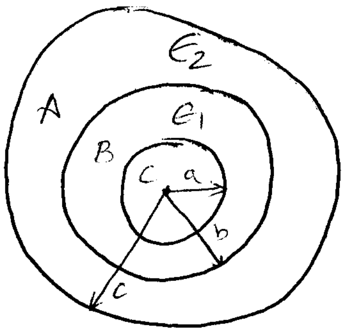
HW #5 Solution

(2)



In region C, $\vec{E} = 0$, $\vec{D} = \epsilon \vec{E} = 0$, $V = 0$.

In region B, $r \in [a, b]$

$$Q_{encl.} = \int D \cdot ds = D \cdot 2\pi r l = \rho_L l.$$

$$\Rightarrow \vec{D} = \frac{\rho_L}{2\pi r} \hat{a}_r$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_1} = \frac{\vec{D}}{4.2\epsilon_0} = \frac{\rho_L}{8.4\epsilon_0 \pi r} \hat{a}_r.$$

$$V = -\int_b^a \frac{\rho_L}{8.4\epsilon_0 \pi r} dr = \frac{\rho_L}{8.4\epsilon_0 \pi \cancel{r}} \int_a^b \frac{dr}{r} = \frac{\rho_L}{8.4\epsilon_0 \pi} \ln \frac{b}{a}.$$

In region A. $r \in (b, c)$

$$\vec{D} = \frac{\rho_L}{2\pi r} \hat{a}_r. \quad \vec{E} = \frac{\vec{D}}{3.7\epsilon_0} = \frac{\rho_L}{7.4\epsilon_0 \pi r} \hat{a}_r.$$

$$V = -\int_c^b \frac{\rho_L}{7.4\epsilon_0 \pi r} dr = \frac{\rho_L}{7.4\epsilon_0 \pi} \ln \frac{c}{b}.$$

A4 ≠ NAO

$$V_{ac} = \frac{\rho_L}{8.4\,\epsilon_0 \pi} \ln \frac{b}{a} + \frac{\rho_L}{7.4\,\epsilon_0 \pi} \ln \frac{c}{b} = 100\text{V}.$$

$$\therefore \quad \rho_L = \frac{100}{\frac{1}{8.4(8.85\times10^{-12})\pi} \ln(2) + \frac{1}{7.4(8.85\times10^{-12})\pi} \ln\left(\frac{3}{2}\right)}$$

$$= 2.025\times10^{-8}\ \text{c/m}.$$

For region B, $\vec{D} = \dfrac{\rho_L}{2\pi r} = \dfrac{3.22\times10^{-9}}{r}\hat{a}_r$

$$\vec{E} = \frac{\vec{D}}{4.2\,\epsilon_0} = \frac{86.7}{r}\hat{a}_r$$

$$V = \frac{\rho_L}{8.4\,\epsilon_0 \pi} \ln\left(\frac{b}{a}\right) = 60.1\ \text{V}.$$

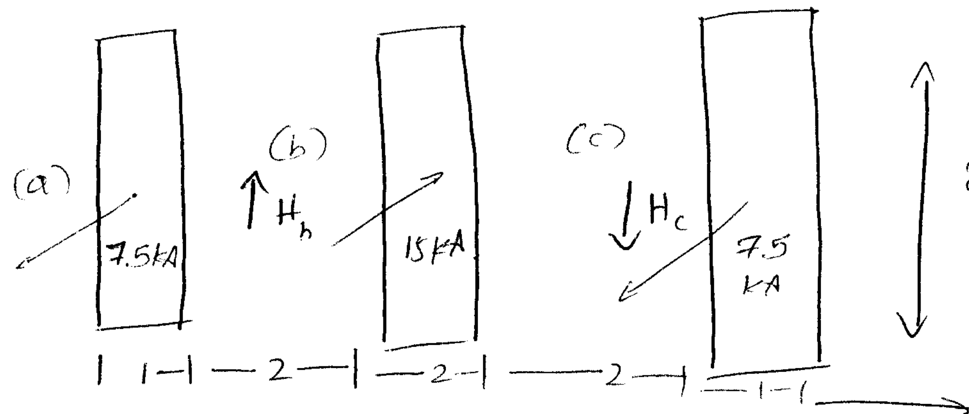
For region C, $\vec{D} = \dfrac{3.22\times10^{-9}}{r}\hat{a}_r.$

$$\vec{E} = \frac{\vec{D}}{3.7\,\epsilon_0} = \frac{98.4}{r}\hat{a}_r.$$

$$V = \frac{\rho_L}{7.4\,\epsilon_0 \pi} \ln\left(\frac{c}{b}\right) = 39.9\text{V}.$$

$$c' = \frac{Q}{V} = \frac{2.025\times10^{-8}}{100} = 202.5\ \text{pF/m}.$$

(3)



Additional information. Not required for problem.

- Fields are parallel to the plates.
- Field is constant between the plates.

We know that $\oint H\,dl = I_{encl}$.

So, in region (a) $H = 0 \Rightarrow B = 0$.

The H's in (b) & (c) are in opposite directions. & equal in magnitude.

$$|H_1| = |H_3| = \frac{7500}{25\,cm} = 30000\ A/m.$$

$$|H_2| = |2H_1| = 60{,}000\ A/m.$$

$B = \mu_0 H \Rightarrow B_1 = 0.0377/_2\,T$

$B_2 = 0.0754/_2\,T$

$B_3 = +0.0377/_2\,T.$

Single sheets of charge/current use $I/2$.

$B_A = -B_1 - B_3 + B_2 = 0\,T.$

$B_B = -B_C = B_1 - B_3 + B_2 = \frac{0.0754}{2} = 0.0377\,T.$

$B_C = -0.0377\,T.$

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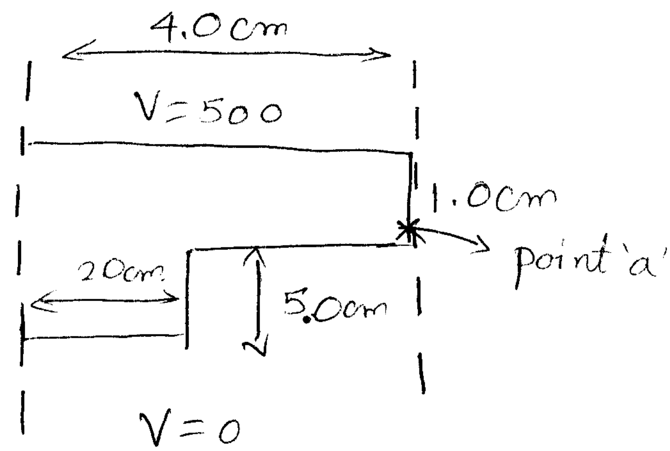

The idea is to use excel/spreadsheet method to solve the Laplace equation.

Using a uniform scale on the excel spreadsheet, say 1 mm = 1 cell, form the grid.

We assume symmetry & hence for the boundary, the value should be coded as $(V_{top} + V_{bottom} + 2 \times V_{left})$ for point 'a' & all points like point 'a'. (see fig.)

The flux lines should look like,