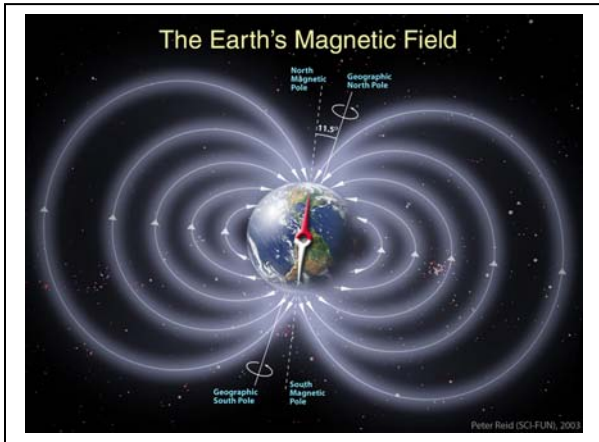


**MAGNETIC FIELDS
CURRENT & RESISTANCE**



Name _____ Solution _____

Section _____ Typos Corrected _____

Multiple Choice

- 1. (8 Pts) _____
- 2. (8 Pts) _____
- 3. (8 Pts) _____
- 4. (8 Pts) _____
- 5. (8 Pts) _____

Regular Questions

- 6. (20 Pts) _____
- 7. (20 Pts) _____
- 8. (Opt 1 – 20 Pts) _____
- 8. (Opt 2 – 20 Pts) _____
- 8. (Opt 3 – 20 Pts) _____

Total (100 Pts) _____

Notes:

- 1. In the multiple choice questions, each question may have more than one correct answer; circle all of them.
- 2. For multiple choice questions, you may add some comments to justify your answer.
- 3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees.

Some Comments and Helpful Info:

In this test, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \quad \hat{a}_y = \hat{y} \quad \hat{a}_z = \hat{z} \quad \hat{a}_r = \hat{r} \quad \hat{a}_\phi = \hat{\phi} \quad \hat{a}_\theta = \hat{\theta}$$

Be sure to show your work for the multiple choice questions.

Draw pictures for each problem to be sure that you understand the problem statement.

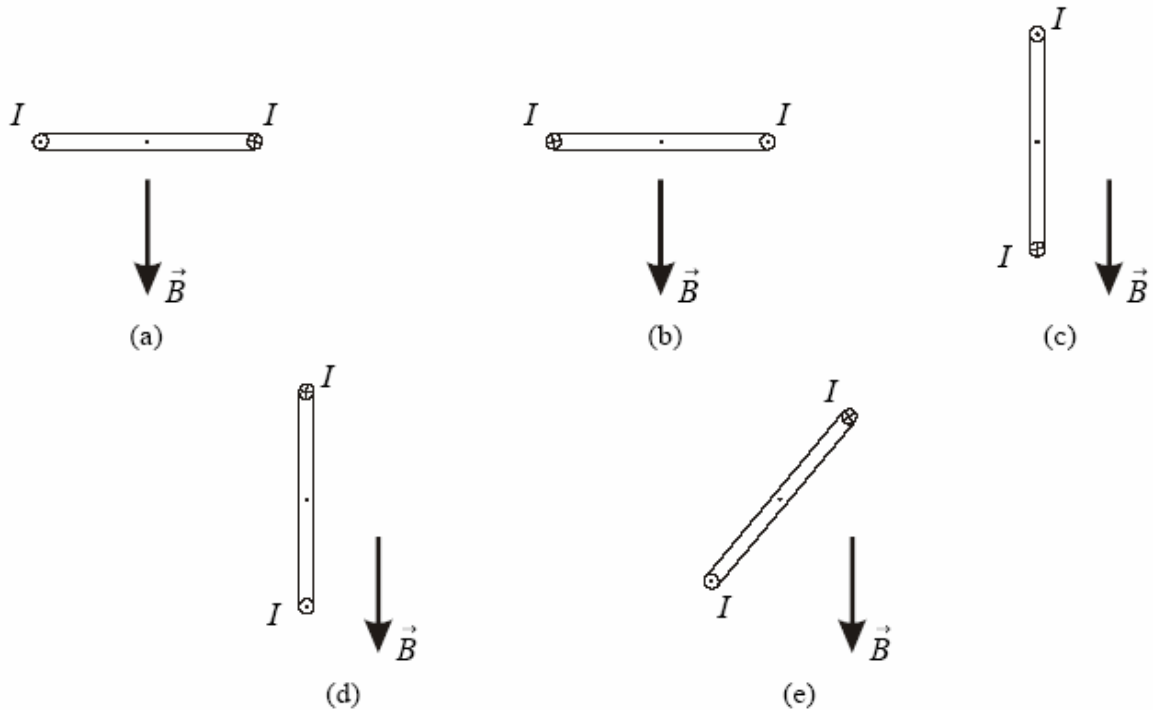
Please note that there are 3 options for the last problem. You should quickly decide which one you want to do and work it through.

MULTIPLE CHOICE QUESTIONS

Except for problem 4, there is only one answer to any of these questions.

1. Force (8 points)

A rectangular loop with a time-invariant current I is placed in a uniform magnetostatic field. The loop can rotate about its axis normal to the page. The magnetic field exerts a torque on the loop for

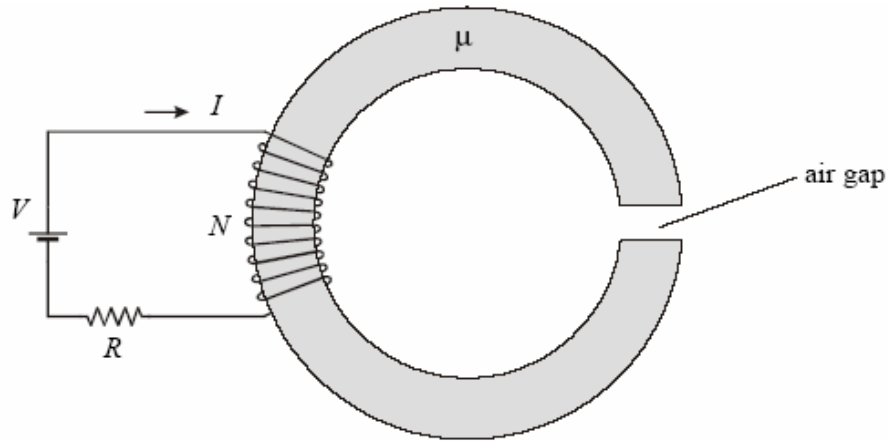


- a. cases (a) and (b) only.
- b. case (c) only.
- c. cases (a), (b) and (c) only.
- d. cases (d) and (e) only.
- e. cases (c), (d) and (e) only.

Quiz 3

2. Magnetic Core (8 Points)

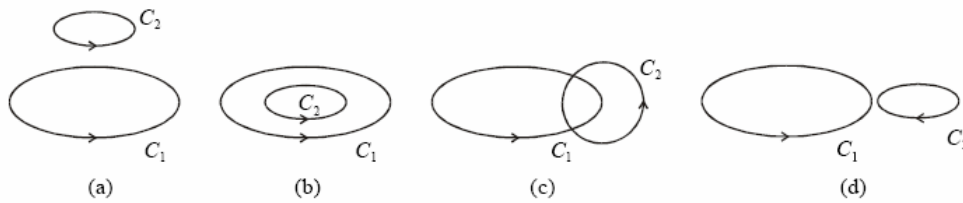
A thin toroidal core, made of a ferromagnetic material of permeability μ , has an air gap, as shown in the figure. There is a time-invariant current through the winding. The magnitude of the magnetic field intensity vector in the ferromagnetic core with respect to the clock-wise reference direction is H_c . The magnitude of the magnetic field intensity vector in the gap H_g with respect to the same reference direction is



- f. $H_g = H_c$
- g. $H_g = 0$
- h. $H_g = \mu_0 H_c$
- i. $H_g = \frac{\mu_0}{\mu} H_c$
- j.** $H_g = \frac{\mu}{\mu_0} H_c$

3. Mutual Inductance (8 Points)

Of the four mutual positions of the two loops shown, the magnitude of the mutual inductance between the loops is largest for the position in



- i.** Figure (a)
- ii. Figure (b)
- iii. Figure (c)
- iv. Figure (d)
- v. Cannot tell

Quiz 3

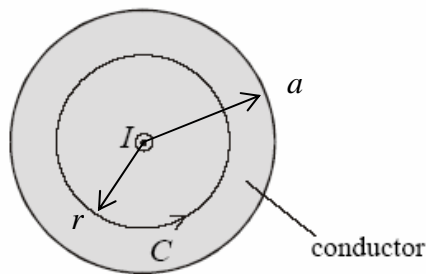
4. Fields and Waves Heroes (8 Points)

Identify which name goes with each equation. Equation (c) does not have a name and one name goes with all of the equations.

- | | |
|--|--------------------|
| a) $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$ | Maxwell <i>All</i> |
| b) $\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \frac{d}{dt} \int \vec{D} \cdot d\vec{S}$ | Faraday <i>(a)</i> |
| c) $\oint \vec{B} \cdot d\vec{S} = 0$ | Gauss <i>(d)</i> |
| d) $\oint \vec{D} \cdot d\vec{S} = \int \rho_v dv$ | Ampere <i>(b)</i> |

5. Ampere's Law (8 points)

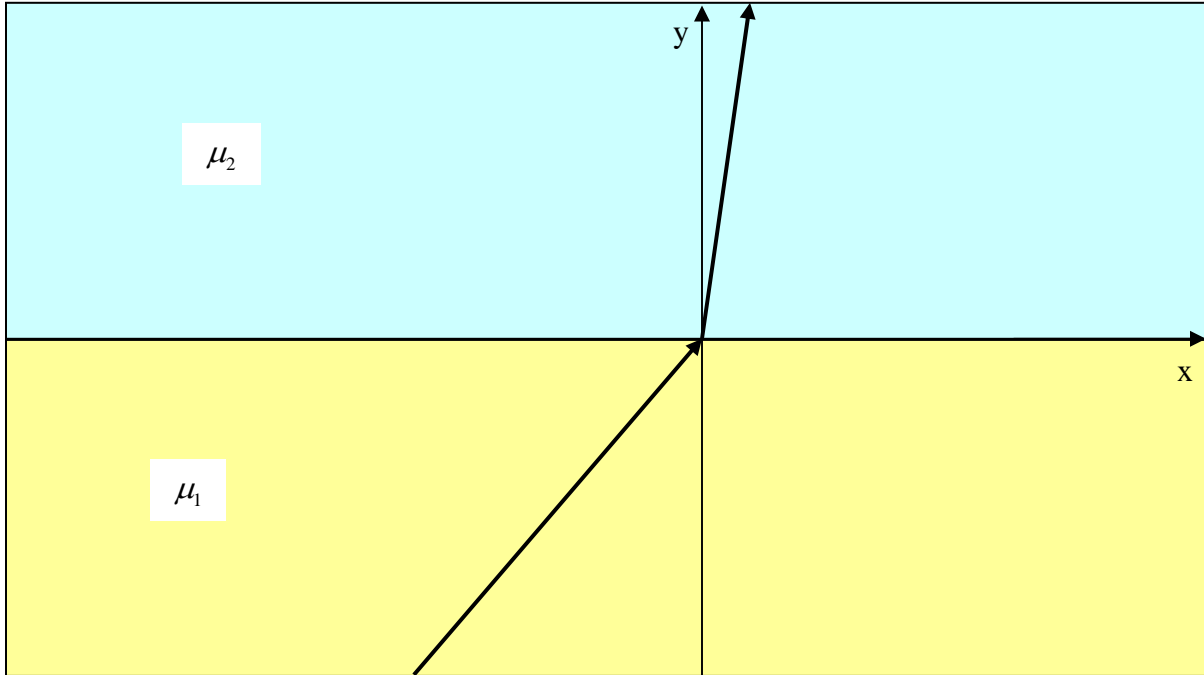
A **cylindrical** conductor of a circular cross section (radius = a) carries a time-invariant current I ($I > 0$) directed **out of the page**. The line integral of the magnetic flux density vector, \vec{B} , along a closed circular contour C positioned inside the conductor (the contour radius r is smaller than the conductor radius a) is



- a) $\mu_o I$
- b) $-\mu_o I$
- c) greater than $\mu_o I$
- d) less than $-\mu_o I$
- e) less than $\mu_o I$ and positive
- f) greater than $-\mu_o I$ and negative
- g) zero

REGULAR QUESTIONS

6. Boundary Conditions (20 points)



The magnetic field in region 1 is $\vec{H}_1 = H_o(\hat{a}_x + \hat{a}_y)$. The magnetic field in region 2 is $\vec{H}_2 = H_o(\hat{a}_x + 1000\hat{a}_y)$. Assuming that one of these regions is free space, what is the permeability μ of the other region? (10)

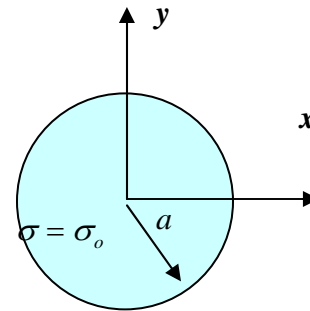
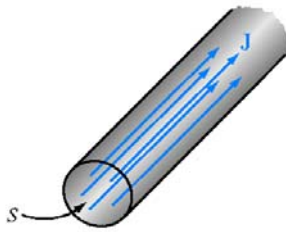
The two boundary conditions are $H_{t1} = H_{x1} = H_o = H_{t2} = H_{x2} = H_o$ and $B_{n1} = \mu_1 H_{y1} = \mu_1 H_o = B_{n2} = \mu_2 H_{y2} = \mu_2 1000 H_o$. Thus, $\mu_1 H_o = \mu_2 1000 H_o$ and $\mu_2 = \mu_o$, $\mu_1 = 1000\mu_o$

Identify which region is free space (air), region 1 or region 2. (10)

Region 2 is air and region 1 is the magnetic material with $\mu_1 = 1000\mu_o$

Quiz 3

7. Ampere's Law (20 points)



A long, straight, solid cylindrical conductor with a radius of a is shown above. The surrounding medium is free space. There is a total current I_o carried by this conductor directed **into the page**.

What is the current density vector? (6)

The current density is $\vec{J} = -\hat{z} \frac{I}{\pi a^2}$. The current is into the page and the z -axis points out of the page. Thus, the current density vector is negative.

What is the magnetic field intensity vector \vec{H} inside the conductor ($r < a$)? (7)

$\oint \vec{H} \cdot d\vec{l} = H_\phi 2\pi r = I_{encl} = -I \frac{\pi r^2}{\pi a^2}$. Solving for \vec{H} , we have $\vec{H} = -\hat{\phi} \frac{Ir}{2\pi a^2}$

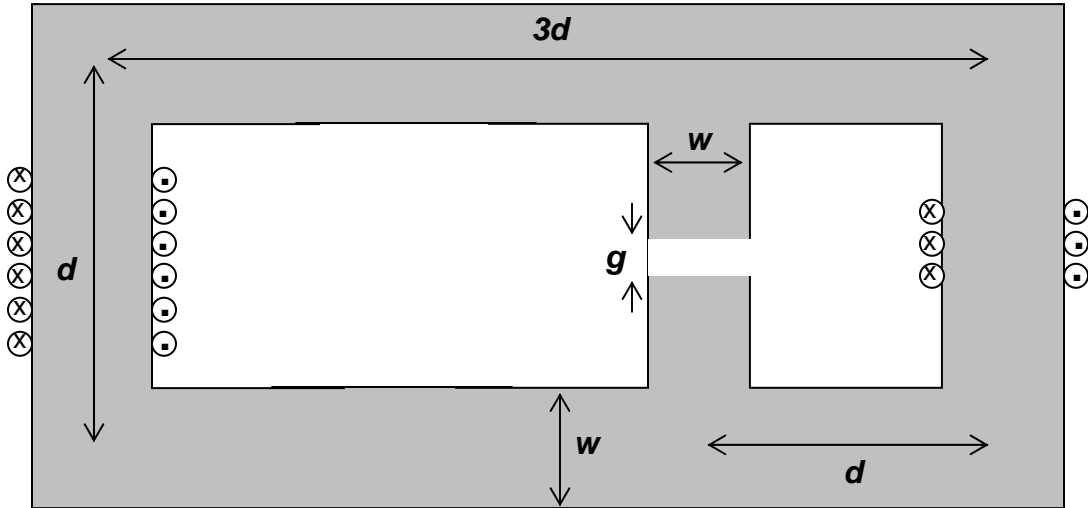
What is the magnetic field intensity vector \vec{H} outside the conductor ($r > a$)? (7)

Outside of the wire, $\oint \vec{H} \cdot d\vec{l} = H_\phi 2\pi r = I_{encl} = -I$. Solving for \vec{H} , we have $\vec{H} = -\hat{\phi} \frac{I}{2\pi r}$

Quiz 3

8. (Option 1) Magnetic Circuits (20 points)

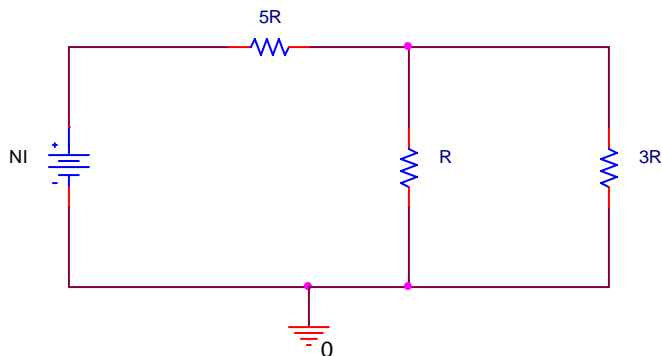
Two windings are wrapped around the magnetic core shown below. The coil at the left carries a current I_1 and the coil at the right carries a current I_2 .



The permeability of the core is μ which is very much larger than μ_o . There are N_1 turns around the left leg and N_2 turns around the right leg. The width and depth of all legs are exactly the same and equal to w . The height of the core is d and the total width of the core is $3d$ as shown. For the first few questions, assume that the gap g does not exist. It will be added in at the end. Also, assume that the current in the right hand coil (I_2) is zero.

a. Find the reluctance of the core seen by the coil at the left. (3)

The circuit diagram for the reluctance looks like



where $R = \frac{d}{\mu w^2}$ since each of the legs of length d have the same reluctance. Thus, the

$$\text{total reluctance seen by the source is } R_{total} = 5R + R \parallel 3R = 5R + \frac{3R^2}{4R} = 5.75R$$

Quiz 3

b. Find the total flux linked by all the turns of this coil. (3)

$$\Lambda = N_1 \psi_1 = \frac{N_1 N_1 I}{R_{total}} = \frac{N_1^2 I \mu w^2}{5.75d}$$

c. Find the inductance of this coil. (3)

$$L = \frac{\Lambda}{I} = \frac{N_1 N_1 I}{I R_{total}} = \frac{N_1^2 \mu w^2}{5.75d}$$

d. Find the flux produced by this coil that links all the turns of the second coil. (3)

The flux of part b divides by the usual current divider relationship. The total flux linking

$$\text{all of the turns of the second coil is given by } N_2 \psi_1 \frac{R}{4R} = \frac{N_2 N_1 I}{4R_{total}} = \frac{N_2 N_1 I \mu w^2}{4(5.75)d}$$

e. Find the mutual inductance between the two coils. (3)

$$\text{The mutual inductance is } M = \frac{N_2 \psi_1}{I} \frac{R}{4R} = \frac{N_2 N_1}{4R_{total}} = \frac{N_2 N_1 \mu w^2}{4(5.75)d}$$

f. Now add in the gap and repeat question a above. Make any reasonable approximations. (3)

Since the gap reluctance will be much larger than R, it will appear as an open circuit in parallel with 3R. Thus the total reluctance seen by the source will be 8R.

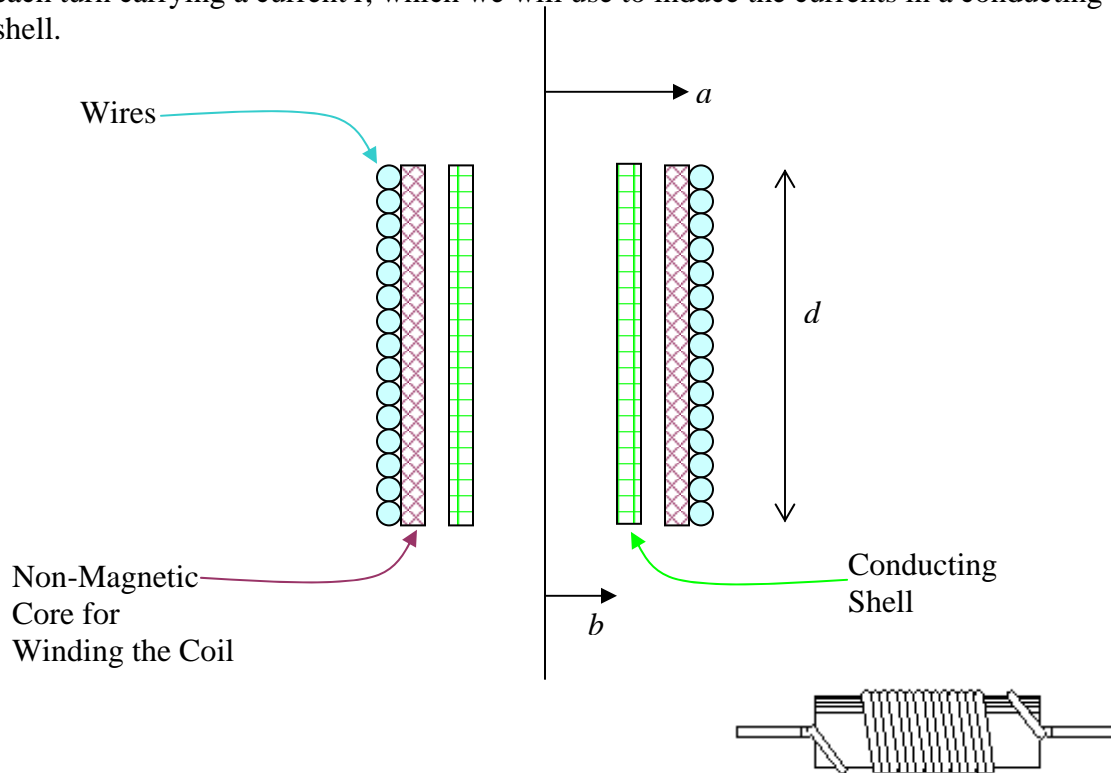
g. Will the self inductance of the left coil increase, decrease or stay the same when the gap is added? Be sure to justify your answer. (2)

Since the reluctance will be larger ($8 > 5.75$), the flux will be smaller for the same current and, thus, the inductance will be smaller.

Quiz 3

8. (Option 2) Faraday's Law (20 Points)

In this problem we will address eddy current heating, which is used extensively in manufacturing to deliver heat to a conducting material. It is also the reason why the can in the can crusher and the coin in the coin flipper are heated by their interaction with the crushing or launching coil. Basically, currents induced in conductors heat the conductors because of their finite resistance. First determine the inductance of an N -turn solenoid, each turn carrying a current I , which we will use to induce the currents in a conducting shell.



- a. Given that the solenoid has N turns and is wound to completely cover a nonmagnetic core (like a toilet paper tube) with one thin layer of wire, find the magnetic field \vec{B} . Assume that you can neglect fringing and that the radius of the solenoid is a and its length is d . Recall that this type of solenoid is called an air core solenoid. (4)

The magnetic field for the solenoid is solved for in many places. It is given by

$$\vec{B} = \hat{z} \frac{\mu_0 N I}{d}$$

Quiz 3

b. Determine the inductance of this solenoid using either the flux method or energy method. Please indicate which method you are using. (4)

$$\text{From the flux method, the inductance is } L = \frac{\Lambda}{I} = \frac{N}{I} \int \vec{B} \cdot d\vec{S} = \frac{\mu_0 N^2 \pi a^2}{d}$$

$$\text{From the energy method, } L = \frac{\int \vec{B} \cdot \vec{H} dv}{I^2} = \frac{\mu_0 \left(\frac{NI}{d}\right)^2 \pi a^2 d}{I^2} = \frac{\mu_0 N^2 \pi a^2}{d}$$

c. A conducting shell is placed inside the solenoid, as shown. This shell has a radius of b , a length d and a thickness Δ . For current flowing around the circumference of the cylinder (in the ϕ -direction) what is the resistance of the shell? Assume that it has a conductivity σ . (4)

$$R = \frac{2\pi r}{\sigma d \Delta}$$

d. If the current driving the solenoid varies sinusoidally, determine the total current induced in the shell. Assume $I(t) = I_o \cos \omega t$. *Hint: determine the induced emf first.* (4)
The induced emf is given by the time derivative of the flux linked. Note that the number of turns in the secondary, in this case, is only 1 since the conducting shell has only one turn.

$$\Lambda = \int \vec{B} \cdot d\vec{S} = \frac{\mu_0 N \pi b^2}{d} I_o \cos \omega t \quad \text{so that}$$

$$\text{emf} = -\frac{d\Lambda}{dt} = -\frac{d}{dt} \frac{\mu_0 N \pi b^2}{d} I_o \cos \omega t = \omega \frac{\mu_0 N \pi b^2}{d} I_o \sin \omega t$$

$$I_{\text{induced}} = \frac{\text{emf}}{R} = \frac{\sigma d \Delta}{2\pi b} \frac{\omega \mu_0 N \pi b^2}{d} I_o \sin \omega t$$

e. Determine the power delivered to the conducting shell. (4)

This can be answered either with power or average power. Either are OK.

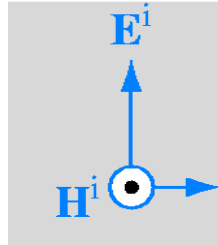
$$P = VI = \left(\omega \frac{\mu_0 N \pi b^2}{d} I_o \sin \omega t \right)^2 \left(\frac{\sigma d \Delta}{2\pi b} \right)$$

For average power, leave out the sin term and divide by 2.

Quiz 3

8. (Option 3) Uniform Plane Waves in Lossless and Lossy Materials (20 Points)

A uniform plane wave is propagating in a material with properties similar to those of distilled water. That is, $\epsilon = 81\epsilon_0$. The frequency of the wave is 100MHz. The average power density of the wave is 10 Watts per square meter.



a. Determine the angular frequency ω , the propagation constant β , the wavelength λ , and the intrinsic impedance η for this wave. (4)

$$\omega = 2\pi f = 2\pi 10^8 \quad \beta = \omega\sqrt{\mu_0\epsilon} = 9\frac{\omega}{c} \quad \lambda = \frac{2\pi}{\beta} \quad \eta = \sqrt{\frac{\mu_0}{\epsilon}} = \frac{120\pi}{9}$$

b. Write both the electric and magnetic fields in phasor notation. (4)

$$\frac{1}{2} \frac{E_m^2}{\eta} = 10 \quad E_m = \sqrt{20\eta} \quad E = E_m e^{-j\beta z} \quad H = \frac{E_m}{\eta} e^{-j\beta z}$$

c. Now some pollutants are added to the water which do not change the real part of ϵ but do result in a small imaginary component so that $\epsilon_c = \epsilon' - j\epsilon''$. In particular, $\epsilon'' = 0.02\epsilon'$. Find the decay constant α and the complex intrinsic impedance η_c . (4)

$$\gamma = j\omega\sqrt{\mu_0\epsilon_c} = j\omega\sqrt{\mu_0\epsilon'}\sqrt{1 - j\frac{\epsilon''}{\epsilon'}} = j\omega\sqrt{\mu_0\epsilon'}\left(1 - j\frac{\epsilon''}{2\epsilon'}\right)$$

$$\beta = j\omega\sqrt{\mu_0\epsilon'}$$

$$\alpha = j\omega\sqrt{\mu_0\epsilon'}\left(-j\frac{\epsilon''}{2\epsilon'}\right) = \frac{\omega\sqrt{\mu_0\epsilon'}\epsilon''}{2\epsilon'}$$

$$\eta_c = \sqrt{\frac{\mu_0}{\epsilon_c}} = \sqrt{\frac{\mu_0}{\epsilon'}} \frac{1}{\sqrt{1 - j\frac{\epsilon''}{\epsilon'}}} = \sqrt{\frac{\mu_0}{\epsilon'}} \frac{1}{1 - j\frac{\epsilon''}{2\epsilon'}} = \sqrt{\frac{\mu_0}{\epsilon'}} \left(1 + j\frac{\epsilon''}{2\epsilon'}\right)$$

Quiz 3

d. Write the phasor form of the electric and magnetic fields. (4)

$$E = E_m e^{-\alpha z} e^{-j\beta z} \qquad H = \frac{E_m}{\eta} e^{-\alpha z} e^{-j\beta z}$$

e. Determine the average power delivered to a cubic meter of this material. (4)

This is the same calculation as on the HW problem.

$$S_{ave} = \frac{1}{2} \frac{E_m^2}{\eta_c^*} e^{-2\alpha z} \quad \text{We can approximate the complex intrinsic impedance with the}$$

lossless value, since the loss is not large. $\eta_c \approx \sqrt{\frac{\mu_o}{\epsilon'}}$. The power delivered per cubic

meter is then: $(1 - e^{-2\alpha z})10\text{Watts} = (1 - e^{-2\alpha})10\text{Watts}$ since the depth is equal to 1.

$$\alpha = \frac{\omega \sqrt{\mu_o \epsilon' \epsilon''}}{2\epsilon'} = \frac{2\pi 10^8 (9)}{(3 \times 10^8)} 0.02 = \frac{2\pi(9)}{2(3)} 0.02 = 0.06\pi$$

$$(1 - e^{-0.12\pi})10\text{Watts} = 3.1\text{Watts}$$