FIELDS AND WAVES I.

HOMEWORK #1 - SOLUTIONS.

(1) For the following wave expressions, indicate if the wave is standing or traveling. Find the direction of propagation & the velocity.

(a) $\sin(500t + 0.3x)$.
This wave is traveling in $-x$ direction.
$\omega = 500$ rad/s.
$\beta = 0.3$.
velocity $= \frac{\omega}{\beta} = 1666.6$ m/s.

(b) $\cos(1.5\times10^5 t - 6\times10^{-1} z)$
This wave is traveling in $+z$ direction,
velocity $= \frac{\omega}{\beta} = \frac{1.5\times10^5 t}{6\times10^{-1}} = 2.5\times10^5$ m/s.

(c) $\cos(120t)\sin(50x)$.
This is a standing wave.

(2.) Find the phasor representation of the following

(a) $v(t) = 4\cos\left(\omega t - \frac{2\hat{x}}{3}\right)$

The phasor form of an expression in the form $\underline{A\cos(\omega t + \phi)}$ is $\underline{Ae^{j\phi}}$

$\Rightarrow \tilde{V} = 4e^{-j2\hat{x}/3}$

(b) $v(t) = 2.5\sin\left(\omega t + \frac{\hat{x}}{3}\right)$

$= 2.5\cos\left(\frac{\hat{x}}{3} - (\omega t + \hat{x})\right) = 2.5\cos(\omega t - \hat{x})$

$$\therefore \tilde{V} = 2.5e^{-j\pi/6}$$

(c) $v(t) = 8\sin\left(\omega t + \frac{2\pi}{3}\right) + 8\cos\left(\omega t - \frac{\pi}{3}\right)$

$= 8\cos\left(\frac{\pi}{2} - \left(\omega t + \frac{2\pi}{3}\right)\right) + 8\cos\left(\omega t - \frac{\pi}{3}\right)$

$= 8\cos\left(\omega t + \frac{\pi}{6}\right) + 8\cos\left(\omega t - \frac{\pi}{3}\right)$

$= 8e^{j\pi/6} + 8e^{-j\pi/3}$

$= 8\cos\pi/6 + j\sin\pi/6 + 8\cos\pi/3 - j\sin\pi/3$

$= 8\left[\cos\pi/3 + \cos\pi/6\right] + j8\left[\sin\pi/6 - \sin\pi/3\right]$

$= 10.928 - 2.928j$

(3) Find the time domain expression of the following.

(a) $\tilde{V} = 3.5 + j1.5\ V$

$r = \sqrt{3.5^2 + 1.5^2} = 3.807$

$\phi = \tan^{-1}\left(\frac{1.5}{3.5}\right) = 23.2^\circ$

$\Rightarrow \tilde{V} = 3.807\angle 23.2^\circ$ $\xleftrightarrow[\text{time domain}]{\text{Phasor to}}$ $3.807\cos(\omega t + 23.2^\circ)$

$= 3.807e^{j(23.2^\circ)}$

(b) $\tilde{V} = 2.0e^{j\pi/4}\ V$

$v(t) = \text{Re}\left\{2.0e^{j\pi/4}e^{j\omega t}\right\} = 2.0\cos\left(\omega t + \frac{\pi}{4}\right)$.

(4) A wave is described by $v(t,z) = 3.5e^{-\alpha z} \sin(2\pi \times 10^8 t - 12\pi z)$. Find the frequency, wavelength and velocity. At $z = 2m$, the magnitude is measured as 1V. Find the attenuation constant.

Ans.) $\omega = 2\pi \times 10^8 = 2\pi f$

$\therefore$ frequency $(f) = 10^8$ Hz.

$\beta = 12\pi$.

Wavelength $\lambda = \frac{2\pi}{\beta} = 0.166$ m.

Velocity $v = \frac{\omega}{\beta} = \frac{2\pi \times 10^8}{12\pi} = 1.67 \times 10^7$ m/s.

At $z = 2m$, magnitude $= 1V$.

$\therefore$ For attenuation constant,

$3.5e^{-2(\alpha)} = 1$.

$\Rightarrow \alpha = \ln\left(\frac{1}{3.5}\right) \times -\frac{1}{2}$

$= 0.626$.