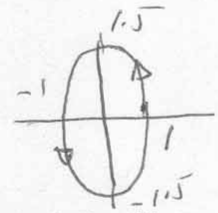
1) (25)

a) Determine the polarization of the following uniform plane wave field



$$E = 1\cos(\omega t + \beta z)\,\hat{a}_x + 1.5\sin(\omega t + \beta z)\,\hat{a}_y$$

(With the holiday season approaching, I always wonder if Santa Clause is polarized!)

LH elliptical

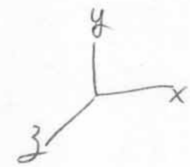
b) You are told that a wave in free space has the following electric field associated with it. $E = E_0 \cos(\omega t - \beta z)\hat{a}_z$. Is this a possible electric field? Explain.

No $\nabla \cdot D = 0$ is violated

c) A wave is traveling in the –y direction in air. It has a wavelength of λ meters and a angular frequency of ω. The magnitude of the fields are E_0 and H_0. The wave is linearly polarized in the z direction.

$\beta = 2\pi/\lambda$

Write the phasor expressions for E and H.



$\tilde{E} = E_0 e^{j\beta y}\,\hat{a}_z$ $\quad E = E_0 \cos(\omega t + \frac{2\pi}{\lambda} y)\,\hat{a}_z$

$\bar{H} = H_0 e^{j\beta y}\,\hat{a}_x$ $\quad H = H_0 \cos(\omega t + \frac{2\pi}{\lambda} y)\,-\hat{a}_x$

Write the time domain expressions for E and H.

Find the velocity

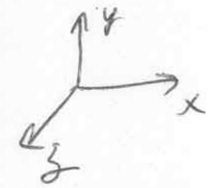
$v_p = \lambda \cdot f = \frac{\lambda \omega}{2\pi}$

Find the wave impedance.

$\eta = \frac{E_0}{H_0}$

2) (15)

A plane wave is propagating in the x direction in a lossless material. The electric field is z polarized and has peak value 100 V/m. The wavelength is 25 cm and the velocity is 2×10^8 m/s. Find the frequency and relative permittivity (assume $\mu = \mu_0$). Write the time domain expressions for E and H.



$$v_p = \frac{c}{\sqrt{\epsilon_r}} \Rightarrow \sqrt{\epsilon_r} = \frac{3}{2}$$

$$\epsilon_r = 9/4$$

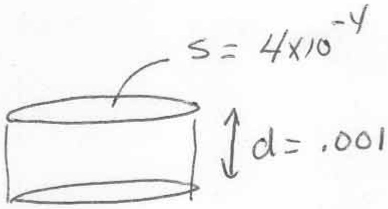
$$v_p = \lambda \cdot f \quad , \quad f = \frac{2 \times 10^8}{0.25} = 8 \times 10^8 \text{ Hz}$$

$$E = 100 \cos\left(2\pi \times 8 \times 10^8 t - \frac{2\pi}{0.25} x\right) \hat{a}_z$$

$$\eta = \eta_0 / \sqrt{\epsilon_r} = 80\pi$$

$$H = \frac{100}{80\pi} \cos\left(2\pi \times 8 \times 10^8 t - \frac{2\pi}{0.25} x\right) -\hat{a}_y$$

3) (15) A parallel plate capacitor has area of 4 cm^2 and spacing of 0.1 cm. The material has $\mu_r = 1, \varepsilon_r = 1.9, \sigma = 2.0\times10^{-7} S/m$ An electric field $E = 200\sin(\omega t)\hat{a}_z$ V/m. is applied. At what frequency does the displacement current equal the conduction current? Calculate the total current at that frequency.



$E = 200 \sin \omega t$

$J_c = \sigma E \quad , \quad J_d = j\omega\epsilon E$

$\sigma = \omega\epsilon \quad$ for $|J_d| = |J_c|$

$$\frac{2\times10^{-7}}{1.9\times\frac{1}{36\pi}\times10^{-9}} = 11904. = \omega$$

$f = 1895$

$I = \sigma E A + j\omega\epsilon E A$

$= 2\times10^{-7}\times 4\times10^{-4}\times 200\ \cancel{\sin\omega t} + j\ 11904\times1.9\times\frac{1}{36\pi}\times10^{-9}\times4\times10^{-4}\times200$

$= 1.6\times10^{-8} + j\,1.6\times10^{-8}$ A

4) (25)
a) Why are conductors made for high frequency operation smaller then those made for dc or low frequency operation? Explain in a few sentences.

$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

we want the current density to be as uniform as possible. The conductor size should be smaller than δ.

b) The displacement current in a material is related to the conduction current in which of the following ways (circle one)

the two currents are in phase
displacement current leads conduction current by 90 degrees
displacement current lags conduction current of 90 degrees.
the currents are 180 out of phase

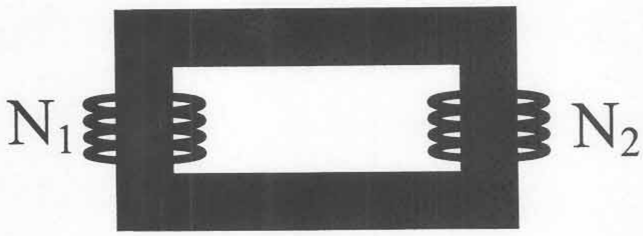
c) A circular antenna with an area of 0.01 square meters is in a uniform field at 300 MHz. The peak voltage induced in 20 mV. What is the peak flux density?

$$\mathcal{E} = -\frac{d\Psi_m}{dt} = j\omega A B$$

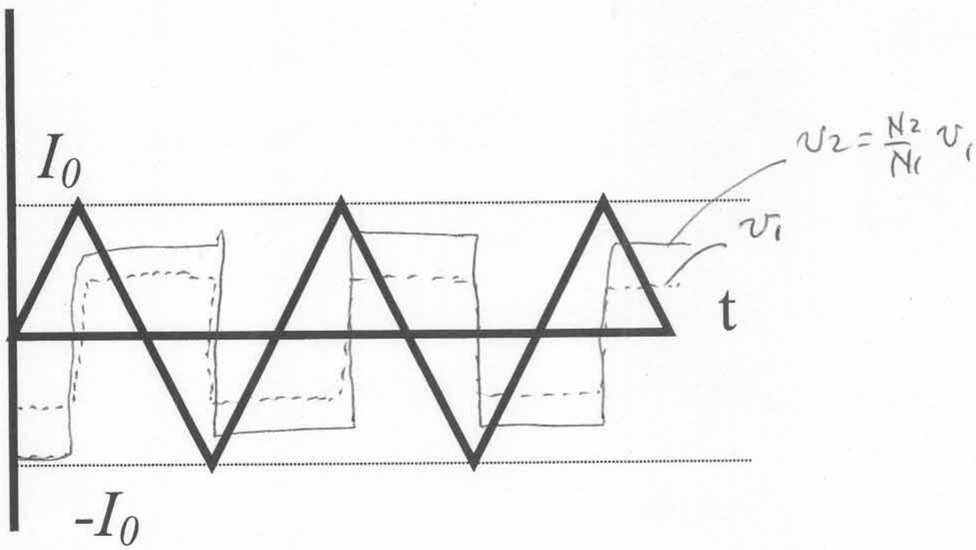
$$.02 = 2\pi \times 300 \times 10^6 \times 0.01\ B$$

$$B = 1.06 \times 10^{-9}\ T$$

5) (20)An ideal transformer has two windings with N1 and N2 turns are on a magnetic core.



a) In the first case winding 2 is open circuited (no current) . The current in winding 1 has the form below:



Sketch the voltage on winding 1 and the induced voltage on winding 2.

b) Assuming that the reluctance of the core is $\Re$, write expressions for the self and mutual inductance of the windings.

$L_1 = \frac{N_1^2}{R}$, $L_2 = \frac{N_2^2}{R}$, $L_{12} = \frac{N_1 N_2}{R}$

c) For the ideal transformer, the primary (winding 1) is now connected to a voltage source $v = V_m \sin \omega t$. The secondary is connected to a load which is a simple resistance R_L. Find the currents in each winding.

$$v_2 = \frac{N_1}{N_2} V_m$$

$$i_2 = \frac{v_2}{R_2} = \frac{N_1}{N_2}\frac{V_m}{R_2}$$

$$i_1 = \frac{N_2}{N_1} i_2$$