Calculus Review

Divergence Theorem – For the following geometries and fields, solve both sides of the equation $\int_V \left(\nabla \cdot \vec{A}\right) dV = \oint_S \vec{A} \cdot d\vec{S}$. You should verify that both sides yield equal results for a given vector and geometry.

These problems are just mathematical exercises and don't have a physical representation. They are a useful exercise in vector calculus and solving them would indicate a solid understanding in the methods we will use in the course.

Cartesian coordinates: $\vec{A} = x^2\hat{x} + (3x+2)\hat{y} + yz\hat{z}$

For a cube with sides of length 2m:

1. The cube is centered at the origin, *(0,0,0)*
2. The cube is centered at *(3,0,0)*
3. The cube is centered at *(1,1,1)*
4. Repeat part 1 for a cube with an arbitrary side length, *a*

Cylindrical coordinates:
$$\vec{A}_1 = r\,\hat{r} + \sin\phi\,\hat{\phi} + 2\,\hat{z} \quad \textit{for } r < 2,$$
$$\vec{A}_2 = \frac{2}{r}\hat{r} + \sin\phi\,\hat{\phi} + 2\,\hat{z} \quad \textit{for } 2 < r$$

1. For a cylinder defined by $-5 < z < 5,\ r < 1$
2. For a cylinder defined by $-5 < z < 5,\ r < 3$
3. For a hollow cylinder defined by $-5 < z < 5,\ 1 < r < 3$

Cylindrical coordinates: $\vec{A}_1 = r\,\hat{r} + \sin\phi\,\hat{\phi} + 2\,z\hat{z}$

1. Repeat part 1 in the previous problem
2. Repeat part 3 in the previous problem

Spherical coordinates: $\vec{A}_1 = r^3\,\hat{r} + \cos\theta\,\hat{\theta} + \sin\phi\,\hat{\phi}$

1. For a sphere with radius 5
2. For a half sphere with radius 5, $0 < \theta < \pi/2$