# Rensselaer ${ }^{\text {Wew }}$ 

Department of Electrical, Computer, \& Systems Engineering

## MAGNETIC FIELDS AND UNIFORM PLANE WAVES



Notes:

1. Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.
2. For short answer questions, you should add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians \& not degrees \& use 4 significant digits.

Name $\qquad$

Section $\qquad$

1. (8 Pts) $\qquad$
2. (8 Pts)
3. (8 Pts) $\qquad$
4. (6 Pts) $\qquad$
5. (6 Pts) $\qquad$
6. (4 Pts)
7. (20 Pts)
8. (20 Pts)
9. (20 Pts)

Total

Some Comments and Helpful Info:
In this course, we use two types of notation for unit vectors. Keep in mind that

$$
\begin{aligned}
& \hat{a}_{x}=\hat{x} \quad \hat{a}_{y}=\hat{y} \quad \hat{a}_{z}=\hat{z} \\
& \hat{a}_{r}=\hat{r} \quad \hat{a}_{\phi}=\hat{\phi} \quad \hat{a}_{\theta}=\hat{\theta}
\end{aligned}
$$

Also, sometimes $R$ is used for spherical radius instead of $r$, so $R$ is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.

## 1. Shielding (8 points)

In order to prevent the electric and magnetic fields from entering or leaving a room, the walls of the room are shielded with metal. For which of the following conditions (if any) will the room be reasonably well shielded?
a. 100 kHz , aluminum walls, 2 mm thick
b. 10 GHz , copper walls, 0.005 mm thick
c. 1 MHz , copper walls, 0.5 mm thick
d. 1 GHz , aluminum walls, 0.02 mm thick
e. None of the above
f. All of the above

## 2. Force (8 points)

Assume that there is a long solenoid of radius $a=5 \mathrm{~cm}$, length $=100 \mathrm{~cm}$, with $N=1000$ turns carrying a current $I=500 \mathrm{~mA}$. This is the outer cylinder in the figure below.


The inner cylinder consists of a single piece of copper of radius $b=3 \mathrm{~cm}$, that extends the full length of the outer cylinder and carries a current $I=-1 A$, where the minus sign indicates that the current is in the opposite direction.

Using any method you wish, determine the force experienced by the inner cylinder due to the magnetic field produced by the outer cylinder.

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## 3. Inductive Coupling (8 points)

Three possible configurations are considered to layout the wires between sources and loads on a single layer printed circuit board. On the diagrams below, indicate which will produce the most inductive coupling and which will produce the least inductive coupling.


By using more than one layer for the board, it is possible to create a configuration similar to twisted pair wires, as shown below. Will this produce less coupling than the other three configurations? Explain your answer.


## 4. Ampere's Law (6 points)

A cylindrical conductor of a circular cross section carries a time-invariant current $I(I>0)$. (In the diagram, the current is coming out of the page.) The line integral of the magnetic flux density vector, $\boldsymbol{B}$, along a closed circular contour $C$ positioned inside the conductor (the contour radius is smaller than the conductor radius) is

a) $\mu_{0} I$
b) $-\mu_{0} I$
c) greater than $\mu_{0} I$
d) less than $-\mu_{0} I$
e) less than $\mu_{o} I$ and positive
f) greater than $-\mu_{o} I$ and negative
g) zero

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## 5. Plane Wave Reflection (6 points)

A dielectric slab inserted between two media with different intrinsic impedances is designed to provide quarter-wave matching (the slab thickness equals a quarter of the wave wavelength) at a given frequency, yielding zero reflection in the incident region. Which of the following changes would not affect the matching properties of the slab:
a. The slab thickness doubled.
b. The wave frequency doubled.
c. Both the thickness and the frequency doubled.
d. The thickness doubled and the frequency halved.
e. The slab thickness halved.



Jean-Baptiste Biot Tony Danza


John Muir


Tony Romo

Shown above are the pictures of four people born on 21 April. Identify the picture that goes with each of the following statements.
a. French physicist who did early theoretical work in the representation of magnetic fields due to currents.
b. Born in Scotland, but grew up in Portage, Wisconsin, his work is associated with the Day we celebrate tomorrow. He was also an industrial engineer.
c. Born in Burlington, Wisconsin, he apparently gets easily distracted by his celebrity and forgets how to play football in the post season.
d. Born in New York (Brooklyn, actually), he once said "In an earthquake, I shouldn't run out of the house - I should run into it."

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## REGULAR QUESTIONS

## 7. Boundary Conditions (20 points)

We have encountered boundary conditions in two contexts. The first involves the change in direction of the field vector at a boundary between two media. The second involves the change in the direction of wave propagation at the boundary between two media. Both will be considered in this problem. Remember that all angles are measured with respect to the normal so that zero degrees or radians will be found perpendicular to the surface between the two media.
a. (6 Points) Determine the unknown angle for each of the plots below, where the magnetic field vector is directed at $45^{\circ}$ on the air side of the boundary. That is, find the angle in the magnetic material for the three cases.




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b. (6 Points) For the configuration shown below, a uniform plane wave propagating in air is incident obliquely at an angle of $\theta_{i}=20^{\circ}$ on the boundary of a dielectric medium with $\varepsilon_{r}=4$. Find the reflected and transmitted angles $\theta_{r}$ and $\theta_{t}$.


Now, assume that the wave is incident on the boundary at $\theta_{i}=20^{\circ}$ from the dielectric side and transmits into air. The dielectric medium still is characterized by $\varepsilon_{r}=4$. Find the reflected and transmitted angles $\theta_{r}$ and $\theta_{t}$.

Comment on the relationship between the two transmitted angles in the two cases.


The magnetic circuit above has three gaps and all parts of the circuit have the same cross sectional area A. The gaps are small compared with either $h$ or $d$. Leave all answers in generic form, simplifying as much as you can.
a. Assuming that the core material has a permeability of $\mu=\mu_{r} \mu_{o}$, find the flux in each of the three vertical legs of the circuit due to current $i_{1}$.
b. Find the self inductance of the coil at the left ( $N_{1}$ turns) and the mutual inductance between the two coils.
c. Simplify (as much as you can) your answer to part a for the case where $\mu \rightarrow \infty$
d. Simplify (as much as you can) your answers to part b for the case where $\mu \rightarrow \infty$

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| CSE |</table-markdown></div> 

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## 9. Uniform Plane Waves in Lossless Materials (20 Points)

A uniform plane wave is propagating in air and incident normally on a lossless dielectric medium. The frequency of the wave is 3 GHz . The average power density of the wave is 10 Watts per square meter. You might find it helpful to draw the vector diagram for each wave to be sure you have the directions correct. You do not have to simplify any expression (you can leave it in terms of parameters), except for part d, which requires a number.

a. Determine the angular frequency $\omega$, the propagation constant $\beta$, and the wavelength $\lambda$ for this incident wave. (3)

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b. Determine the magnitude of the incident electric field and both the full phasor expressions for incident electric and magnetic fields. (5)
c. Write the phasor vector form of the reflected and transmitted electric fields. (5)
d. Determine the average transmitted power density (in region 2). (7)

## Material Properties (from Ulaby Appendices)

## Dielectric Materials

| Material | $\varepsilon_{r}$ | Material | $\varepsilon_{r}$ |
| :--- | :---: | :--- | :---: |
| Vacuum | 1 | Dry Soil | $2.5-3.5$ |
| Air (sea level) | 1.006 | Plexiglas | 3.4 |
| Styrofoam | 1.03 | Glass | $4.5-10$ |
| Teflon | 2.1 | Fused Quartz | $3.8-5$ |
| Petroleum Oil | 2.1 | Bakelite | 5 |
| Wood (Dry) | $1.5-4$ | Porcelain | 5.7 |
| Paraffin | 2.2 | Formica | 6 |
| Polyethylene | 2.25 | Mica | $5.4-6$ |
| Polystyrene | 2.6 | Ammonia | 22 |
| Paper | $2-4$ | Sea Water | $72-80$ |
| Rubber | $2.2=4.1$ | Distilled Water | 81 |
|  | 1 | All are low frequency values at room <br> temperature $\left(20^{\circ} \mathrm{C}\right)$ |  |
| Most Metals |  |  |  |
|  |  |  |  |

## Conductors

| Material | $\sigma \mathrm{S} / \mathrm{m}$ | Material | $\sigma \mathrm{S} / \mathrm{m}$ |
| :--- | :---: | :--- | :---: |
| Conductors |  | Semiconductors |  |
| Silver | $6.2 \times 10^{7}$ | Pure Germanium | 2.2 |
| Copper | $5.8 \times 10^{7}$ | Pure Silicon | $4.4 \times 10^{-4}$ |
| Gold | $4.2 \times 10^{7}$ | Insulators |  |
| Aluminum | $3.5 \times 10^{7}$ | Wet Soil | $10^{-2}$ |
| Tungsten | $1.8 \times 10^{7}$ | Fresh Water | $10^{-3}$ |
| Zinc | $1.7 \times 10^{7}$ | Distilled Water | $10^{-4}$ |
| Brass | $1.5 \times 10^{7}$ | Dry Soil | $10^{-4}$ |
| Iron | $1 \times 10^{7}$ | Glass | $10^{-12}$ |
| Bronze | $1 \times 10^{7}$ | Hard Rubber | $10^{-15}$ |
| Tin | $0.9 \times 10^{7}$ | Paraffin | $10^{-15}$ |
| Lead | $0.5 \times 10^{7}$ | Mica | $10^{-15}$ |
| Mercury | $0.1 \times 10^{7}$ | Fused Quartz | $10^{-17}$ |
| Carbon | $3 \times 10^{4}$ | Glass | $10^{-17}$ |
| Sea Water | 4 |  | All are low frequency values at room |
| Animal Body | 0.3 | temperature $\left(20^{\circ} \mathrm{C}\right)$ |  |

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Magnetic Materials

| Material | $\mu_{r}$ | Material | $\mu_{r}$ |
| :---: | :---: | :---: | :---: |
| Diamagnetic |  | Titanium | $1.0002 \approx 1$ |
| Bismuth | $0.99983 \approx 1$ | Platinum | $1.0003 \approx 1$ |
| Gold | $0.99996 \approx 1$ | Ferromagnetic | Nonlinear |
| Mercury | $0.99997 \approx 1$ | Cobalt | 250 |
| Silver | $0.99998 \approx 1$ | Nickel | 600 |
| Copper | $0.99999 \approx 1$ | Mild Steel | 2000 |
| Water | $0.99999 \approx 1$ | Iron (Pure) | 4000-5000 |
| Paramagnetic |  | Silicon Iron | 7000 |
| Air | $1.000004 \approx 1$ | Mu Metal | ~100,000 |
| Aluminum | $1.00002 \approx 1$ | Purified Iron | $\sim 200,000$ |
| Tungsten | $1.00008 \approx 1$ | Only typical values, actual values depend on material variety |  |
|  |  |  |  |

