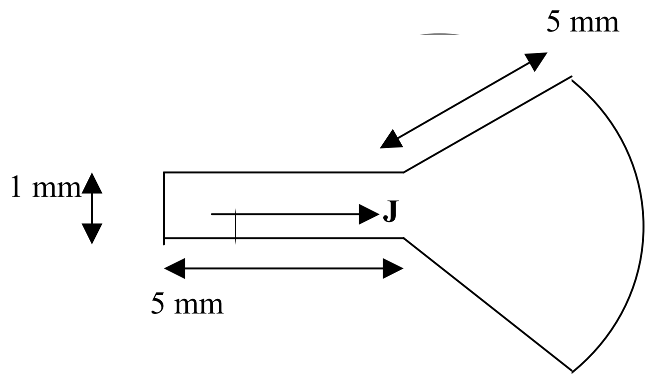
Fields and Waves I

Name _______________ ECSE-2100 Spring 2003 Section ____________

Homework 5

Due Thursday 14, March, 2003

1) Resistance

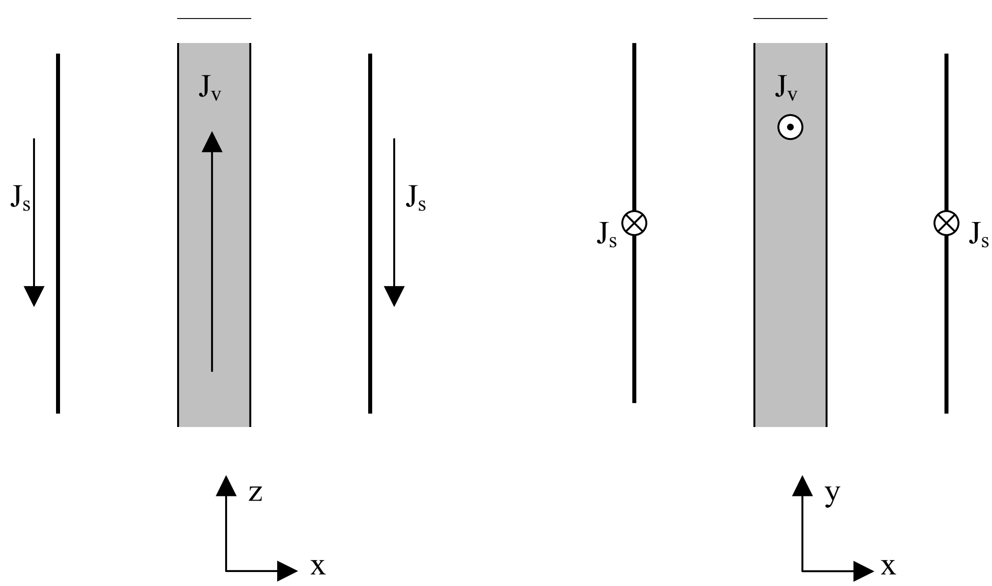


The above figure represents a copper conductor that is comprised of a short straight section and a flare with a 75° angle. There is a thickness of 0.5 [mm] into the paper. Approximate the resistance of this geometry with current flowing in the direction indicated. What assumptions are sources of error for your estimate?

2) Ampere's Law

The following current densities exist in a Cartesian coordinate system, $a < b$,

$$\vec{J} = \begin{cases} \vec{J}_s = -J_{so}\ \hat{z} & [A/m] & x = -b \\ \vec{J}_v = J_o x^2\ \hat{z} & [A/m^2] & -a < x < a \\ \vec{J}_s = -J_{so}\ \hat{z} & [A/m] & x = b \end{cases}$$



(Notice, there are two figures above. They are different viewpoints, one is the *xz* plane and the other is the *xy* plane.)

A current density exists in the region $-a < x < a$ and a surface current density exists at the location $x = -b$ and $x = b$. Furthermore, the current densities are given such that the total current in the domain is zero. We often using the phrase, "the current is equal and opposite", meaning $I_{vol} = -I_{surf}$, in this geometry. Capital 'I' indicates the total current (per unit length) for that distribution. You may assume the entire domain is free space.

1) In terms of geometry and the known expression for the current density, $\vec{J}_v$, determine an expression for the surface current, $\vec{J}_v$. (Determine what $\vec{J}_v$, must equal.)

2) What are the regions of the problem? (Indicate the regions where you will find the field)

3) What coordinate system applies to this problem (this better be easy).

4) What direction are the field lines? (just provide the coordinate direction, not the sign).

5) Given the answer to part 3, what shape would be appropriate for an Ampere's loop? Your loop should be constructed such that the sides are either parallel or perpendicular to the field line. Make your loop closed, not infinite in length. What is $d\vec{S}$ for the sides of that loop? What is $d\vec{S}$ for the surface defined by those lines?

6) Set up, (do not solve) the integral, $\oint_l \vec{H} \bullet d\vec{l}$, for the closed loop that you would use to solve the Ampere's Law problem. Include limits, the $d\vec{l}$, and the result from the dot product. (Your equation should have 4 integrals, two of which are trivial.)

Set up the integral(s) for current passing through the surface as well, $\oint_S \vec{J} \bullet d\vec{S}$. Depending on your region, you need to be careful. A surface current is a "special" type of current, requiring a different integral than the surface integral.

7) For each region, determine the magnetic field, $\vec{H}$. Pay attention to what currents are passing through your surface. When you have finished determining the field, your solution should include all information: the field in that region, the units, and the specification of the region (similar to the format of the charge distribution given at the top of the previous page).

8) What can you say about the magnetic field when you cross the surface at $x = b$?

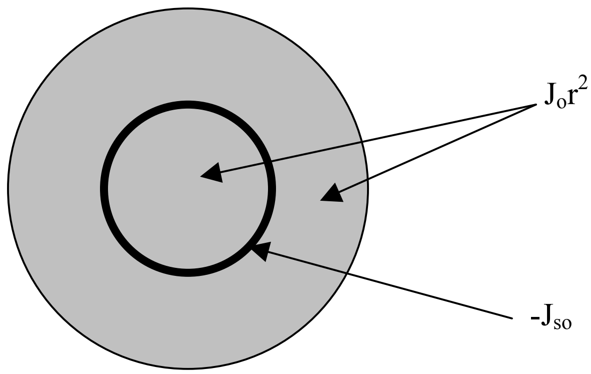
9) Verify your solution is correct for each region by using the differential form of Maxwell's Laws as applied to electrostatics, $\nabla \times \vec{H} = \vec{J}$ and $\nabla \times \vec{B} = 0$.
(Alternatively, $\nabla \times \vec{B} = \mu \vec{J}$ and $\nabla \bullet \vec{H} = 0$)

10) What is the total flux, Ψ, per unit length passing through the *xy* plane?

3) Ampere's Law

The following current exist in a *cylindrical* region, where $a < b$

$$\vec{J} = \begin{cases} J_o r^2 \hat{z} & [A/m^2] \quad r < b \\ -J_{so} \hat{z} & [A/m] \quad r = a \end{cases}$$



Repeat questions 1-9) of the previous problem.

10) What is the total flux, Ψ, per unit length in this geometry?