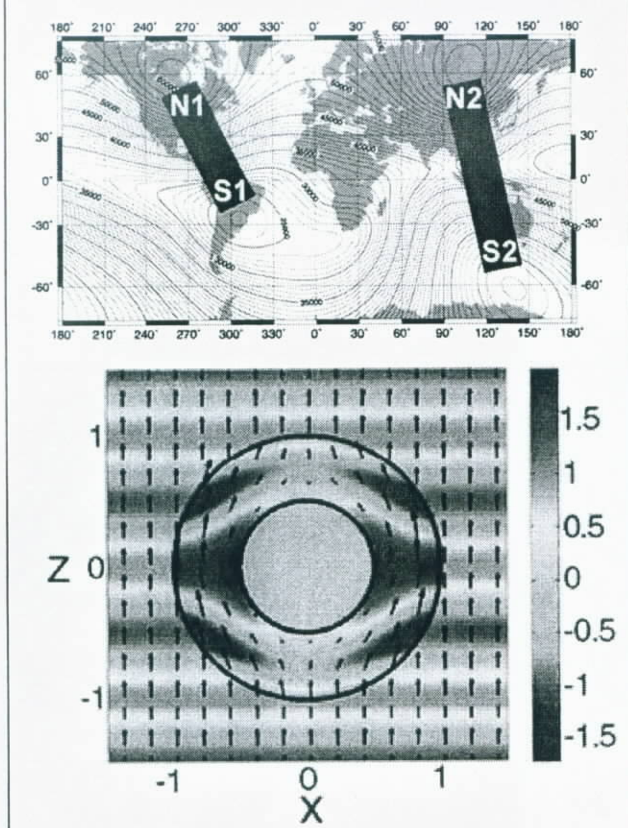
MAGNETIC FIELDS AND UNIFORM PLANE WAVES

Name Solution

Section ______________



1. (8 Pts)	______
2. (8 Pts)	______
3. (8 Pts)	______
4. (6 Pts)	______
5. (6 Pts)	______
6. (4 Pts)	______
7. (20 Pts)	______
8. (20 Pts)	______
9. (20 Pts)	______
Total	______

Notes:
1. **Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.**
2. For short answer questions, you should add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use 4 significant digits.

Some Comments and Helpful Info:
In this course, we use two types of notation for unit vectors. Keep in mind that

$\hat{a}_x = \hat{x}$ $\quad \hat{a}_y = \hat{y}$ $\quad \hat{a}_z = \hat{z}$

$\hat{a}_r = \hat{r}$ $\quad \hat{a}_\phi = \hat{\phi}$ $\quad \hat{a}_\theta = \hat{\theta}$

Also, sometimes R is used for spherical radius instead of r, so R is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.

1. **Shielding (8 points)**

In order to prevent the electric and magnetic fields from entering or leaving a room, the walls of the room are shielded with metal. For which of the following conditions (if any) will the room be reasonably well shielded?

a. 100 kHz, aluminum walls, 2mm thick
b. 10 GHz, copper walls, 0.005mm thick
c. 1 MHz, copper walls, 0.5mm thick
d. 1 GHz, aluminum walls, 0.02mm thick
e. ~~None of the above~~
f. All of the above

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

a. $\delta = 2.7 \times 10^{-4}$ c. $\delta = 6.6 \times 10^{-5}$

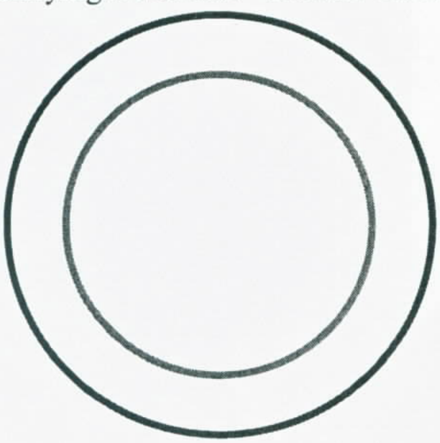
b. $\delta = 6.6 \times 10^{-7}$ d. $\delta = 2.7 \times 10^{-6}$

all are much less than the thickness

in fact $e^{-\alpha z} \approx 0.0005$ or 0.0006 for all 4 cases

2. **Force (8 points)**

Assume that there is a long solenoid of radius *a=5cm, length=100cm,* with *N=1000 turns* carrying a current *I=500mA.* This is the outer cylinder in the figure below.



$$B = \frac{\mu_0 N I}{\ell} = \frac{\mu_0 (1000)(0.5)}{1}$$
$$= 500 \mu_0$$
$$I L B = (1)(2\pi\, 0.03)(500\mu_0)$$
$$= 30\pi \mu_0 = \text{Force}$$

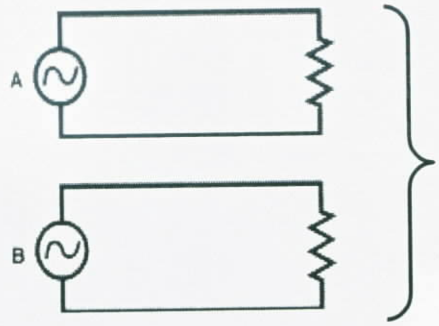
The inner cylinder consists of a single piece of copper of radius $b=3cm$, that extends the full length of the outer cylinder and carries a current $I=-1A$, where the minus sign indicates that the current is in the opposite direction.

Using any method you wish, determine the force experienced by the inner cylinder due to the magnetic field produced by the outer cylinder.

Can do magnetic pressure also.

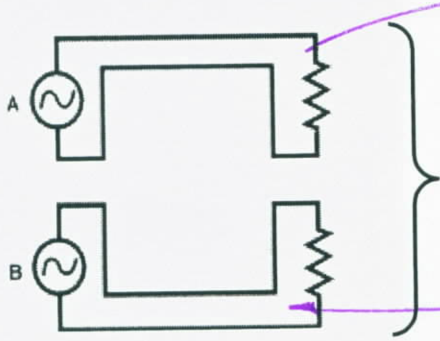
3. Inductive Coupling (8 points)

Three possible configurations are considered to layout the wires between sources and loads on a single layer printed circuit board. On the diagrams below, indicate which will produce the most inductive coupling and which will produce the least inductive coupling.



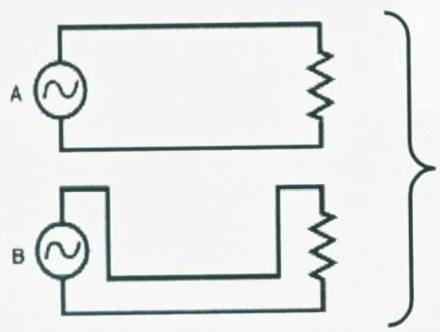
Configuration 1 Most

cross sectional area is least so there is less area to link flux



Configuration 2 Least

having wires near one another causes the stray field to cancel more.



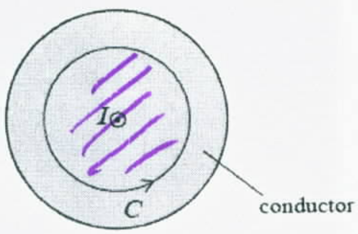
Configuration 3

By using more than one layer for the board, it is possible to create a configuration similar to twisted pair wires, as shown below. Will this produce less coupling than the other three configurations? Explain your answer.

Less
Twisted pairs produce less stray field & what links the other set of wires mostly cancels due to the alternating direction of the areas

4. **Ampere's Law (6 points)**

A cylindrical conductor of a circular cross section carries a time-invariant current I ($I > 0$). (In the diagram, the current is coming out of the page.) The line integral of the magnetic flux density vector, $\boldsymbol{B}$, along a closed circular contour C positioned inside the conductor (the contour radius is smaller than the conductor radius) is



only some of the current is linked & C is oriented to give a + value for this current.

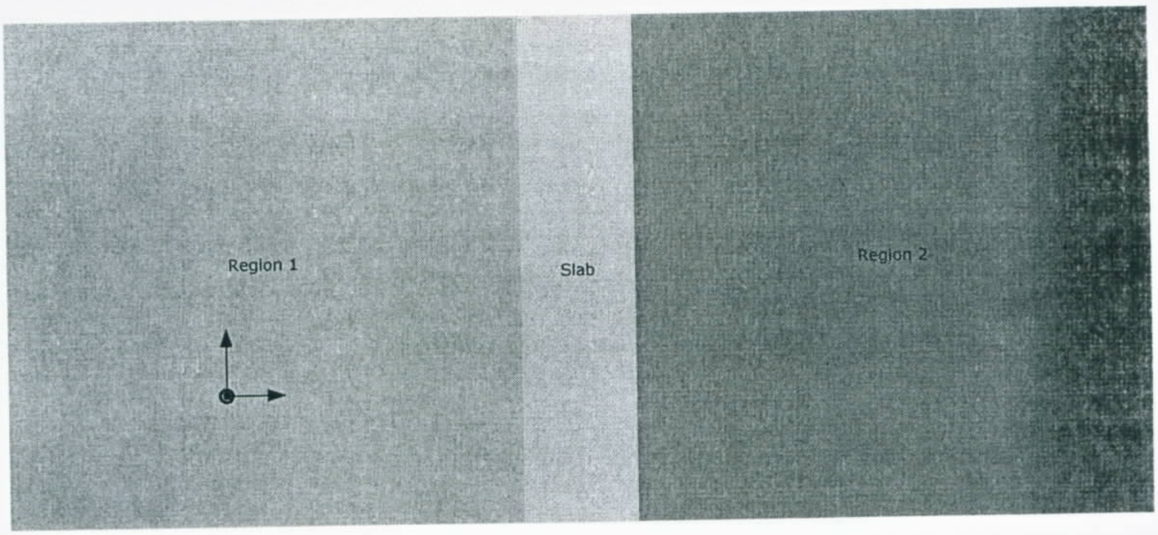
a) $\mu_o I$
b) $-\mu_o I$
c) greater than $\mu_o I$
d) less than $-\mu_o I$
e) less than $\mu_o I$ and positive
f) greater than $-\mu_o I$ and negative
g) zero

5. Plane Wave Reflection (6 points)

A dielectric slab inserted between two media with different intrinsic impedances is designed to provide quarter-wave matching (the slab thickness equals a quarter of the wave wavelength) at a given frequency, yielding zero reflection in the incident region. Which of the following changes would not affect the matching properties of the slab:

a. The slab thickness doubled.
b. The wave frequency doubled.
c. Both the thickness and the frequency doubled.
d. The thickness doubled and the frequency halved.
e. The slab thickness halved.

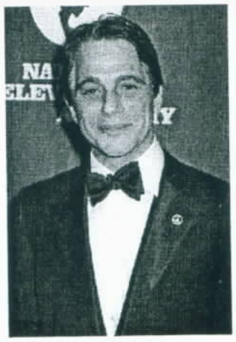
$\lambda = \frac{v}{f} \Rightarrow$ double for half $f \Rightarrow \lambda/4$ remains satisfied



half $f \Rightarrow$ doubled λ

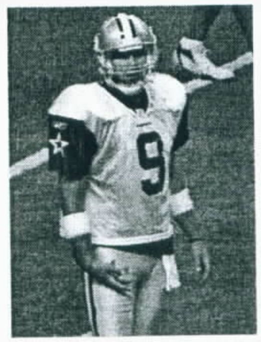
6. Famous Birthdays – 21 April (4 points)

Jean-Baptiste Biot a



Tony Danza d

John Muir b



Tony Romo c

Shown above are the pictures of four people born on 21 April. Identify the picture that goes with each of the following statements.

a. French physicist who did early theoretical work in the representation of magnetic fields due to currents.

b. Born in Scotland, but grew up in Portage, Wisconsin, his work is associated with the Day we celebrate tomorrow. He was also an industrial engineer.

c. Born in Burlington, Wisconsin, he apparently gets easily distracted by his celebrity and forgets how to play football in the post season.

d. Born in New York (Brooklyn, actually), he once said "In an earthquake, I shouldn't run out of the house - I should run into it."

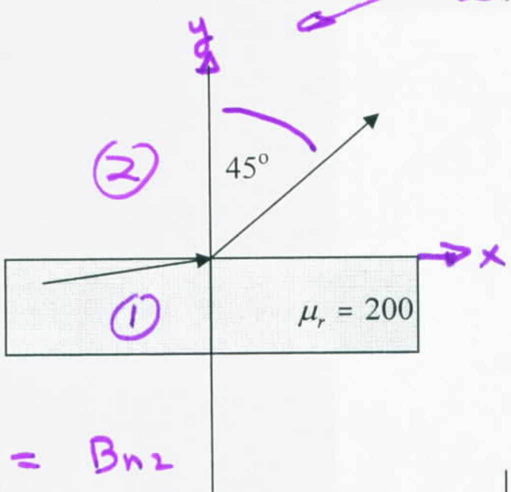
REGULAR QUESTIONS

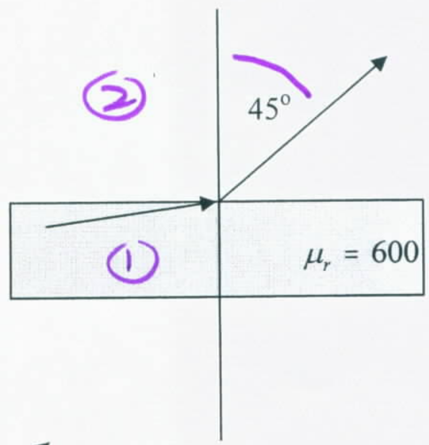
7. Boundary Conditions (20 points)

We have encountered boundary conditions in two contexts. The first involves the change in direction of the field vector at a boundary between two media. The second involves the change in the direction of wave propagation at the boundary between two media. Both will be considered in this problem. Remember that all angles are measured with respect to the normal so that zero degrees or radians will be found perpendicular to the surface between the two media.

a. (6 Points) Determine the unknown angle for each of the plots below, where the magnetic field vector is directed at 45° on the air side of the boundary. That is, find the angle in the magnetic material for the three cases.

always label diagrams for clarity

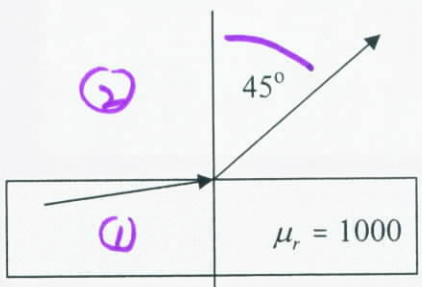




$B_{n1} = B_{n2}$

$\mu_1 H_{n1} = \mu_2 H_{n2}$

$H_{t1} = H_{t2}$



$\mu_1 H_{n1} = \mu_2 H_{n2} = \mu_2 H_{t2} = \mu_2 H_{t1}$

$\frac{H_{n1}}{H_{t1}} = \frac{\mu_2}{\mu_1} = \frac{1}{200}, \frac{1}{600}, \frac{1}{1000}$

$\tan\theta_1 = \frac{H_{t1}}{H_{n1}} = 200, 600, 1000$

$\theta_1 = \underbrace{89.7°, 89.9°, 89.94°}_{\text{all almost } 90°}$

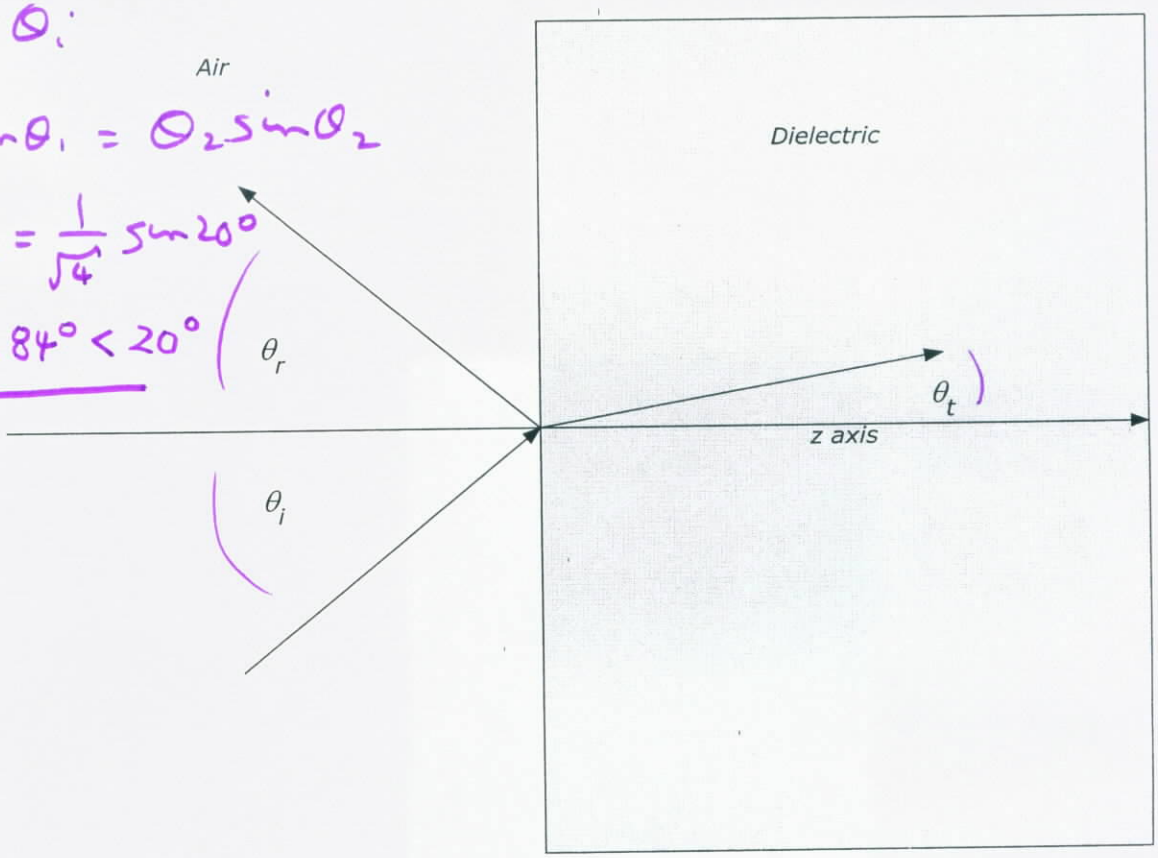
b. (6 Points) For the configuration shown below, a uniform plane wave propagating in air is incident obliquely at an angle of $\theta_i = 20^o$ on the boundary of a dielectric medium with $\underbrace{\varepsilon_r = 4}_{\text{note}}$. Find the reflected and transmitted angles θ_r and θ_t.

$\theta_r = \theta_i$

$\theta_1 \sin\theta_1 = \theta_2 \sin\theta_2$

$\sin\theta_2 = \frac{1}{\sqrt{4}} \sin 20^\circ$

$\theta_2 = 9.84^\circ < 20^\circ$



Now, assume that the wave is incident on the boundary at $\theta_i = 20^o$ from the dielectric side and transmits into air. The dielectric medium still is characterized by $\underbrace{\varepsilon_r = 4}_{\text{note}}$. Find the reflected and transmitted angles θ_r and θ_t.

$\sin\theta_2 = 2 \sin 20^\circ$

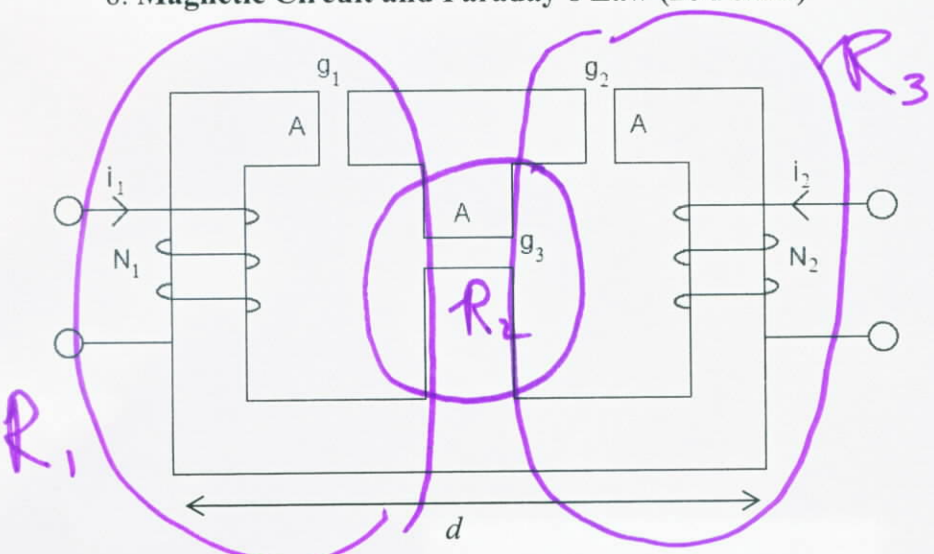
$\theta_2 = 43.16^\circ > 20^\circ$

Comment on the relationship between the two transmitted angles in the two cases.

For case 1 $\quad \theta_t < \theta_i$

For case 2 $\quad \theta_t > \theta_i$

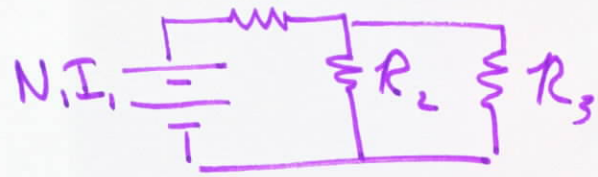
8. Magnetic Circuit and Faraday's Law (20 Points)



Leave answers in generic form

The magnetic circuit above has three gaps and all parts of the circuit have the same cross sectional area A. The gaps are small compared with either h or d.

a. Assuming that the core material has a permeability of $\mu = \mu_r \mu_o$, find the flux in each of the three vertical legs of the circuit due to current i_1.



$$R_1 = \frac{\frac{d}{2} + \frac{d}{2} + h}{\mu A} + \frac{g_1}{\mu_0 A}$$

$$R_2 = \frac{h}{\mu A} + \frac{g_3}{\mu_0 A} \qquad R_3 = \frac{\frac{d}{2} + \frac{d}{2} + h}{\mu A} + \frac{g_2}{\mu_0 A}$$

$$R_{TOTAL} = R_1 + R_2 \| R_3$$

$$\Psi_{MT} = \frac{N_1 I_1}{R_{TOTAL}} \qquad \Psi_{m\,center} = \frac{R_3}{R_2 + R_3} \Psi_{m\,TOTAL}$$

$$\Psi_{m\,right} = \frac{R_2}{R_2 + R_3} \Psi_{m\,TOTAL}$$

b. Find the self inductance of the coil at the left (N_1 turns), ~~the self inductance of the coil at the right (N_2 turns)~~ and the mutual inductance between the two coils.

$$L_1 = \frac{\Lambda}{I_1} = \frac{N_1 \Psi_{m\,TOTAL}}{I_1} = \frac{N_1^2}{\mathcal{R}_{TOTAL}}$$

$$M = \frac{\Lambda}{I_1} = \frac{N_2 N_1}{\mathcal{R}_{TOTAL}} \frac{\mathcal{R}_2}{\mathcal{R}_2 + \mathcal{R}_3}$$

c. Simplify your answer to part a for the case where $\mu \to \infty$

$$\mathcal{R}_1 = \frac{g_1}{\mu_0 A} \quad \mathcal{R}_2 = \frac{g_3}{\mu_0 A} \quad \mathcal{R}_3 = \frac{g_2}{\mu_0 A}$$

Otherwise The Same.

$$\mathcal{R}_{TOTAL} = \frac{g_1}{\mu_0 A} + \frac{g_3}{\mu_0 A} \Big\| \frac{g_2}{\mu_0 A} = \frac{g_1}{\mu_0 A} + \frac{1}{\mu_0 A}\left(\frac{g_3 g_2}{g_3 + g_2}\right) = \frac{g_1 g_2 + g_1 g_3 + g_2 g_3}{(g_3 + g_2)\mu_0 A}$$

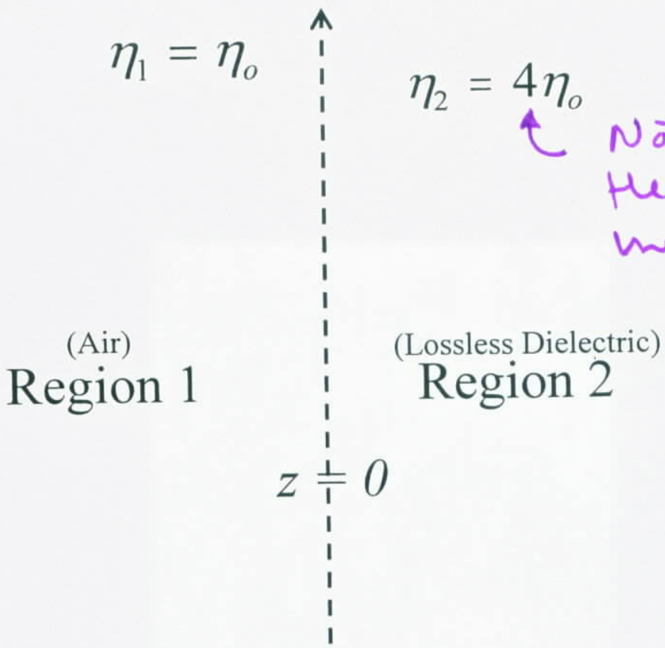
d. Simplify your answers to part b for the case where $\mu \to \infty$

They look the same except we use the new answers for the reluctances & $\frac{\mathcal{R}_2}{\mathcal{R}_2 + \mathcal{R}_3} = \frac{g_3}{g_2 + g_3}$

Answers do not have to be in identical form to these answers, however they should be reasonably simple

9. Uniform Plane Waves in Lossless Materials (20 Points)

A uniform plane wave is propagating in air and incident normally on a lossless dielectric medium. The frequency of the wave is 3GHz. The average power density of the wave is 10 Watts per square meter. You might find it helpful to draw the vector diagram for each wave to be sure you have the directions correct. You do not have to simplify any expression (you can leave it in terms of parameters), except for part d which requires a number.



Note that this is not the usual case since it implies that $\varepsilon_r = \frac{1}{16}$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\frac{1}{\sqrt{\varepsilon_r}}$$

$$\frac{1}{\sqrt{\varepsilon_r}} = 4 \Rightarrow \varepsilon_r = \frac{1}{16}$$

a. Determine the angular frequency ω, the propagation constant β, and the wavelength λ for this incident wave. (3)

$$\omega = 2\pi f = 2\pi\ 3\times10^9 = 6\pi\times10^9$$

$$\beta_o = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c} = \frac{6\pi\times10^9}{3\times10^8} = 20\pi$$

$$\lambda = \frac{2\pi}{\beta_o} = \frac{2\pi}{20\pi} = \frac{1}{10}$$

also $$\frac{c}{f} = \lambda = \frac{3\times10^8}{3\times10^9} = \frac{1}{10}$$

b. Determine the magnitude of the incident electric field and both the full phasor expressions for incident electric and magnetic fields. (5)

$$S_{ave} = 10\,\frac{W}{m^2} = \frac{1}{2}\frac{E_m^2}{\eta_0} \qquad E_m^2 = 2\eta_0 10 \Rightarrow E_m = \sqrt{2400\pi} = 86.8\,\frac{V}{m}$$

$$\vec{E}_i = \hat{x} E_m e^{-j\beta_0 z}$$

$$\vec{H}_i = \hat{y}\,\frac{E_m}{\eta_0} e^{-j\beta_0 z}$$

c. Write the phasor vector form of the reflected and transmitted electric fields. (5)

$$\beta = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r} = \beta_0/4 = 5\pi \qquad \eta = 4\eta_0$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{4-1}{4+1} = \frac{3}{5} \qquad \tau = \frac{2\eta_2}{\eta_2+\eta_1} = \frac{8}{5}$$

$$\vec{E}_t = \hat{x} E_m \tau e^{-j\beta z} \qquad \vec{E}_r = \hat{x}\,\Gamma E_m e^{+j\beta_0 z}$$

$$\vec{H}_t = \hat{y}\,\frac{E_m\tau}{\eta} e^{-j\beta z} \qquad \vec{H}_r = -\hat{y}\,\frac{\Gamma E_m}{\eta_0} e^{+j\beta_0 z}$$

d. Determine the average transmitted power density (in region 2). (7)

$$\vec{S} = \frac{1}{2}\mathrm{Re}\,\vec{E}\times\vec{H}^* \qquad S = \frac{1}{2}\frac{E_m^2\tau^2}{\eta} = \frac{1}{2}\frac{E_m^2}{\eta}\tau^2$$

$$= \underbrace{\frac{1}{2}\frac{E_m^2}{\eta_0}}_{10}\frac{\tau^2}{4} = 10\,\frac{(8/5)^2}{4}$$

$$= 6.4\,\frac{W}{m^2}$$

Checking the reflected power $= \Gamma^2 10 = 3.6\,\frac{W}{m^2}$

so $6.4 + 3.6 = 10$ as it should ✓