Quiz 3

MAGNETIC FIELDS &
UNIFORM PLANE WAVES

Name ________________________

Section _______________________

Multiple Choice

1. (8 Pts) ____________

2. (8 Pts) ____________

3. (8 Pts) ____________

4. (8 Pts) ____________

5. (8 Pts) ____________

Regular Questions

6. (20 Pts) ____________

7. (20 Pts) ____________

8. (20 Pts) ____________

Total (100 Pts) ____________

Notes:
1. In the multiple choice questions, each question may have more than one correct answer; circle all of them.
2. For multiple choice questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees.

Some Comments and Helpful Info:
In this test, we use two types of notation for unit vectors. Keep in mind that

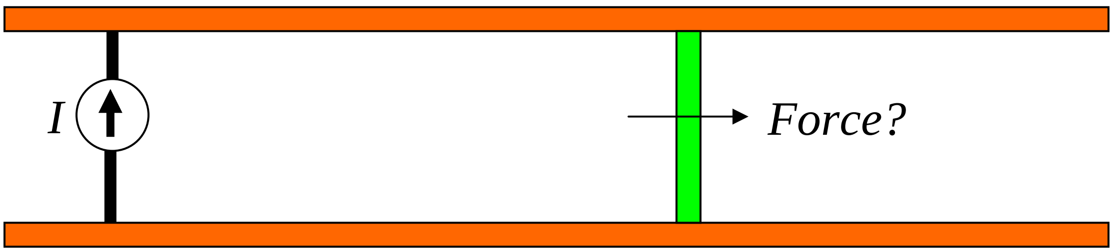
$\hat{a}_x = \hat{x} \qquad \hat{a}_y = \hat{y} \qquad \hat{a}_z = \hat{z} \qquad \hat{a}_r = \hat{r} \qquad \hat{a}_\phi = \hat{\phi} \qquad \hat{a}_\theta = \hat{\theta}$

Be sure to show your work for the multiple choice questions.
Draw pictures for each problem to be sure that you understand the problem statement.

MULTIPLE CHOICE QUESTIONS

Except for problem 4, there is only one answer to any of these questions.

1. Force (8 points)
A rectangular current loop is formed using two parallel conducting rails, a current source with a time-invariant current *I*, and a sliding contact between the rails. The force on the sliding contact will be directed to the right for which of the following cases?



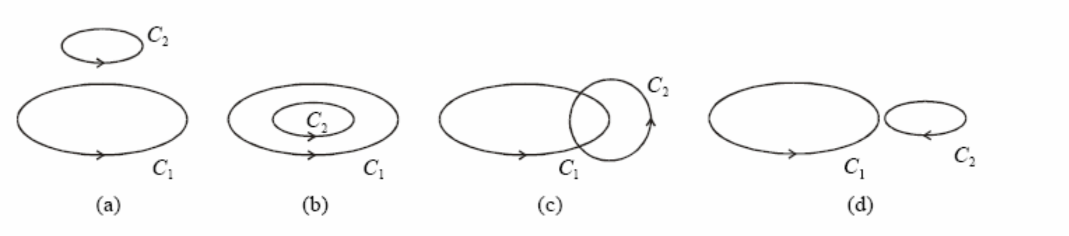
a. when the current is as shown but not when the current is in the opposite (downward) direction. *4 points for this answer*
b. not when the current is as shown but when the current is in the opposite (downward) direction. *4 points for this answer*
c. when the current is as shown and when the current is in the opposite (downward) direction.. *8 points for this answer*
d. never, there is no force on the sliding contact.

2. Shielding (8 points)
In order to prevent the electric and magnetic fields from entering or leaving a room, the walls of the room are shielded with 1-mm thick aluminum foil. The best protection is achieved at

a. 1 Hz
b. 1 kHz
c. 1 MHz
d. 1 GHz
e. No difference

Shielding works better at higher frequencies.

3. Mutual Inductance (8 Points)
Of the four mutual positions of the two loops shown, the magnitude of the mutual inductance between the loops is largest for the position in



i. Figure (a)
ii. Figure (c)
iii. Figure (d)
iv. Figure (b) *Coupling works better when the coils are as similar as possible. Also, the area is maximized where the field is largest.*
v. Cannot tell

4. Fields and Waves Heroes (8 Points)
Identify which name goes with each equation. Equation (c) does not have a name and one name goes with all of the equations.

a) $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{S}$ Maxwell

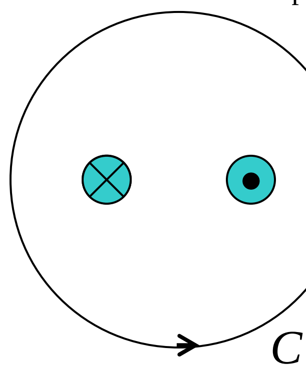
b) $\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \frac{d}{dt}\int \vec{D} \cdot d\vec{S}$ Faraday

c) $\oint \vec{B} \cdot d\vec{S} = 0$ Gauss

d) $\oint \vec{D} \cdot d\vec{S} = \int \rho_v dv$ Ampere

5. Ampere's Law (8 points)

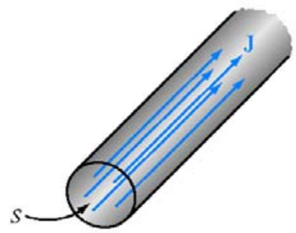
Two **cylindrical** conductors of a circular cross section (radius = a) carry a time-invariant current I ($I > 0$) directed **into the page** at the left **out of the page** at the right. The line integral of the magnetic flux density vector, $\vec{B}$, along a closed circular contour C positioned as shown is

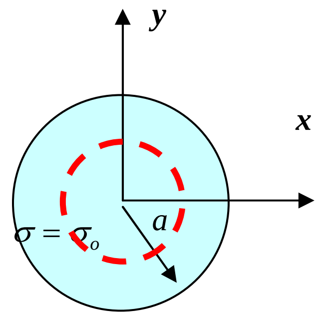


a) $\mu_o I$
b) $-\mu_o I$
c) greater than $\mu_o I$
d) less than $-\mu_o I$
e) less than $\mu_o I$ and positive
f) greater than $-\mu_o I$ and negative

zero *the contour surrounds both conductors so that the total current enclosed is zero. This uses Ampere's Law, which is equation b in the previous problem. Because the current is time-invariant, we can drop the last term in the equation.*

REGULAR QUESTIONS
6. Ampere's Law (20 points)





A long, straight, solid cylindrical conductor with a radius of a is shown above. The surrounding medium is free space. There is a total current I_o carried by this conductor directed **into the page**.

a. What is the current density vector? (5)

The current flows into the page, so it flows in the negative z direction. The area of the conductor is πa^2. Thus, the current density is given by $\vec{J} = -\hat{z}\dfrac{I_o}{\pi a^2}$

b. What is the magnetic field intensity vector $\vec{H}$ inside the conductor ($r<a$)? (5)

Inside the conductor, at some radius r, the enclosed current will be given by the fraction of the conductor area within that radius. Using the red dashed circular contour shown above, Ampere's law looks like $\oint \vec{H}\cdot d\vec{l} = H_\phi 2\pi r = I_{enclosed} = -I_o \dfrac{r^2}{a^2}$

The magnetic field intensity $\vec{H}$ *is then given by* $\vec{H} = -\hat{\phi}\dfrac{I_o r}{2\pi a^2}$

c. How much energy is stored per unit length in the magnetic field of the region inside the conductor ($r<a$)? (5)

The stored energy in a region is given by

$$W_m = \frac{1}{2}\int \vec{B}\cdot\vec{H}dv = \frac{1}{2}\int_0^1 dz\int_0^{2\pi} d\phi\int_0^a \left(\frac{\mu_o I_o r}{2\pi a^2}\right)\left(\frac{I_o r}{2\pi a^2}\right) = \frac{\mu_o I_o^{\,2}}{4\pi a^4}\int_0^a r^3 dr = \frac{\mu_o I_o^{\,2}}{4\pi a^4}\frac{a^4}{4} = \frac{I_o^{\,2}}{2}\frac{\mu_o}{8\pi}$$

note that there is a missing rdr in the above expression. It should be located before the third equal sign.

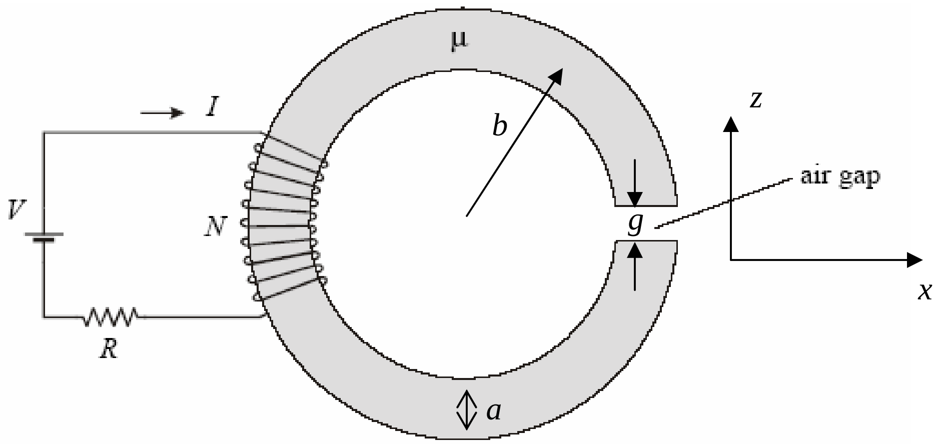
d. What is the internal inductance of the conductor? That is, what is the inductance associated with the region inside the conductor ($r<a$)? (5)

Since $W_m = \dfrac{1}{2}LI^2$ *we have that* $L = \dfrac{\mu_o}{8\pi}$

7. **Magnetic Circuit and Faraday's Law (20 Points)**

A thin toroidal core, made of a ferromagnetic material of permeability μ, has an air gap, as shown in the figure. There is a time-invariant current through the winding. The current-carrying wires are perfect conductors. The depth of the core is w.

a. Which of the following is correct for the magnitude of the magnetic field H_g in the gap? (6)



- o $H_g = H_c$
- o $H_g = 0$
- o $H_g = \mu_o H_c$
- o $H_g = \dfrac{\mu_o}{\mu} H_c$
- o $H_g = \dfrac{\mu}{\mu_o} H_c$ *since* $B_g = \mu_o H_g = B_c = \mu H_c$

b. Now determine the magnetic field in the gap in terms of V, R, a, b, g, w and μ. Any reasonable approximation will be accepted. (8)

This can be done either by the magnetic circuit method or directly from Ampere's Law. Since most will probably use the magnetic circuit method, we will follow that approach.

First evaluate the reluctance for the core and gap. These are $R_{gap} = \dfrac{g}{\mu_o wa}$ *and*

$R_{core} = \dfrac{2\pi b - g}{\mu wa} \approx \dfrac{2\pi b}{\mu wa}$. *The MMF for this circuit is NI. The flux is the given by*

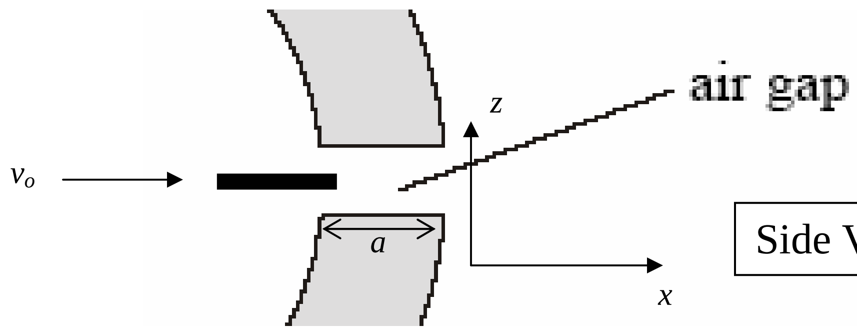
$\psi_m = \dfrac{NI}{\dfrac{2\pi b - g}{\mu wa} + \dfrac{g}{\mu_o wa}}$. *The flux is directed clockwise because of the way the current carrying wires are wrapped around the core. The magnetic field is then given by*

$$B = \frac{\psi_m}{wa} = \frac{NI}{\left(\dfrac{2\pi b - g}{\mu wa} + \dfrac{g}{\mu_o wa}\right) wa} = \frac{NI}{\left(\dfrac{2\pi b - g}{\mu} + \dfrac{g}{\mu_o}\right)} \approx \frac{\mu_o NI}{g}$$

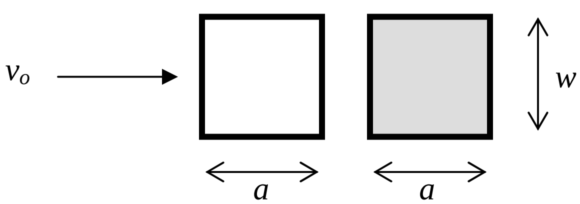
Finally, to find I, we note

that $I = \frac{V}{R}$ *and* $B \approx \frac{\mu_o NV}{gR}$. *Either form of the solution is fine. The magnetic flux density is directed downward through the gap or in the negative z direction.* $\vec{B} \approx \frac{\mu_o NV}{gR}\hat{z}$

c. Now assume that a one turn square conducting loop passes through the gap as shown at a velocity v_o. Determine the voltage induced around the loop as a function of time. Note that the area of the loop is exactly equal to the area of the gap (a times w). (6)



Side View

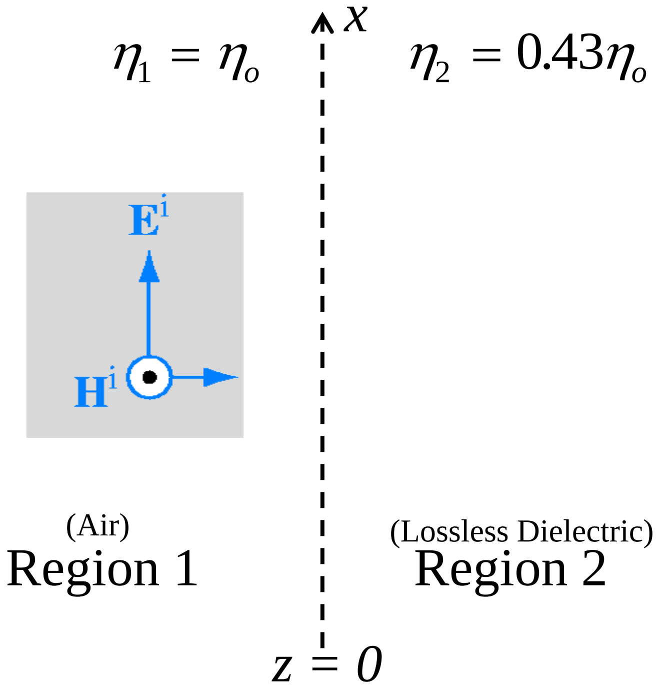


Top View

The time when the coil reaches the magnetic field is not specified so the exact timing is not required for the answer. For simplicity, we will assume that the induced voltage starts at t = 0. For our answer, we will assume no fringing. Thus, the flux that passes through the coil will rise linearly from zero to the full flux and then immediately fall linearly back to zero. The rise time and the fall time are given by the core width divided by the velocity or $\tau = \frac{a}{v_o}$. *Thus the flux linked is given by* $\psi_m(t) = Bwa\left(\frac{t}{\tau}\right)$ *for* $0 \le t \le \tau$ *and* $\psi_m(t) = Bwa\left(1 - \frac{t}{\tau}\right)$ *for* $\tau \le t \le 2\tau$. *The induced voltage is given by the negative time derivative (the sign is not important here because we have not precisely specified the connections to the coil) of the flux.* $V = -\frac{d}{dt}\psi_m(t) = -Bwa\left(\frac{1}{\tau}\right)$ *for* $0 \le t \le \tau$ *and* $V = -\frac{d}{dt}\psi_m(t) = Bwa\left(\frac{1}{\tau}\right)$ *for* $\tau \le t \le 2\tau$.

8. Uniform Plane Waves in Lossless and Materials (20 Points)

A uniform plane wave is propagating in air and incident normally on a lossless dielectric medium. The frequency of the wave is 300MHz. The average power density of the wave is 53 Watts per square meter. You might find it helpful to draw the vector diagram for each wave to be sure you have the directions correct. You do not have to simplify any expression (you can leave it in terms of parameters), except for part f, which requires a number.



a. Determine the angular frequency ω, the propagation constant β_o and the wavelength λ_o for this wave. (3)

$$\omega = 2\pi f = \pi\left(6x10^8\right) \qquad \lambda_o = \frac{c}{f} = 1 \qquad \beta_o = \frac{2\pi}{\lambda_o} = 2\pi$$

b. Determine the magnitude of the electric field E_o and both the electric and magnetic fields in phasor vector notation. (4)

$$E_o^{\ 2} = 2\eta_o(53) \quad \Rightarrow \quad E_o = \sqrt{2\eta_o(53)} = 200$$

$$\vec{E} = \hat{x}E_o e^{-j\beta z} \qquad \vec{H} = \hat{y}\frac{E_o}{\eta_o}e^{-j\beta z}$$

c. Find the reflection coefficient Γ for the electric field. (3)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0.43 - 1}{0.43 + 1} = -0.4$$

d. Write the phasor vector form of the reflected and transmitted electric fields. (4)

$$\vec{E}_r = \hat{x}\Gamma E_o e^{+j\beta_o z} \qquad \vec{E}_t = \hat{x}\left(1+\Gamma\right)E_o e^{-j\beta z}$$

where $\beta = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.43} = 4.65\pi$

e. Find the phasor vector form of the transmitted wave magnetic field. (2)

$$\vec{H}_t = \hat{y}\frac{\left(1+\Gamma\right)E_o}{\eta}e^{-j\beta z}$$

f. Find the transmitted power. (4) (Be sure you provide a number for this part.)

$$\vec{P}_{ave} = \frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right) = \hat{z}\,\frac{(1-0.4)^2(200)^2}{2(0.43)(120\pi)} = \hat{z}44.4$$