Fields and Waves I

Name _______________ ECSE-2100 Fall 1999 Section ____________

Preparation Assignments

Due at the start of class.

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

Due 1 September (2 points) Lesson 1.2 & Lesson 1.3 Problem 4

1. $\mathbf{A} = 3x\,\mathbf{a_x} + 2\,\mathbf{a_y} + (4xy + z^2)\,\mathbf{a_z}$ and $\mathbf{B} = 4\,\mathbf{a_x} - 2z\,\mathbf{a_y}$

   What is **A•B?**

   What is the unit vector in the direction of **B**? (This one was left off in class.)

2. What is the differential volume element in spherical coordinates?

Due 3 September (2 points) Lesson 1.3 & Lesson 1.4 Problem 1

$\mathbf{A} = 5x^2\,\mathbf{a_x} + 3y\,\mathbf{a_y} + (5y + z^2)\,\mathbf{a_z}$

1. What is the line integral $\int \vec{A} \bullet dl$ along the path from the point (1,2,3) to the point (1,4,3)?
2. What is $\nabla \times \vec{A}$?

Due 8 September (2 points) Lesson 1.4 & Lesson 2.1

Using the same expression for **A** as in the previous assignment,

1. What is $\nabla \bullet \vec{A}$?
2. If $\int \nabla \cdot \vec{D} dv$ integrated over some volume is equal to 7 coulombs, what is the value of $\oint \vec{D} \cdot d\vec{s}$ integrated over the surface of the volume?

Class time 10 September

Open shop to work on Homework 1. Due at 5 pm on 10 September.

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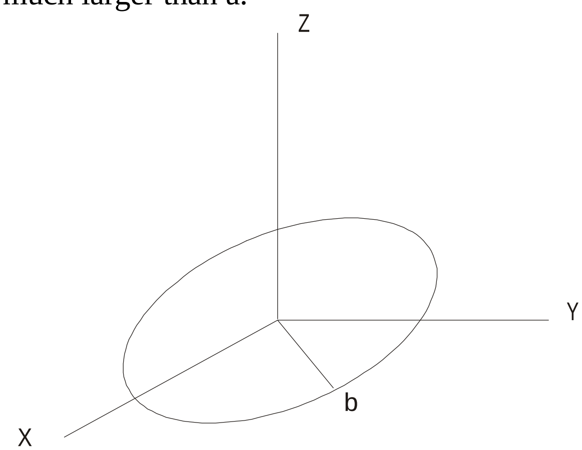
Homework #1

Revised To Correct Typo And To Add Figures

Problem 1 – (10 points)

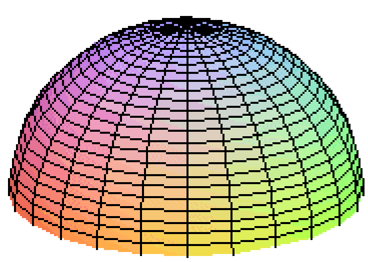
The magnetic vector potential from a small current loop is given approximately by $\vec{A} = \frac{cI\sin\theta}{r^2} a_\phi$ where $c = \mu_o a^2/4$, a is the radius of the loop and μ_o is a constant characteristic of free space, called the permeability.

a. Calculate the integral $\oint \vec{A} \cdot d\vec{l}$ along the circular path of radius b lying in the x-y plane (z=0). Here b is very much larger than a.



b. Determine $\nabla \times \vec{A}$ at all positions in space. This will be the expression for the magnetic field $\vec{B} = \nabla \times \vec{A}$. However, like the vector potential, this expression will only be approximately correct.

c. Calculate the surface integral $\int \nabla \times \vec{A} \cdot d\vec{s}$ over the surface of the hemisphere of radius b in the region z > 0 whose edge is given by the circle of radius b we used in part a. This surface looks like the top half of a ball resting on the x-y plane, as shown below. Compare your answers to parts a and c and explain how they relate to Stoke's Theorem.



Problem 2 – (10 points)

Consider a sphere of radius a, composed of an electret material. This sphere has a permanent electric field outside, $\vec{E}_o = \frac{k}{r^3}(2\cos\theta)a_r + \frac{k}{r^3}(\sin\theta)a_\theta$, where k = $(P_o a^3)/3\varepsilon_o$ and a field inside, $\vec{E}_i = k_2(\cos\theta)a_r - k_2(\sin\theta)a_\theta$, where $k_2 = -P_o/3$. That is, the first expression holds for the region r < a while the second holds for the region r > a.

(Spheres such as this, with a radius of a few microns, are used in the pharmaceutical industry to aid in chemical separation. P_o and ε_o are constants.

a. Find the divergence of the electric field inside and outside of the sphere.

b. Prove that these electric fields are conservative. (Check the book for the definition of conservative fields and select any reasonable path for your integral.)

c. Show that the field outside the sphere can be derived from the scalar function (called the electric scalar potential) $V = k\frac{\cos\theta}{r^2}$ using the relation $\vec{E} = -\nabla V$.

d. Did you see any similarities between the magnetic field seen in problem 1 and the electric field in problem 2?