VECTOR CALCULUS ELECTROSTATICS CAPACITANCE

Name

	Section
INVERSITY TRAJA	1. (5 Pts)
A GINE E	2. (12 Pts)
Ser Con	3. (28 Pts)
Lots of Physics Cetting here's a Lots of Math Problem solving is what we do.	4. (20 Pts)
	5. (25 Pts)
Notes:	6. (10 Pts)
1. In the multiple choice questions, each question may have more than one correct answer; circle all	7. (Ex Cred)
correct answers. 2. For multiple choice questions, you may add some comments to justify your answer.	Total (100 Pts)

3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees. 4. Draw pictures for each problem to be sure that

you understand the problem statement.

Some Comments and Helpful Info: In this course, we use two types of notation for unit vectors. Keep in mind that

 $\hat{a}_{v} = \hat{y}$ $\hat{a}_{z} = \hat{z}$ $\hat{a}_{r} = \hat{r}$ $\hat{a}_{\phi} = \hat{\phi}$ $\hat{a}_{\theta} = \hat{\theta}$ $\hat{a}_{x} = \hat{x}$

Also, sometimes R is used for spherical radius instead of r, so R is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.

1. The Electric Field (5 pts)

While studying for this quiz, you decide that you need the expression for the electric field due to a point charge. Since you are working with five classmates, your group comes up with six possible expressions. Which one of the following is correct and why? Use one of Maxwell's equations to show why your choice is correct.

a.
$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_o r^2}$$

b. $\vec{E}(\vec{r}) = \frac{q}{2\pi\varepsilon_o r}$
c. $\vec{E}(\vec{r}) = \frac{q}{2\pi r}\hat{r}$
d. $\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_o r^2}\hat{r}$
e. $\vec{E}(\vec{r}) = \frac{q}{2\pi\varepsilon_o r^2}\hat{r}$
f. $\vec{E}(\vec{r}) = \frac{q}{4\pi r^2}\hat{r}$

2. Equipotentials and Electric Field Lines (12 pts)

Shown below are some drawings of electric field lines and equipotentials. The first drawing shows the E field lines and constant voltage lines around a positive electrode located over a ground plane. The ground plane is at the bottom with the slanted lines. Identify a small number of electric field lines by putting arrows on them showing direction. If the positive electrode voltage is V_o , label the equipotential at half this voltage. (4 pts)



The following figure shows dashed equipotentials and solid field lines. Since they are sketches, they are not perfect. What relationship should exist where the two types of lines cross? Circle at least one point where this is shown correctly. (4 pts)



The following figure shows the equipotentials and field lines for three electrodes. The top plane is one electrode and the bottom plane is divided into two electrodes. Two of the three electrodes are grounded and one is connected to a positive voltage source. Identify the positive electrode and the two grounded electrodes. (4 pts)



3. Numerical Determination of Fields and Capacitance (28 pts)

On the following pages are found the spreadsheet solutions for a series of parallel plate capacitor configurations. Each of the configurations consists of a pair of conducting plates, one connected to a positive voltage V^+ and the other connected to a negative voltage V. Between the plates are three slabs of insulator of equal thickness, with dielectric constants ε_1 , ε_2 , and ε_3 , respectively. The width of the plates is w, the distance between the plates is d and the plates have unit depth (*depth* = 1).



The cases shown on the following pages include:

a. $\varepsilon_1 = \varepsilon_2 < \varepsilon_3$ b. $\varepsilon_1 = \varepsilon_3 < \varepsilon_2$ c. $\varepsilon_2 = \varepsilon_3 < \varepsilon_1$ d. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_0$ e. $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 > \varepsilon_o$ f. $\varepsilon_1 = \varepsilon_2 > \varepsilon_3$ g. $\varepsilon_1 = \varepsilon_3 > \varepsilon_2$ h. $\varepsilon_2 = \varepsilon_3 > \varepsilon_1$

(8 pts) Identify which figures go with each case by labeling them with the appropriate letter.

(5 pts) For each case, the smaller values of $\varepsilon = \varepsilon_o$. From the over abundance of information in these figures, determine the value of the larger $\varepsilon > \varepsilon_o$. The larger values of ε are all the same so you are only looking for a single number.

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There are some additional questions after the figures.





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(10 pts) Assuming that the cell size is 0.01m = 1cm, determine the capacitance per unit length for cases *c* and *e*. You may either do this from the spreadsheet information or analytically.

(5 pts) For which of the other cases will the capacitance be the same as for cases cand e?





A spherical capacitor consists of two conducting electrodes. The inner solid spherical electrode has a radius a and a charge +Q on it. The outer hollow spherical electrode has an inner radius b and a charge -Q on it. The region between the two conductors is filled with an insulator with permittivity ε .

a. What is the surface charge density on the inner and outer conductors? (4 pts)

b. Find the electric flux density vector $\vec{D}(\vec{r})$ for this configuration. (4 pts)

c. Find the corresponding electric field vector $\vec{E}(\vec{r})$. (4 pts)

d. Find the scalar potential function $V(\vec{r})$ (4 pts)

e. Using the information you have so far, determine the capacitance of this configuration. You may use either the voltage method or the energy method. Be sure you indicate which method you are using. (4pts)





For the same spherical capacitor addressed in the previous problem, the insulator has been replaced by a material whose permittivity is a function of

radius $\varepsilon(r) = \varepsilon \frac{a^2}{r^2}$ where ε is the same constant we have used in the previous

problem. Repeat the same 5 steps for the new insulating material. *Hint: Some of the answers will be the same*.

a. What is the surface charge density on the inner and outer conductors? (4 pts)

b. Find the electric flux density vector $\vec{D}(\vec{r})$ for this configuration. (4 pts)

c. Find the corresponding electric field vector $\vec{E}(\vec{r})$. (4 pts)

d. Find the scalar potential function $V(\vec{r})$ (4 pts)

e. Using the information you have so far, determine the capacitance of this configuration. You may use either the voltage method or the energy method. Be sure you indicate which method you are using. (4 pts)

f. The new capacitance should be different than the old capacitance. Explain why one is larger than the other. (5 pts)

6. Boundary Conditions (10 pts)



The electric field in region 1 is $\vec{E}_1 = E_o(\hat{a}_x + \hat{a}_y)$. The electric field in region 2 is $\vec{E}_2 = E_o(\hat{a}_x + 4\hat{a}_y)$.

a. Assuming that one of these regions is free space, what is the dielectric constant ε of the other region? (6 Points)

b. Identify which region is free space (air), region 1 or region 2. (4 Points)

7. Extra Credit (10 pts)

a. Shown below is a chart of the electrical properties of several materials. Circle all the relative permittivities shown that have reasonable values. Explain your answer. (5 pts)

Material	Plastic	Distilled Water	Wood	Glass
Relative	$\varepsilon_r = 2.25$	$\mathcal{E}_r = 80$	$\varepsilon_r = 4$	$\varepsilon_r = 22$
Permittivity	-			

b. Starting from the differential forms of Maxwell's equations, derive Poisson's Equation. (5 pts)