Fields and Waves 1

Quiz 3

Fall 2002

1. (20) ________________

2. (5) ________________

3. (5) ________________

4. (5) ________________

5. (25) ________________

6. (25) ________________

7. (15) ________________

Total ________________

Name ________________(2)

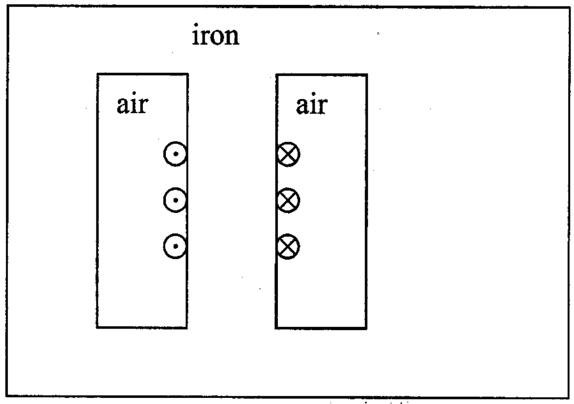
Notes:

1) In the Multiple Choices section, each question may have more than one correct answer. Circle all of them.
2) For the Multiple Choice questions, you may add some comments to justify your answer.
3) Make sure your calculator is set to perform trigonometric functions in radians, not degrees.
4) All solutions should include the units. Points will be deducted if they are missing.
5) If you are stuck on part of a problem, use general expressions for the following sections. To earn the maximum credit, include all information that you know ($d\vec{l}$, $d\vec{S}$, limits, etc.)

Fields and Waves I

Name _______________ ECSE-2100 Fall 2002 Section ___________

1) Multiple Choice (20)-Circle all correct answers



i) The above figure is a transformer (engineering, not cartoon) with current carrying wires wrapped around the middle section. The geometry has some finite thickness, *t*, into the paper:

a. More field lines will be looping through the right section than the left section

b. If the right side cracks and a significant gap is introduced on the right section, there will more field lines through the right section than the left section.

c. The reluctance of the left and right sections are approximately in parallel, $\Re = \left(\dfrac{1}{\Re_{left}} + \dfrac{1}{\Re_{right}} \right)^{-1}$

d. The reluctance of the left and right sections are approximately in series, $\Re = \Re_{left} + \Re_{right}$

ii) For a current carrying wire with uniform current density:

a. $\nabla \bullet \vec{B} = 0$ inside the wire.

b. $\nabla \bullet \vec{B} = 0$ outside the wire.

c. $\nabla \times \vec{H} = 0$ inside the wire.

d. $\nabla \times \vec{H} = 0$ outside the wire.

iii) The self-inductance of an air-filled geometry:

a. Decreases as the applied current increases.

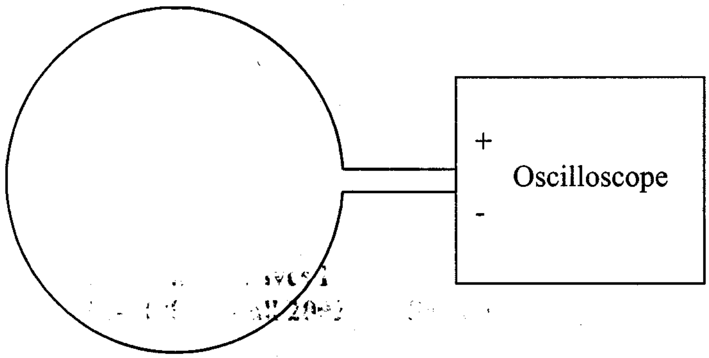
b. Increases significantly when iron is added inside the loops.

c. Increases significantly when copper is added inside the loops.

d. Increases significantly when plastic is added inside the loops.

iv) Hysteresis:

a. Describes the nonlinear relationship between $\vec{H}$ and $\vec{B}$.
b. Describes the nonlinear relationship between $\vec{D}$ and $\vec{E}$.
c. Applies to ferromagnetic materials.
d. Applies to paramagnetic materials
e. Describes my experience taking Fields and Waves tests.



$\otimes$

B field into the paper – uniform field, but increasing with time

v) In the above figure, a circular loop is oriented such that it is perpendicular to an externally applied uniform magnetic field. The magnitude of the field is increasing with time:

a. The current induced in the loop will be in the clockwise direction.
b. The current induced in the loop will be in the counterclockwise direction.
c. The oscilloscope will measure a positive voltage.
d. The oscilloscope will measure a negative voltage.

vi) Which of the following are true:

a. Maxwell's equations describe the proper technique to brew coffee.
b. Gauss's Law is what Southerners use to choose answers on a T/F test.
c. A high mu material is a reference to cow feed.

Fields and Waves I

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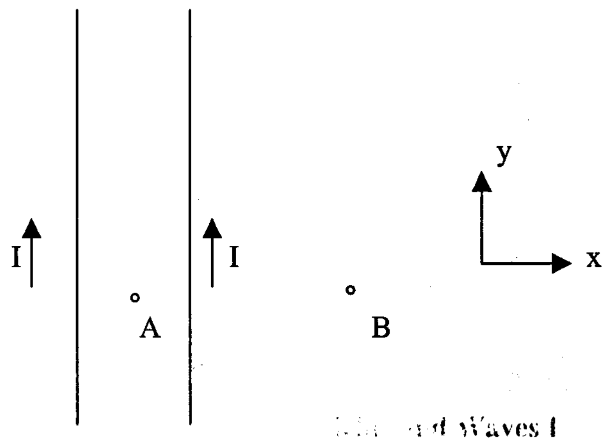
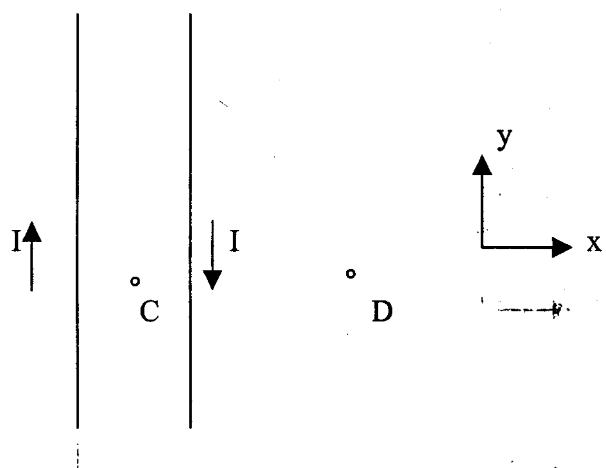
Field Direction (aka Right Hand Rule) (5)

For the following three geometries, indicate the direction of the field at the points indicated. Include the sign. If the field is zero, indicate that. All points lie in the same plane as the wires or loops.



Point A (midway between the two wires) ____ 0 ____ (1)

Point B ____ $-\hat{z}$ ____ (1)



Point C (midway between the two wires) ____ $-\hat{z}$ ____ (1)

Point D ____ $\hat{z}$ ____ (1)

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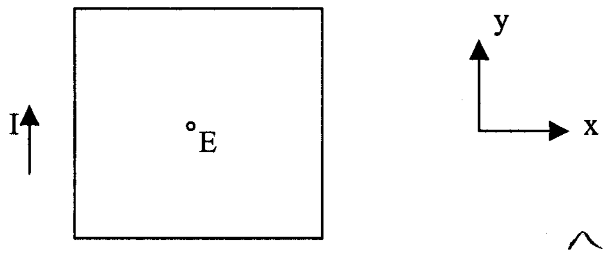
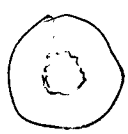


Point E (exact center of square loop of current) $-\hat{z}$ (1)

Simple Ampere's Law (5)

A current carrying wire has a uniform current density J_o and radius a. Determine the magnetic field, $\vec{H}$, inside the wire. Draw the physical geometry and the surface you would use to apply Ampere's Law.



$$\int \bar{H}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{S}$$

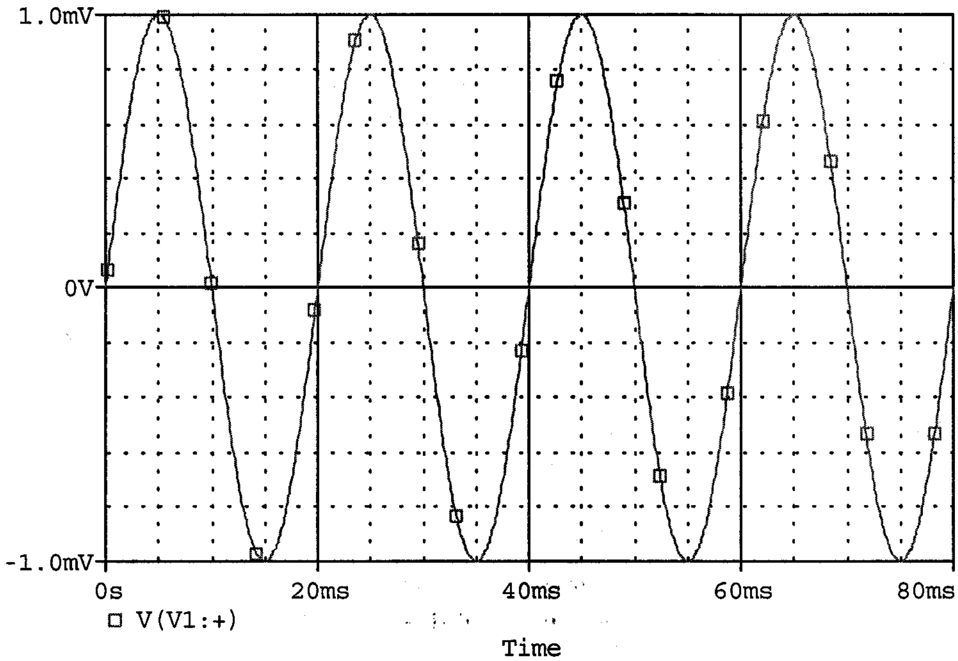
$$\cancel{2\pi r H_\phi =}$$

$$\int_0^{2\pi} H_\phi r\, d\phi = \int_0^{2\pi}\int_0^{r} J_0\, r\, dr\, d\phi$$

$$2\pi r H_\phi = \pi r^2 J_0$$

$$H_\phi = \frac{J_0 r}{2}\hat{\phi} \quad [A/m]$$

Measurements (5)



The above figure is the measured EMF of a loop spinning in a 3E-7 [Wb/m^2] uniform magnetic field. The loop is oriented such that it will couple maximum flux. What is the surface area of the loop? (5)

for a loop

$$EMF = \frac{-d}{d\cancel{t}}\left[B\,(\text{area})\cos\omega t\right]$$

$$= B\,(\text{area})\,\omega\,\underset{\cancel{\cos}}{\sin}\,\omega t$$

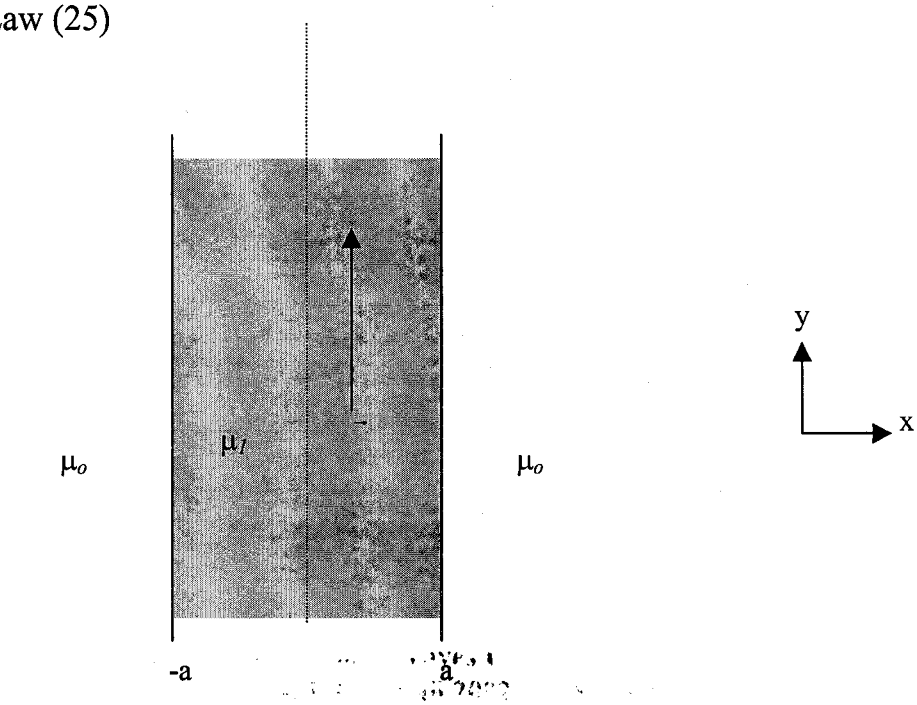
$$B\,(\text{area})\,\omega = 1\,mV$$

$$\omega = 2\pi f \qquad f = 50\ Hz = \frac{1}{0.02\,s}$$

$$= 314$$

$$\text{area} = \frac{0.001}{(314)(3\times10^{-7})}$$

Ampere's Law (25)



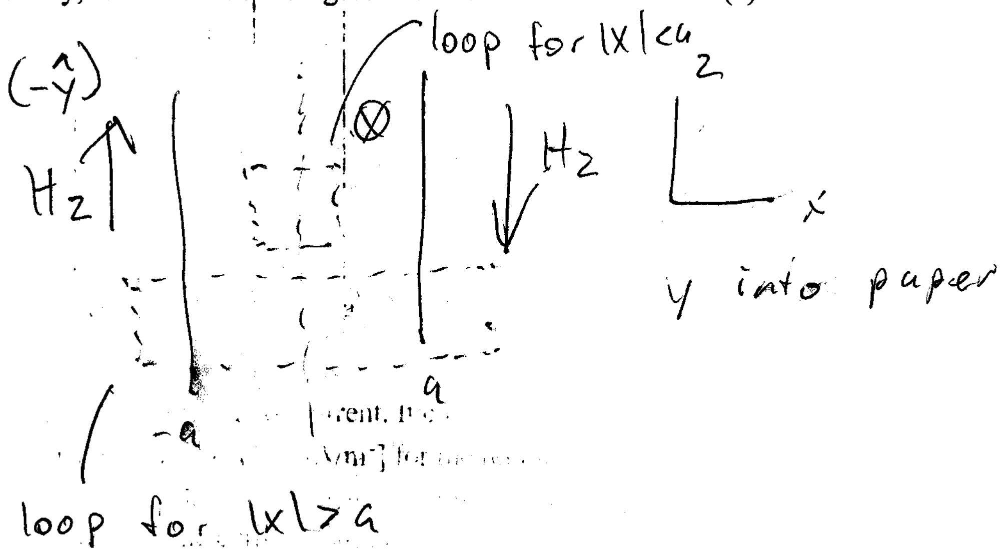
The above figure represents an infinite slab of current. It continues in the *y*-direction and *z*-direction. The current density is $\vec{J} = J_o x^2\,\hat{y}$ [A/m^2] for the region $-a < x < a$ and zero elsewhere. Additionally, the current density is located within a magnetic material of permeability μ_1. The region outside the current density is free space. When applying Ampere's Law to determine the magnetic field in this problem, consider what symmetry applies.

Draw the surface you would use to apply Ampere's Law. Indicate the $d\vec{S}$ and the $d\vec{l}$ (s) for this surface. If necessary, redraw the above figure with a different orientation. (5)

$d\bar{s} = dx\,dz\,(-\hat{y})$

$d\bar{l}_3 = d\bar{l}_1 = dx\,\hat{x}$

$d\bar{l}_4 = d\bar{l}_2 = dz\,\hat{z}$



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Determine the magnetic field, $\vec{H}$, everywhere. (10)

$|H_z|$ has symmetry about $x=0$ plane, the sign changes

$|x|<a$

$$\oint \bar{H}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{S}$$

$$2Hl = -\int_0^l \int_{-x}^{x} J_0 x^2 \, dx\, dz$$

$$= 2 l J_0 \frac{x^3}{3}$$

$$H_z = -J_0 \frac{x^3}{3} \hat{z} \quad |x|<a \quad [A/m]$$

$|x|>a$

$$2Hl = -\int_0^l \int_{-a}^{a} J_0 x^2 \, dx\, dz$$

$$H_z = -J_0 \frac{a^3}{3} \hat{z} \quad \cancel{|x|>a \ [A/m]}$$

$$x>a \quad H_z = -J_0 \frac{a^3}{3} \hat{z} \quad [A/m]$$

$$x<a \quad H_z = +J_0 \frac{a^3}{3} \hat{z} \quad [A/m]$$

Determine the magnetic flux, $\vec{B}$, everywhere. (5)

$$B = -J_o \frac{\mu_o x^3}{3} \quad |x| < a \quad [Wb/m^2]$$

$$B = -J_o \frac{\mu_o a^3}{3} \quad x > a \quad [Wb/m^2]$$

$$B = J_o \frac{\mu_o a^3}{3} \quad x < a \quad [Wb/m^2]$$

Verify your solutions using the differential forms of both Maxwell's equations that apply to Magnetostatics. (5)

$\nabla \cdot B = 0$ by observation, function is variable of x but $\hat{z}$ directed

$|x| < a$

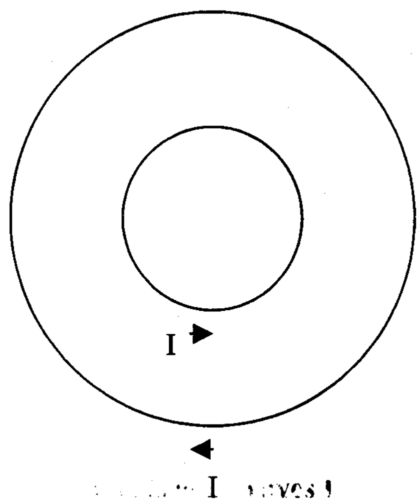
$$\nabla \times H = -\hat{y} \frac{\partial}{\partial x}\left(-J_o \frac{x^3}{3}\right)$$

$$= J_o x^2 \hat{y} \quad ✓$$

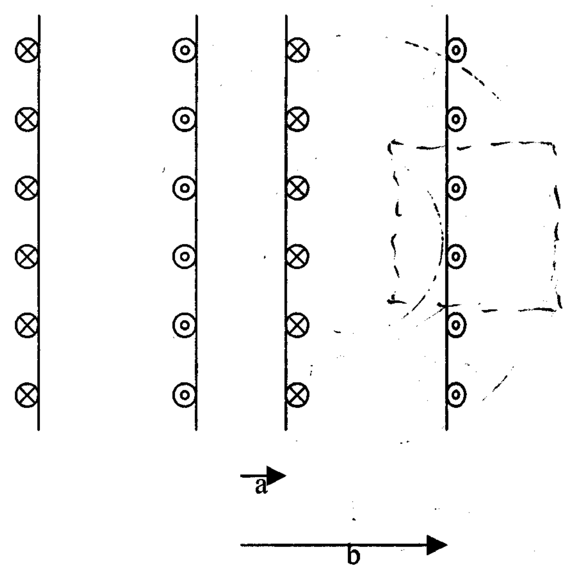
$|x| > a$

$$\nabla \times H = -\hat{y} \frac{\partial}{\partial x}\left(\pm J_o \frac{a^3}{3}\right) = 0 \quad ✓$$

Field, Inductance, and Energy (25)



Overhead cross-sectional view of two concentric wire wrapped solenoids
(positive z-direction out of the paper)



$d\bar{S} = r\,dr\,d\cancel{z}\,\hat{\phi}$

$d\bar{l}_1 = d\bar{l}_3 = dr\,\hat{r}$

z

$d\bar{l}_2 = d\bar{l}_4 = dz\,\hat{z}$

Lengthwise cross-sectional view of two concentric wire wrapped solenoids

⊗ - into the paper ⊙ - out of the paper

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On the previous page, the two figures represent different viewpoints of a two-solenoid configuration. The geometries are concentric (have the same origin) and the current in the outer solenoid is equal and opposite to the current of the inner solenoid. They both have a wire density of $n = \frac{N}{l}$ [wires/m] and the current in the wire is I [A].

On the previous page, draw the figure you would use to apply Ampere's Law (4).

on page

Determine the magnetic field, $\vec{H}$, in all regions. (6)

$r > b \qquad H \simeq 0$

$r < a \qquad H = 0$

$a < r < b$

$$\oint \bar{H} \cdot d\bar{l} = \int \bar{J} \cdot d\bar{S} = n\, l\, I$$

$$-H_z\, l = n\, l$$

$$H_z = -nI = \frac{-NI}{l}\,\hat{z} \quad [A/m]$$

Determine the total flux, Ψ, that exists in this geometry. (4)

$$B_z = -\frac{NI}{l}\mu_0\,\hat{z}$$

$$\int_0^{2\pi}\int_a^b -\frac{NI}{l}\mu\, r\, dr\, d\phi = \pi\,\frac{NI}{l}\mu\,(b^2 - a^2)$$

What is the inductance, L, of this geometry? (4)

$$L = \frac{\Sigma \Psi}{I} = \frac{\pi N^2}{l} \mu \left(b^2 - a^2\right) \quad [H]$$

Using the integral equation relating magnetic energy and field, what is the total energy, W_m, per unit length? (4)

$$W_m = \int_0^{l}\int_0^{2\pi}\int_a^{b} \left(\frac{N I}{l}\right)^2 \mu \; r \, dr \, d\phi \, dz$$

$$= \frac{N^2 I^2}{l^2} \mu \left(\frac{b^2 - a^2}{2}\right) 2\pi l$$

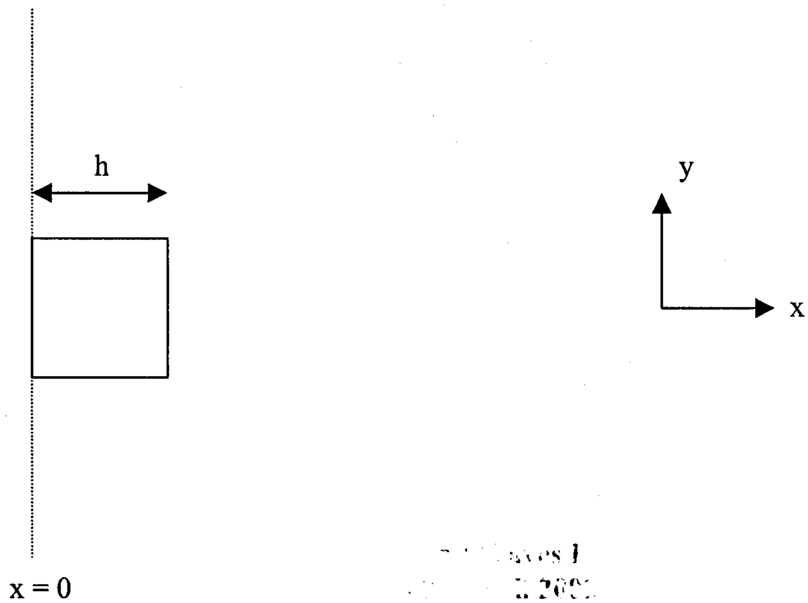
$$= \frac{\pi N^2 I^2}{2 l} \mu \left(b^2 - a^2\right) \quad [J]$$

Verify that this result is consistent with your inductance calculation. (3)

$$L = \frac{2 W_m}{I}$$

$$= \frac{\pi N^2}{l} \mu \left(b^2 - a^2\right) \quad [H] \quad \checkmark$$

Faraday's Law (15)



In the above figure, a square loop with sides of length, h, lies in the x-y plane. At time $t = 0$, the loop as at the location shown, with the left side on the y-axis. The magnetic flux is given as $\vec{B} = B_o x\ \hat{z}$ [W/m^2] (a linearly varying field). (You may consider that there is a small gap in the loop so that it will not be a short circuit.)

Determine the total flux through the loop at time, $t = 0$. (5)

$$\psi = \int_0^h \int_0^h B_o x \, dx \, dy$$

$$= B_o h \frac{h^2}{2} = B_o \frac{h^3}{2}$$

If the loop is given a constant velocity, $\vec{v} = v_o\ \hat{y}$, what is the EMF induced around the loop? (5)

Ψ = constant as a function of time

$$\frac{d\Psi}{dt} = 0 = EMF$$

If the loop is given a constant velocity, $\vec{v} = v_o\ \hat{x}$, what is the EMF induced around the loop? (5)

$$EMF = -\frac{d}{dt}\int \bar{B} \cdot d\bar{S}$$

$$= -\frac{d}{dt}\left[\int_0^h \int_{V_0 t}^{V_0 t + h} B_0 x \, dx \, dz\right]$$

$$= -\frac{d}{dt} B_0 h \left[\frac{(V_0 t + h)^2}{2} - \frac{(V_0 t)^2}{2}\right]$$

$$= -B_0 h \left[\cancel{2}(V_0 t + h) V_0 - V_0^2 t\right] \quad [V]$$

$$= -B_0 V_0 h^2$$