \epsilon_3$
- b.  $\epsilon_1 = \epsilon_3 < \epsilon_2$
- c.  $\epsilon_2 = \epsilon_3 < \epsilon_1$
- d.  $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_0$
- e.  $\epsilon_1 = \epsilon_2 = \epsilon_3 > \epsilon_0$
- f.  $\epsilon_1 = \epsilon_2 > \epsilon_3$
- g.  $\epsilon_1 = \epsilon_3 > \epsilon_2$
- h.  $\epsilon_2 = \epsilon_3 > \epsilon_1$

(8 pts) Identify which figures go with each case by labeling them with the appropriate letter. *Each plot is labeled. The easy way to do this is that the slope is small in dielectric.*

(5 pts) For each case, the smaller values of  $\epsilon = \epsilon_0$ . From the over abundance of information in these figures, determine the value of the larger  $\epsilon > \epsilon_0$ . The larger values of  $\epsilon$  are all the same so you are only looking for a single number. *See page 12.*







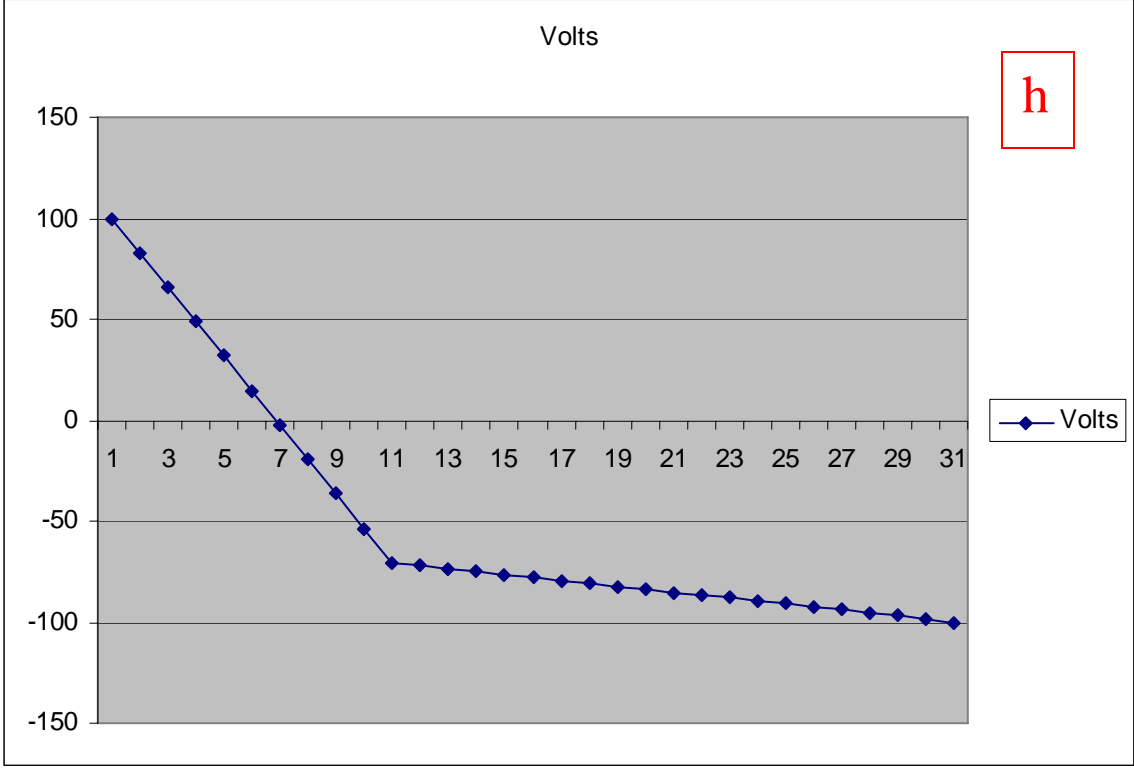






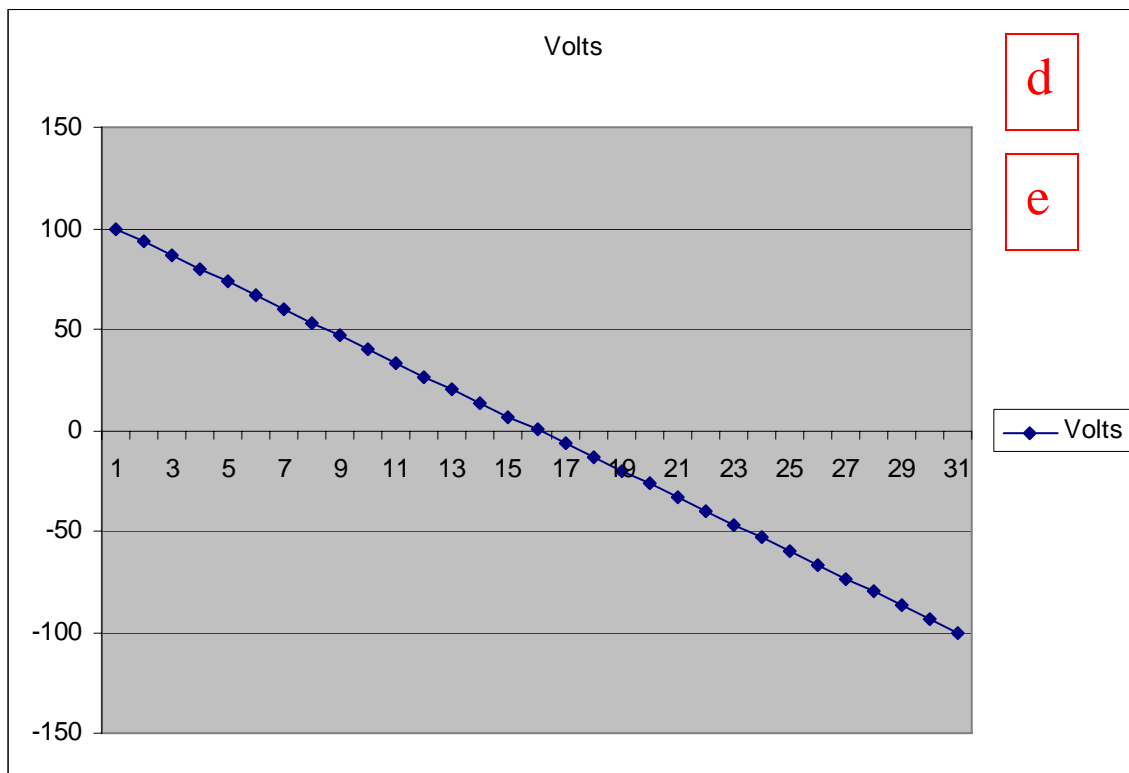
Quiz 2  
19 March 2007

AP10	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE	
1	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2	83	83	83	83	83	83	83	83	83	83	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	82.9	
3	66	66	66	66	65.9	65.9	65.9	65.9	65.9	65.9	65.9	65.9	65.9	65.9	65.9	65.9	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	65.8	
4	48.9	48.9	48.9	48.9	48.9	48.9	48.9	48.9	48.9	48.9	48.8	48.8	48.8	48.8	48.8	48.8	48.8	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	48.7	
5	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.9	31.8	31.8	31.8	31.8	31.8	31.7	31.7	31.7	31.7	31.7	31.7	31.7	31.6	31.6	31.6	31.6	31.6	31.6	31.6	31.6	31.6	31.6	31.6	
6	14.9	14.9	14.9	14.9	14.9	14.9	14.8	14.8	14.8	14.7	14.7	14.7	14.7	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5	
7	-2.1	-2.1	-2.1	-2.1	-2.2	-2.2	-2.2	-2.2	-2.3	-2.3	-2.3	-2.4	-2.4	-2.4	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	-2.6	
8	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-19	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	
9	-36	-36	-36	-36	-36	-36	-36	-36	-36	-36	-36	-36	-36	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	-37	
10	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	
11	-70	-70	-70	-70	-70	-70	-70	-70	-70	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	-71	
12	-71	-71	-71	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	
13	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	-74	
14	-74	-74	-74	-74	-74	-74	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	
15	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-76	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	
16	-77	-77	-77	-77	-77	-77	-77	-77	-77	-77	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	
17	-78	-78	-78	-78	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-79	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	
18	-79	-79	-80	-80	-80	-80	-80	-80	-80	-80	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	-81	
19	-81	-81	-81	-81	-81	-81	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	
20	-81	-81	-82	-82	-83	-83	-83	-83	-83	-83	-83	-83	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	-84	
21	-82	-82	-84	-84	-84	-84	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	-85	
22	-84	-84	-85	-85	-86	-86	-86	-86	-86	-86	-86	-86	-86	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	
23	-87	-87	-87	-87	-87	-87	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	
24	-88	-88	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	-89	
25	-90	-90	-90	-90	-90	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	
26	-92	-92	-92	-92	-92	-92	-92	-92	-92	-92	-92	-92	-92	-92	-92	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	
27	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	-94	
28	-95	-95	-95	-95	-95	-95	-95	-95	-95	-95	-95	-95	-95	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	
29	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	
30	-98	-98	-98	-98	-98	-98	-98	-98	-98	-98	-98	-98	-98	-98	-98	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	
31	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	



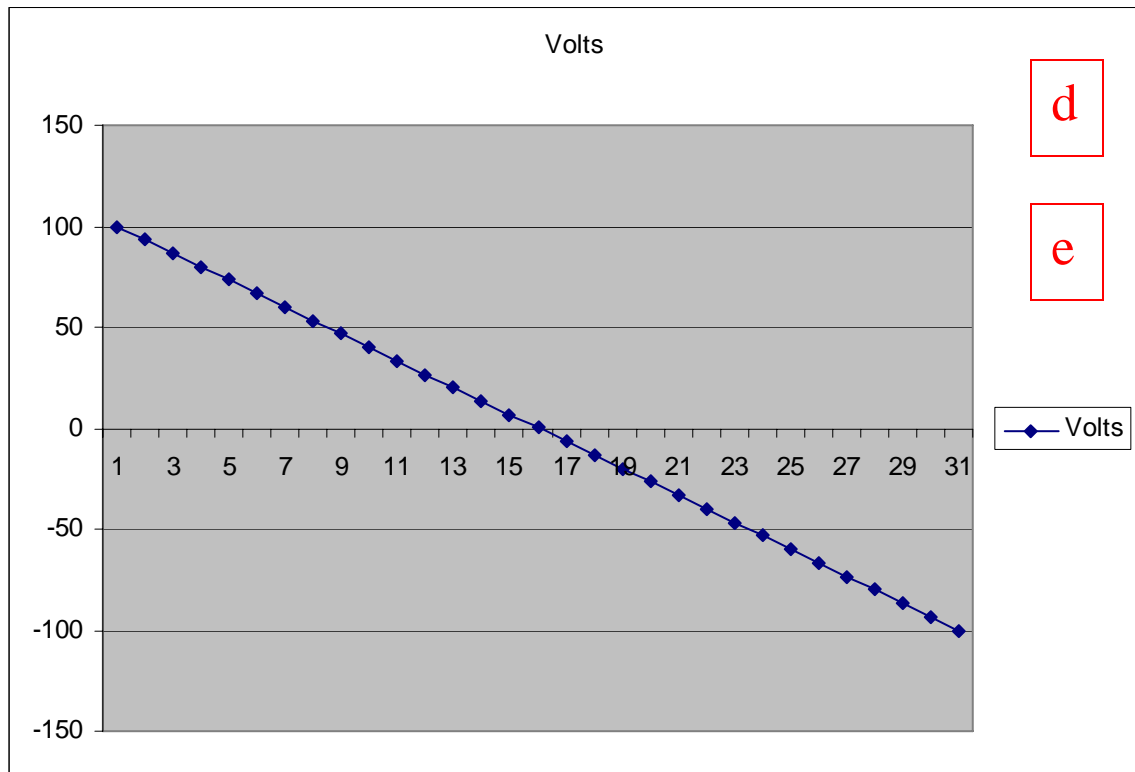
Quiz 2  
19 March 2007

Atto	%																														
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
2	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	
3	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	
4	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	
5	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	
6	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	
7	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	
8	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	
9	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	
10	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	
11	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	
12	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	
13	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	
14	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	
15	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
17	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	
18	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	
19	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	
20	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	
21	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	
22	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	
23	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	
24	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	
25	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	
26	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	
27	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	
28	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	
29	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	
30	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	
31	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	



Quiz 2  
19 March 2007

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	93.3	
86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	86.7	
80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80	
73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	73.3	
66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	66.7	
60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	
53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	53.3	
46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	46.7	
40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	
33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	33.3	
26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	26.7	
20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	
13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	13.3	
6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	6.67	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7	-6.7		
-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13	-13		
-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20	-20		
-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27	-27		
-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33	-33		
-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40	-40		
-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47	-47		
-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53		
-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60	-60		
-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67		
-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73	-73		
-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80		
-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87	-87		
-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93	-93		
-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100	-100		



*The slope change can be used to find the dielectric constant. Because of the moderately bad resolution of the spreadsheet. The slope change is about 190/2 vs 9 so a number between 10 and 13 is reasonable. The actual answer is 12.*

Quiz 2  
19 March 2007

(10 pts) Assuming that the cell size is  $0.01\text{m} = 1\text{cm}$ , determine the capacitance per unit length for cases *c* and *e*. You may either do this from the spreadsheet information or analytically.

Analytically, the *D* vector is the same throughout the region and the *E* vector changes with the dielectric constant. Since there are three regions and the total voltage is twice the voltage on each plate,  $\int \vec{E} \cdot d\vec{l} = E_1 d_1 + E_2 d_2 + E_3 d_3 = 2V$  and since the three regions

are the same size  $E_1 \frac{d}{3} + E_2 \frac{d}{3} + E_3 \frac{d}{3} = 2V$ . Since  $D = \rho_s = \frac{Q}{A} = \epsilon E$ , the three sections act like 3 capacitors in series. It does not matter which capacitor is added first. Thus, we have  $2E_1 \frac{d}{3} + E_3 \frac{d}{3} = 2V$  and since  $D = \epsilon_o E_1 = \epsilon E_3$  we have that

$$2E_1 \frac{d}{3} + \frac{E_1 d}{\epsilon_r 3} = \left( \frac{E_1 d}{3} \left( 2 + \frac{1}{\epsilon_r} \right) \right) = 2V \text{ and } \frac{E_1 2d}{3} = \frac{4V}{\left( 2 + \frac{1}{\epsilon_r} \right)} = \frac{4(100)}{\left( 2 + \frac{1}{12} \right)} = 192.$$

From the plots, we see approximately 192 or so volts, so this comes out pretty close. For the

capacitance,  $E_1 = \frac{6V}{\left( 2 + \frac{1}{\epsilon_r} \right) d} = \frac{\rho_s}{\epsilon_o} = \frac{Q}{\epsilon_o A}$  or  $\frac{3\epsilon_o A}{\left( 2 + \frac{1}{\epsilon_r} \right) d} = \frac{Q}{2V} = C$  since the total voltage

is 2V. Note that when the relative permittivity is 1, the capacitance is that of two parallel plates with air insulation. For the capacitance per unit length for this configuration, the plate area is  $wl$  and the separation is  $w$  so that the capacitance per unit length is

$$C = \frac{3\epsilon_o}{\left( 2 + \frac{1}{12} \right)} = 1.44\epsilon_o. \text{ The capacitance of } e \text{ comes from the simple expression for a}$$

parallel plate capacitor  $C = \frac{\epsilon A}{d} = 12\epsilon_o$ . For the other reasonable values of the permittivity, the *C* values change a little. A range of values is OK.

From the data in the spreadsheet, the average *E* for case *e* is  $6.7/0.01=670$ . The area per unit length is  $30(.01)=.3$ . The charge on the top plate is

$$Q = \rho_s A = \epsilon EA = 12\epsilon_o 670(.3) = 2412$$

so we divide this by 200 to get the capacitance and get  $C = 12\epsilon_o$  as before.

For case *c*, choose the top or the bottom plate. For the top plate, the average *E* is  $0.8/0.01=80$ . The area is the same. The charge on the top plate is

$$Q = \rho_s A = \epsilon EA = 12\epsilon_o 80(.3) = 288$$

so we divide this by 200 to get the capacitance and get  $C = 1.44\epsilon_o$ , so both sets of answers agree.

(5 pts) For which of the other cases will the capacitance be the same as for cases *c* and *e*? *c* is the same as *a* and *b*, nothing is the same as *e*.

Quiz 2  
19 March 2007

*Analytical solution of capacitance for all cases:*

*The integration of the E field gives the total voltage difference*

$$E_1 \frac{d}{3} + E_2 \frac{d}{3} + E_3 \frac{d}{3} = 2V$$

*Can replace the E field using the surface charge density*

$$D = \epsilon E = \rho_s \text{ so that } E = \frac{\rho_s}{\epsilon} \text{ thus}$$

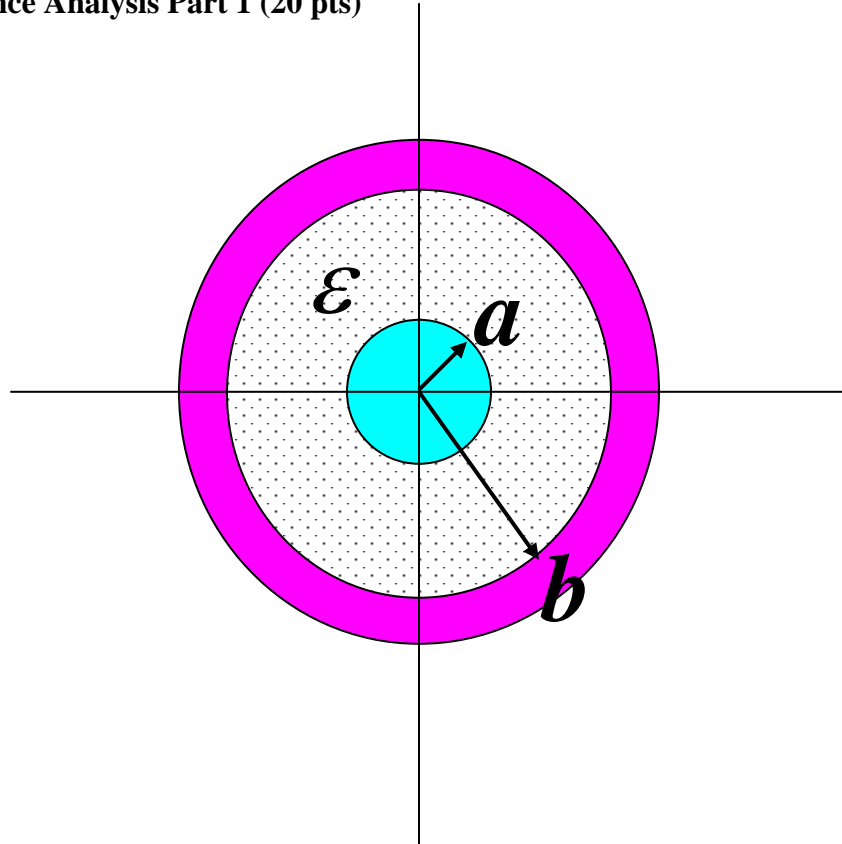
$$2V = \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{1}{\epsilon_3} \right) \frac{\rho_s d}{3} = \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} + \frac{1}{\epsilon_3} \right) \frac{Qd}{3A}$$

*finally*

$$C = \frac{3A}{d} \frac{\epsilon_1 \epsilon_2 \epsilon_3}{\epsilon_1 \epsilon_2 + \epsilon_2 \epsilon_3 + \epsilon_1 \epsilon_3}$$

Quiz 2  
19 March 2007

4. Capacitance Analysis Part 1 (20 pts)



A spherical capacitor consists of two conducting electrodes. The inner solid spherical electrode has a radius  $a$  and a charge  $+Q$  on it. The outer hollow spherical electrode has an inner radius  $b$  and a charge  $-Q$  on it. The region between the two conductors is filled with an insulator with permittivity  $\epsilon$ .

- a. What is the surface charge density on the inner and outer conductors? (4 pts)

$$\rho_{sa} = \frac{Q}{4\pi a^2} \qquad \rho_{sb} = \frac{-Q}{4\pi b^2}$$

- b. Find the electric flux density vector  $\vec{D}(\vec{r})$  for this configuration. (4 pts)

$$\vec{D}(\vec{r}) = \hat{r} \frac{Q}{4\pi r^2} \text{ in the region between the plates. It is zero elsewhere.}$$

Quiz 2  
19 March 2007

- c. Find the corresponding electric field vector  $\vec{E}(\vec{r})$ . (4 pts)

$$\vec{E}(r) = \hat{r} \frac{Q}{4\pi\epsilon r^2}$$

- d. Find the scalar potential function  $V(\vec{r})$  (4 pts)

$$V = -\int_b^r \vec{E} \cdot d\vec{l} = -\int_b^r \hat{r} \frac{Q}{4\pi\epsilon r^2} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon r} - \frac{Q}{4\pi\epsilon b}$$

- e. Using the information you have so far, determine the capacitance of this configuration. You may use either the voltage method or the energy method. Be sure you indicate which method you are using. (4pts)

$$V(a) = \frac{Q}{4\pi\epsilon a} - \frac{Q}{4\pi\epsilon b} \text{ using the voltage method we get } C = 4\pi\epsilon \frac{ab}{b-a}$$

*Using the energy method,*

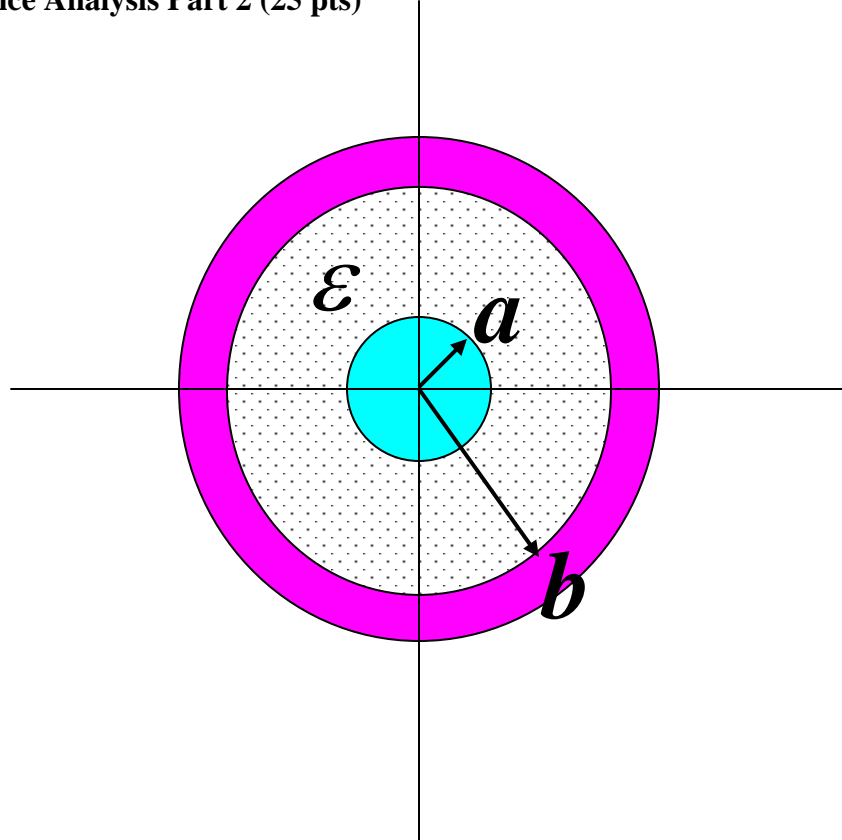
$$W_E = \frac{1}{2} \int \epsilon E^2 dv = \frac{1}{2} 4\pi \int_a^b \frac{Q}{4\pi r^2} \frac{Q}{4\pi\epsilon r^2} r^2 dr = \frac{1}{2} \frac{Q^2}{4\pi} \left( \frac{1}{a} - \frac{1}{b} \right) = \frac{1}{2} \frac{Q^2}{C}$$

*which gives the same result.*



Quiz 2  
19 March 2007

5. Capacitance Analysis Part 2 (25 pts)



For the same spherical capacitor addressed in the previous problem, the insulator has been replaced by a material whose permittivity is a function of

radius  $\epsilon(r) = \epsilon \frac{a^2}{r^2}$  where  $\epsilon$  is the same constant we have used in the previous

problem. Repeat the same 5 steps for the new insulating material. *Hint: Some of the answers will be the same.*

- a. What is the surface charge density on the inner and outer conductors? (4 pts)

*This is the same*  $\rho_{sa} = \frac{Q}{4\pi a^2}$  and  $\rho_{sb} = \frac{-Q}{4\pi b^2}$

- b. Find the electric flux density vector  $\vec{D}(\vec{r})$  for this configuration. (4 pts)

*This is also the same*  $\vec{D}(\vec{r}) = \hat{r} \frac{Q}{4\pi r^2}$  in the region between the plates. It is zero elsewhere.

Quiz 2  
19 March 2007

- c. Find the corresponding electric field vector  $\vec{E}(\vec{r})$ . (4 pts)

*The new dielectric cancels out the spatial variation so that  $\vec{E}(\vec{r}) = \hat{r} \frac{Q}{4\pi\epsilon a^2}$*

- d. Find the scalar potential function  $V(\vec{r})$  (4 pts)

*This integral is quite simple  $V = -\int_b^r \vec{E} \cdot d\vec{l} = -\int_b^r \hat{r} \frac{Q}{4\pi\epsilon a^2} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon a^2} (b - r)$*

- e. Using the information you have so far, determine the capacitance of this configuration. You may use either the voltage method or the energy method. Be sure you indicate which method you are using. (4 pts)

*The voltage method is by far the simplest approach. For this*

*$V = -\int_b^a \vec{E} \cdot d\vec{l} = -\int_b^a \hat{r} \frac{Q}{4\pi\epsilon a^2} \cdot d\vec{l} = \frac{Q}{4\pi\epsilon a^2} (b - a)$  so that the capacitance is*

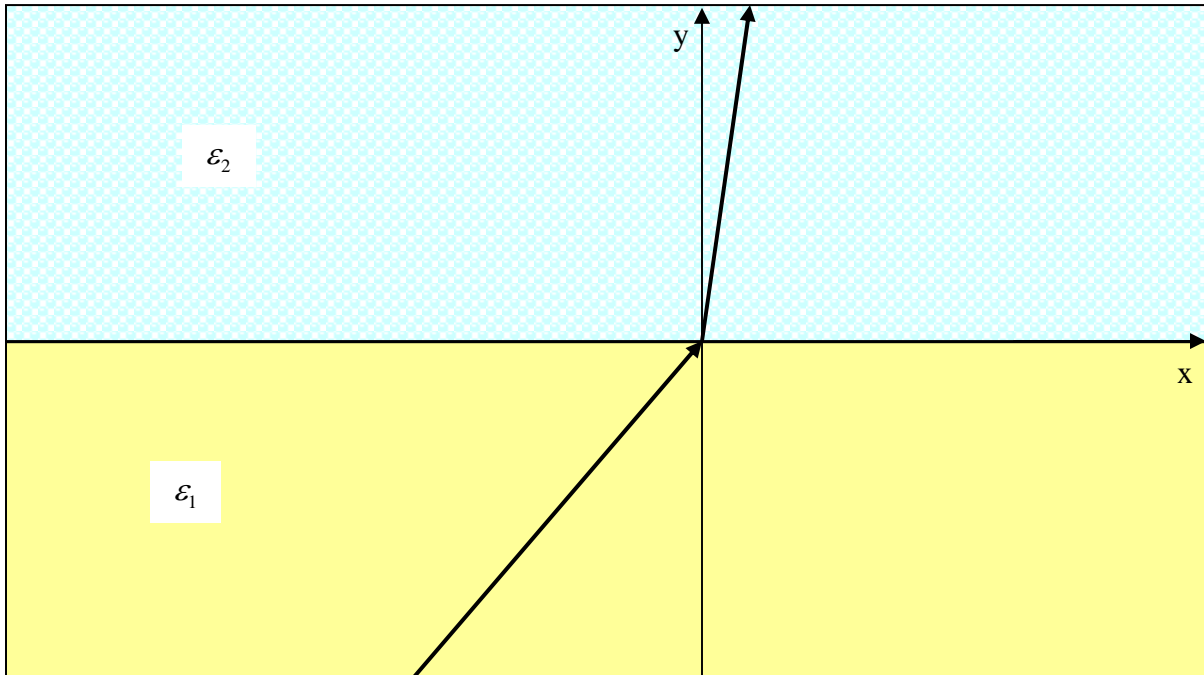
$$C = \frac{4\pi\epsilon a^2}{b - a}$$

- f. The new capacitance should be different than the old capacitance. Explain why one is larger than the other. (5 pts)

*the second capacitance is smaller since  $a^2$  is smaller than  $ab$ . This is because the average permittivity is smaller now since it decays with  $r^2$ .*

Quiz 2  
19 March 2007

**Boundary Conditions (10 pts)**



The electric field in region 1 is  $\vec{E}_1 = E_o(\hat{a}_x + \hat{a}_y)$ . The electric field in region 2 is  $\vec{E}_2 = E_o(\hat{a}_x + 4\hat{a}_y)$ .

a. Assuming that one of these regions is free space, what is the dielectric constant  $\epsilon$  of the other region? (6 Points)

*$\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$  so that the region with the larger value for  $E$  must have the smaller value for the dielectric constant. Thus, region 2 must be air. Since it is 4 times larger, the dielectric constant for the other region must be 4.*

*Note that this also answers the next question.*

b. Identify which region is free space (air), region 1 or region 2. (4 Points)

Quiz 2  
19 March 2007

**6. Extra Credit (10 pts)**

- a. Shown below is a chart of the electrical properties of several materials. Circle all the relative permittivities shown that have reasonable values. Explain your answer. (5 pts)

Material	Plastic	Distilled Water	Wood	Glass
Relative Permittivity	$\epsilon_r = 2.25$	$\epsilon_r = 80$	$\epsilon_r = 4$	$\epsilon_r = 22$

*Glass is typically in the range of 4-7 and 22 is too big. The others are indeed typical.*

- b. Starting from the differential forms of Maxwell's equations, derive Poisson's Equation. (5 pts)

*Starting from Gauss' Law*  $\nabla \cdot \vec{D} = \epsilon \nabla \cdot \vec{E} = \epsilon \nabla \cdot (-\nabla V) = -\epsilon \nabla^2 V = \rho$

*Dividing by the permittivity gives Poisson's Equation*  $\nabla^2 V = -\frac{\rho}{\epsilon}$