Fields and Waves I

Name _______________ ECSE-2100 Fall 1999 Section ____________

Preparation Assignments for Homework #2

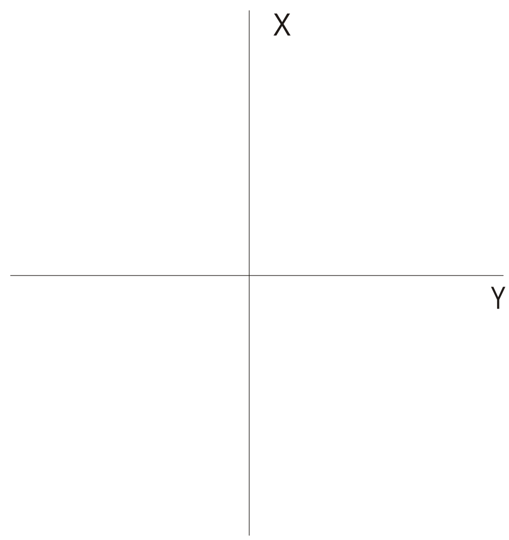
Due at the start of class.

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

Due 13 September (2 points) Lesson 2.2

1. One of the fundamental source distributions we will be working with in this course is the infinite sheet of charge. Assume that there is such a sheet located in the plane x = 0. Assume also that the surface charge density $\rho_s = \rho_{so}$ is a constant everywhere in the plane. Shown below is a two dimensional plot of the surface charge and its nearby region. Show where the sheet charge is located and then draw a few representative E field lines on this plot. Also write the vector expression for E in each region (x > 0 and x < 0).



2. State Gauss' Law for electric fields in your own words.

Due 15 September (2 points) Lesson 2.3

1. Assume that the studio classroom is a parallel plate capacitor with one plate on the ceiling and one on the floor. If the electric field in the room is vertical and approximately equal to 100 volts per meter, what will the voltage be on the floor and the ceiling?

2. I a point charge has a charge Q = 1 coulomb, how far away from it must you be to experience a voltage of 1 volt?

Class time 17 September

Open shop to work on Homework 2. Due at 5 pm on 17 September.

Homework #2

Problem 1 – (10 points)

A charge distribution consists of a point charge +Q at the origin and a volume charge density

$$\rho_v = (-Qr/\pi a^4) \text{ for } r < a \text{ and } \rho_v = 0 \text{ for } r > a.$$

where r is the distance from the origin.

a) From the symmetry of the problem, identify the appropriate coordinate system, the direction of the electric field and the coordinates the field depends upon.

b) Find the electric field at all locations (both r > a and r < a) using Gauss' law. Be sure to identify or show the integrating surface that you use.

c) Using your answer for the electric field, find the voltage everywhere. (set $V(\infty) = 0$)

d) Check your answer using Poisson's equation, $\nabla^2 V = -\rho_v/\varepsilon$

e) Draw the geometry of this problem. On your drawing, include a small number of electric field lines and equipotentials (enough to be representative).

f) Plot the magnitude of the electric field and the voltage as a function of radius. (You should have a figure of the form shown in figure (b) for example 3.12.

Problem 2 – (10 points)

If we knew exactly where all the charges are in some electrostatic configuration, it is relatively straight forward, if possibly tedious, to determine the resulting field. All we need to is to find E or V from Coulomb's law for each charge and then add up all the contributions. Computers allow us to actually approach problem solving in this manner, since they can free us from much of the work involved in this very general approach to evaluating fields. Still, it is difficult to know everything about the charges before we know everything about the fields. Many times, however, it is possible to guess a good deal about the charges before we try to figure out the fields.

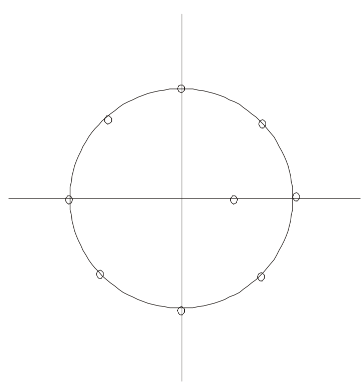
One of the counterintuitive results we can obtain from Gauss' law is the following. If we have either a sphere or a cylinder of uniform charge with a concentric empty space at its center, the electric field in the empty region will be zero. This is discussed on pages II-25 and II-26 of the notes and in examples like 3.12 of the text. As a general rule, if there is sufficient symmetry in the problem to solve it using Gauss' law, then an empty region such as these will have no field in it. To see how this can be, we will use Coulomb's law to show that the electric field inside a spherical sheet of charge is zero. This is sufficient to demonstrate that the general principle holds, since any spherically symmetric charge distribution can be approximated, with arbitrarily good accuracy, using a series of such charge sheets. To be sure that we understand how this is done, we will begin by determining the equivalent surface charge density ρ_{so} that represents the charge in the region between r = a and r = 0.9a in the first problem above. How much charge is contained in this region?

Now, assume that this charge is distributed on the surface at r = a. What is the surface charge density ρ_{so}? Since we have not specified numerical values for either Q or a, your answer should be in terms of these parameters.

It will soon be easier to work with real numbers rather than parameters. Thus, we will need to choose some values for Q and a. Let a be 1 cm. Choose Q such that the maximum potential produced by the distribution in problem 1 is just 1 volt. You will need to use your answer to part c) of problem 1 for this purpose.

We will begin by crudely modeling the spherical shell of charge at r = a by six equally spaced point charges located at (a,0,0), (-a,0,0), (0,a,0), (0,-a,0), (0,0,a) and (0,0,-a). Then we want to determine either the electric field at some point inside the sphere or the potential at two arbitrary points. If we choose to evaluate the electric field, its magnitude should be found to be very small. It would be zero if we had a perfect model, but six charges is far from perfect. If there is no electric field in this region, then the potential must be a constant. Thus, the potential at some finite radius should be equal to the potential at the origin. At this point, it is up to you to choose which approach you prefer. The first approach requires evaluation at only one point, but the field you will be finding is a vector. The second approach requires evaluation at two points, but you will only be finding a scalar in each case. To be sure we are all doing about the same thing, evaluate the fields at the point x = a/2, y = z = 0 and the origin, if you choose the second approach. Begin by writing out the contributions to the electric field or potential from each point charge. Then evaluate your expressions by hand or use something like Maple. Remember that this is a three dimensional problem, so you will have to use expressions involving x, y, and z. Use the symmetries of the problem to simplify your work.

Now, we want to go one step better in resolution by having a charge every 45^{o} rather than



every 90^o. How many charges will there be? The figure above shows a single cross-section of the charges. To address this configuration, it is probably better to write a small program using Matlab rather than trying to do it all by hand. However, the approach you choose is up to you. You will be given some hints on how to do this during class. If you find that you are particularly good at this kind of thing, you can earn up to 5 extra points on this homework by finding the value of the field or the potential at several points in the region inside of the sphere. However, brute force techniques are not what we are looking for here. You will need to figure out how to have Matlab or Maple do the work for you.