Fields and Waves I

Name _______________ ECSE-2100 Fall 1999 Section <u>(The Only One)</u>

Preparation Assignments for Homework #3

Due at the start of class.

Reading Assignments

Please see the handouts for each lesson for the reading assignments.

8 October Lessons 3.3 and 3.4

1. What is the magnetic flux density B of a simple solenoid? Assume that the solenoid has N turns of wire carrying a current I. The area of the solenoid is S and the length is L. We have not gotten to magnetic materials yet, so assume that this is an air core solenoid.

2. What is the magnetic flux produced by this coil?

12 October Lesson 3.4

1. For the wire loop discussed in example 5.2, pick parameters such that the peak-to-peak voltage across the resistor is 0.02 volt.

2. For example B.1, assume that the magnetic field is comparable to that we experience from the earth here in Troy (assume it is all vertical) and that the two wires are railroad tracks and the sliding wires move with the speed of a typical train. What emf will be induced? Make any reasonable assumptions. Data on the Earth's Magnetic Field can be found at http://www.ngdc.noaa.gov/cgi-bin/seg/gmag/igrfpg.pl

Class time 13 October (Note the date – Wednesday)

Open shop to work on Homework 4. Due at 5 pm on 13 October.

Homework #4

Problem 1. Magnetic Field Calculation

An ion beam has a current density flowing in the +z direction given by

$$\vec{J} = J_o \left\{ 1 - \frac{r^2}{a^2} \right\} \hat{a}_z$$

in the cylindrical region r < a and a current density of zero elsewhere.

a. Find the magnetic field **H**(r,ϕ,z) of the beam everywhere in space.

b. Verify that your answer is correct using the differential form of Maxwell's equations.

c. Determine the total current in the beam.

d. Assume that the beam is now directed down the axis of a grounded conducting cylindrical tube of radius 2a and thickness 0.01a = a/100. Assume also that the entire beam current is returned to its source in this conducting cylinder. Write the mathematical expression for the current density vector in the conductor. (Be sure that you write your answer as a vector.)

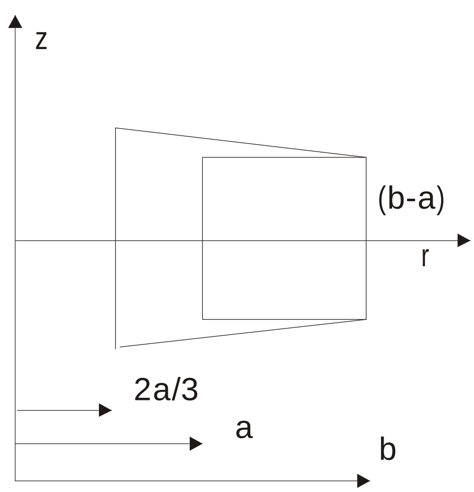
e. The addition of the conducting cylinder around the beam will leave the magnetic field of the beam at least partially unchanged. In which of the following regions will the **H** field be the same with and without the added cylinder? (Circle the correct answer(s).) Explain your answer.

(r < a) (a < r < 2a) (2a < r < 2.01a) (2.01a < r < ∞)

2. Air Core Toroid

Design an air core toroid using a 10 meter length of wire. You can use any standard gauge of wire. Below, assume that the wire diameter is d. Assume that the torus has a square cross-section and that the outer surface of the torus has either a single, double or triple layer of wire. Create the maximum possible magnetic field and dissipate less than 25 watts when you drive current in the wires. It is not necessary to come up with a perfect design, just come up with a consistent design. However, you must keep the power dissipation below 25 watts. You should look at Figure 4.17 in the text to see what this kind of geometry looks like.

The figure below shows some of the geometric issues associated with winding a torus. Assume that there is a single layer of wire at the outer surface (r = b). The inner surface (r = a) is smaller, so the wires will pile up into more than one layer. This is shown by the slanted lines above and below the square field region (also known as the air core, since our torus will not be wound on any magnetic material). All of the wires that form the outer layer must pass through the hole in the center of the torus. The inner radius (r = a) will be as small as possible when the wires more-or-less fill up this hole. If we assume they fill the hole, the average radius of the wire will be 2a/3. You should verify this by taking the average of the radius over the area of a circle. Thus, the trapezoidal shape surrounding the square field region is a typical path for a wire. The total number of turns you can get from a 10 meter long wire will be 10 divided by the length of this path.



An approximate expression for the average wire length is

$$length = 2(b - a + nd) + d + 2\sqrt{\left(b - a + \frac{a}{3} + nd\right)^2 + \left(\frac{d}{2}\right)^2}$$

where n is the number of layers at r = b and $d = \left(b - a + \frac{a}{3} + nd\right)\frac{bnd}{a(b-a)}$ is the additional length of the wire due to the build up of layers at r = a/3. While these expressions have many terms in them, they result from considering the simple geometry of the wire path. They are not perfect, but should be good enough for a simple design. If you want to improve on them, go ahead. However, it will probably take a lot more time than it is worth.

The wires will mostly fill the hole in the center of the torus, but practically it not possible to fill it more than about half full, because the windings are cylindrical and do not go in perfectly. Thus, you should assume for a torus with N windings that the condition that sets the inner radius a is $\frac{\pi a^2}{2} = N\frac{\pi d^2}{4}$ or that the total area of the wires (N times the area of each wire) fills about half the area of the hole. The outer radius can be determined from the number of layers $2\pi b = Nnd$, where again n is the number of layers at r = b. *Note: Be sure that you understand where these expressions come from. You might find it useful to sketch what the wires look like.*

The current in the coil is limited by the condition on power dissipation using the coil resistance (which you can calculate). The maximum magnetic field can then be determined from the coil geometry and the current.

Just so you don't have to try every possible condition to see what gives the maximum possible magnetic field, you will find below the parameters for a particular torus design. Your task is to improve on this design.

Example:

From http://hibp7.ecse.rpi.edu/~connor/education/ElecInst.html#wires solid 18 gauge wire has a diameter of approximately 1mm. Thus, using d = 0.001m, we find that a = 0.01m, b = 0.017m, N = 210, I = 10.8A, B = 0.045T for a coil with two layers at r = b.

You should be able to do better than this with a different wire gauge and possibly a different number of layers. Remember that you will be trying to obtain the maximum magnetic field, not the maximum *average* magnetic field.

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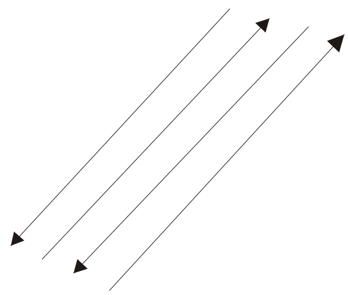
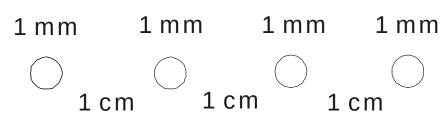
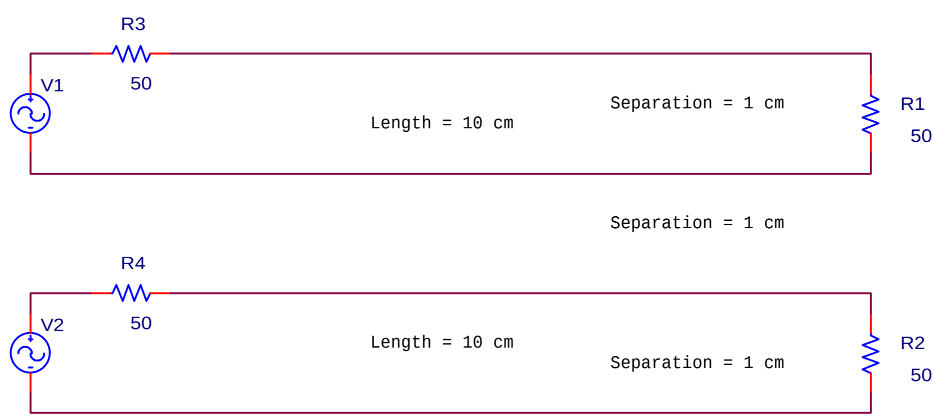
3. Faraday's Law

Assume that you have two parallel traces on a printed circuit board carrying a current of 10 mA at a frequency of 10 kHz. Nearby, you have two more parallel traces carrying a current of 50 mA at a frequency of 50 kHz. For simplicity, assume that the wires are cylinders with a diameter of 1mm and that the space between the wires is 1cm. The wires are 10cm long and are terminated in a resistance of 50 ohms. What voltage is observed at the 50 ohm resistor at the end of the first set of traces (the ones carrying the current of 10 mA)? You must take into account both the signal that is directly applied to these wires and the voltage induced by the current in the other set of traces







Hint: Let the wires be in the y = 0 plane and the coordinate along the wires be z. Then locate the first wire at x = 0, the second at x = a, the third at x = 2a and the fourth at x = 3a. You will have to figure out what the numerical value is for a. The vector area between the wires will then be in the y-direction. Before trying to evaluate the flux passing between the wires, convert the expressions for the magnetic field of each wire into rectangular coordinates.