

Homework 5
Due Thursday March 28, 2002

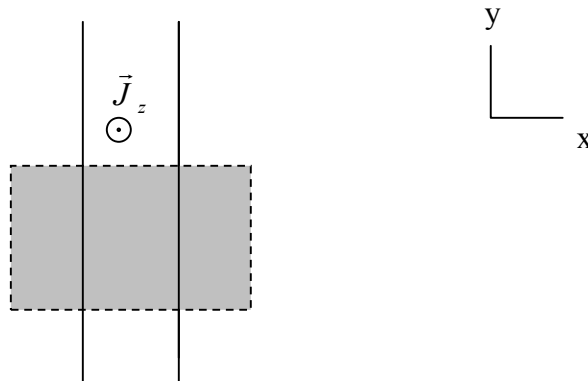
Ampere's Law:

The relationship between DC current and magnetic field is determined by Maxwell's Law $\nabla \times \vec{H} = \vec{J}$, the differential form of Ampere's Law. Integration over a surface and applying Stoke's Theorem, we can form the integral relationship, $\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{S}$.

The differential form can be used to determine the current density if we know the magnetic field. Also, it is useful for confirming solutions we obtain from the integral form. The integral form is used to determine the magnetic field when we know the current density. Similar to application of Gauss's Law, we can solve the integral equation analytically for only a very few geometries. These geometries have a high degree of symmetry so that we can determine that the magnetic field reduces to one coordinate direction.

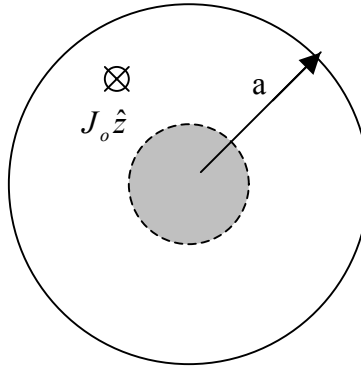
As we discussed in class, the following geometries are within our reach:

A current carrying slab (or surface current):



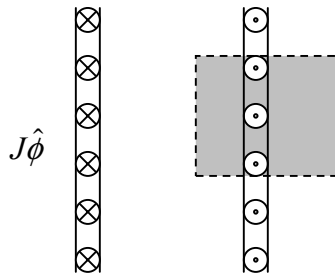
In this geometry, with the current in the z direction, the magnetic field to the left of the slab will be in the $-y$ direction and the field to the right will be in the $+y$ direction.

A current carrying cylinder (wires and transmission lines are applications of this geometry). The current is perpendicular to the plane of the paper. In other words, the figure represents a cross-section.



The current is distributed in the region $r < a$, and by symmetry, the field will be in the ϕ direction.

A solenoid. In practice, the current on the surface is carried by discrete wires which are wrapped around the device. Alternatively, we may represent the current as a density. The figure is a lengthwise cross section. In other words we have cut the solenoid on a full plane defined by $\phi = constant$. It is important to note that the current wraps around the solenoid, so the current direction on the left is out of the page and the current direction on the right is into the page.

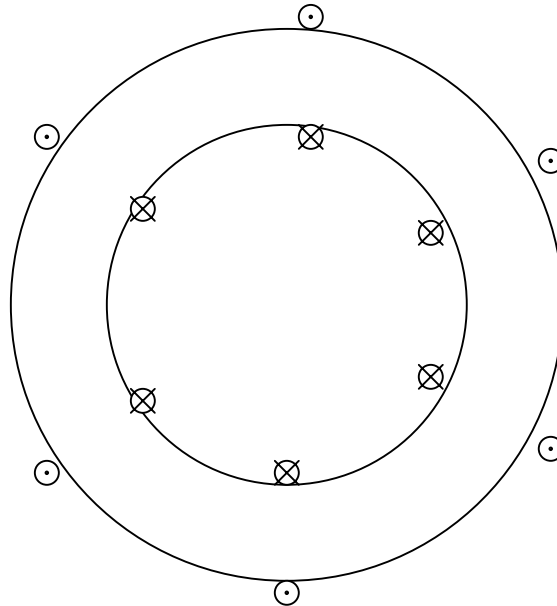


As in the example in class, the field is in the z direction and for long (infinite) solenoids is confined inside the solenoid.

Fields and Waves I

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A toroid. Again we can consider a wire wrapped toroid or and a current density on the surface. The figure represents an overhead view where we have cut the toroid in half. The wires wrap the solenoid, so the direction of the current on the inner radius is opposite of the direction on the outer radius.



As discussed in class, the field is along the axis of the toroid and confined within it.

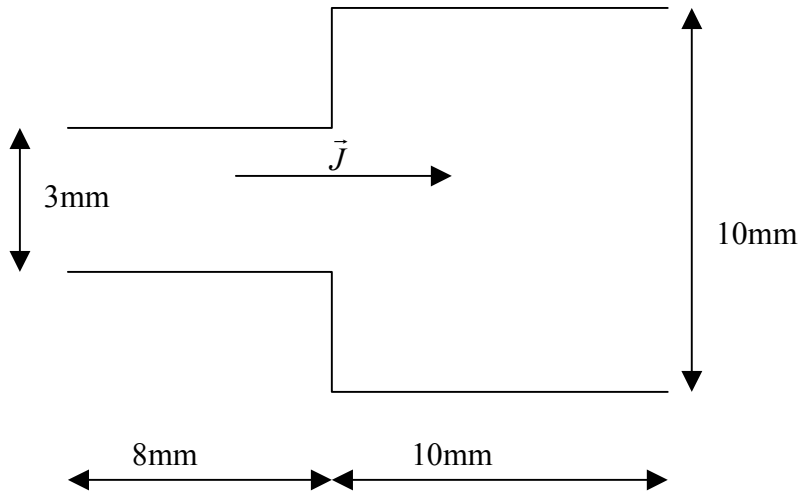
For each of the figures, we have included a shaded area that represents a surface used to apply Ampere's Law in integral form. The surface is perpendicular to the direction of the current and the shape is such that the line that bounds the surface is either perpendicular or parallel to the direction of the magnetic field lines. For example, in the slab of current, the field lines are in the y direction. On the top and bottom of the rectangle, the line that bounds the surface is perpendicular to the magnetic field. The left and right sides are parallel to the magnetic field. By symmetry, if we place those edges equidistant from the center of the slab, we can say that the magnitude of the field at those edges is the same. In the solenoid figure, we place the surface such that it only intersects the current once. Extending the surface to the left ($-r$ direction), would cause the total current through the surface to be zero and not useful.

Another important aspect to keep in mind is making sure your integration limits are consistent with both the surfaces you have chosen and the known magnitudes of current density. Since our surfaces are arbitrary, in the cylindrical wire figure, the current passes through the surface $0 < r < r$ where r is the radius of the circle. If the surface is larger than the radius of the wire, then current exists in $0 < r < a$.

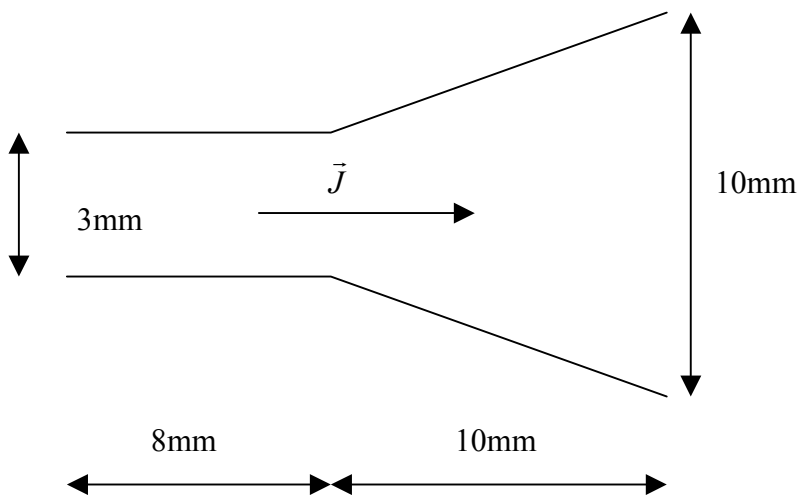
With the above figures in mind, the following steps are useful for solving Ampere's Law.

1. Determine the coordinate system
2. Using symmetry, determine the direction of the field in all regions of interest.
3. Define a surface perpendicular to the current.
4. Choose a shape of the surface such that the field lines are either perpendicular or parallel to the lines that bounds that shape.
5. Determine the current through that surface, $\iint_S \vec{J} \cdot d\vec{S}$.
6. Determine (all) $d\vec{l}$ that bound that surface.
7. Apply the closed loop integration, $\oint \vec{H} \cdot d\vec{l}$.
8. Equate steps 5 and 7 to determine the magnetic field. Be careful to include direction at this point.

1. Resistance



The figure represents a thin 1mm (dimension into the page) strip of copper connector with current in the direction shown. Under ideal conditions, what is the resistance of the geometry? Would you consider this resistance to be larger or smaller than the actual effective resistance of this connection? Is there a significant difference? Justify your reasoning. It is a good idea to draw the current lines.



Repeat the above problem with this figure.

2. Ampere's Law

A current density is defined as $\vec{J} = J_o(r - a)\hat{z}$ in the cylindrical region $a < r < b$ and zero elsewhere. Also, there is a surface current in the $-z$ direction at $r = b$ such that it is equal and opposite to the total inner current density.

What direction is the magnetic field?

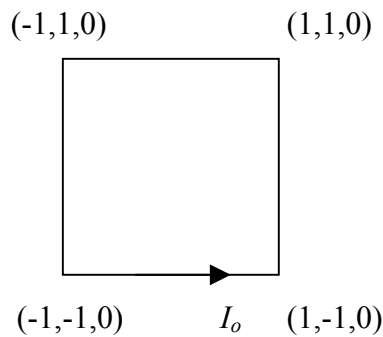
What is the magnitude of the surface current density?

Sketch the surface you would use to apply Ampere's Law.

What are \vec{H} and \vec{B} everywhere?

What is the total flux passing through the surface $z = \text{constant}$ for $r < b$?

3. Biot Savart Law



A square loop with sides of length 2mm is shown in the figure. (The coordinates have been scaled to mm). If the current through the loop is I_o , what is the direction and magnitude of the field at $(2, 0, 0)$?

4. Inductance

The side view of a toroid with 500 turns is shown in the figure. The cross-section of the toroid has radius a such that $a \ll R$. (This is a *thin* torus, like a bicycle tire.)

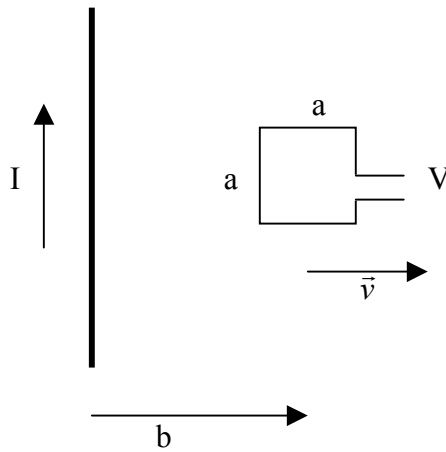


Approximately, what is the magnetic field in the toroid if the wire carries current I_0 ? The current flows counter-clockwise around the right cross-section (you should recognize that the current in the left cross-section will therefore flow clockwise).

What is the total flux through the cross-section?

What is the self-inductance?

5. Faraday's Law



A square loop is moving away from a current carrying wire with a constant velocity. At the time, $t = 0$, the center of the loop is at the location $r = b$. What is the voltage measured for $t > 0$? You may use Maple for this problem.