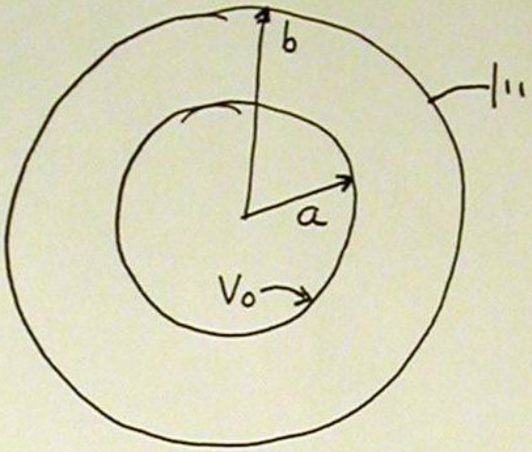
A spherical capacitor has inner sphere radius a and outer sphere radius b. The potential is given as $V = \frac{b-r}{b-a}\frac{a}{r}V_0$



Find $\vec{E}$.

$$E = -\nabla V = -\frac{\partial V}{\partial r}\hat{a}_r - \frac{1}{r}\cancel{\frac{dV}{d\theta}}^{\,0}\hat{a}_\theta - \frac{1}{r\sin\theta}\cancel{\frac{dV}{d\phi}}^{\,0}\hat{a}_\phi$$

$$= -\frac{\partial V}{\partial r}\hat{a}_r = \frac{ab}{b-a}\frac{V_0}{r^2}\hat{a}_r$$

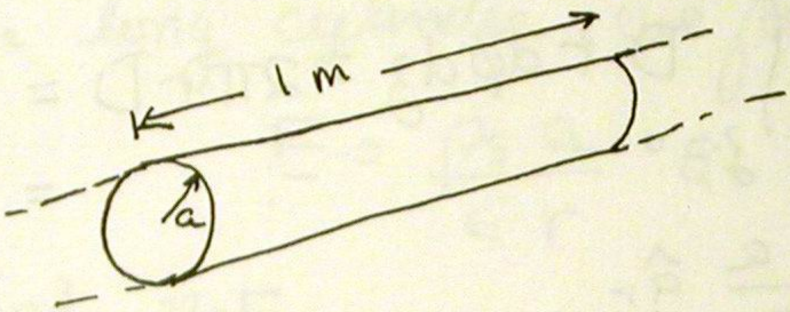
For the spherical capacitor problem, you found $E = \frac{ab}{b-a}\frac{1}{r^2}V_0\,\hat{a}_r$

Find $\nabla \times E$

$$\nabla \times E = \left\{ \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\cancelto{0}{E_\phi \sin\theta}\right) - \cancelto{0}{\frac{\partial E_\theta}{\partial\phi}}\right]\right\}\hat{a}_r$$

$$+ \left\{ \frac{1}{r\sin\theta}\cancelto{0}{\frac{\partial E_r}{\partial\phi}} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \cancelto{0}{E_\phi}\right)\right\}\hat{a}_\theta$$

$$+ \left\{ \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r \cancelto{0}{E_\theta}\right) - \cancelto{0}{\frac{\partial E_r}{\partial\theta}}\right]\right\}\hat{a}_\phi = 0$$



a cylindrical surface radius a and for 1 meter long, has linear charge $\rho_l = \rho_s 2\pi a$ C/m.

Find E.

$$\oint D \cdot dS = \int_{\phi=0}^{2\pi}\int_{z=0}^{1} D\, r\, d\phi\, dz = 2\pi r D = Q_{enclosed}$$
$$= \rho_s 2\pi a$$

$$D = \frac{\rho_s a}{r}\hat{a}_r$$

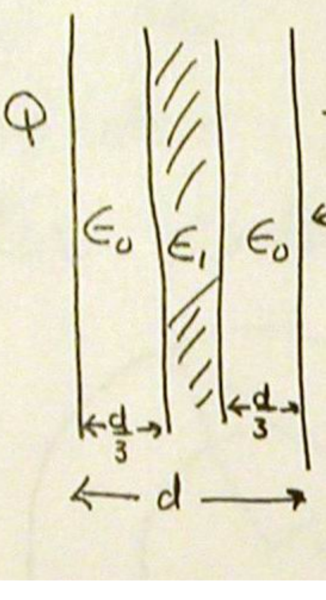
For the long cylinder, we found

$$E = \frac{\rho_s a}{\epsilon r}\,\hat{a}_r$$

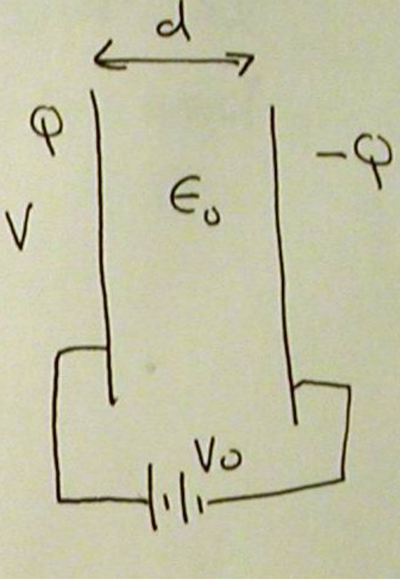
find $\nabla \cdot E$

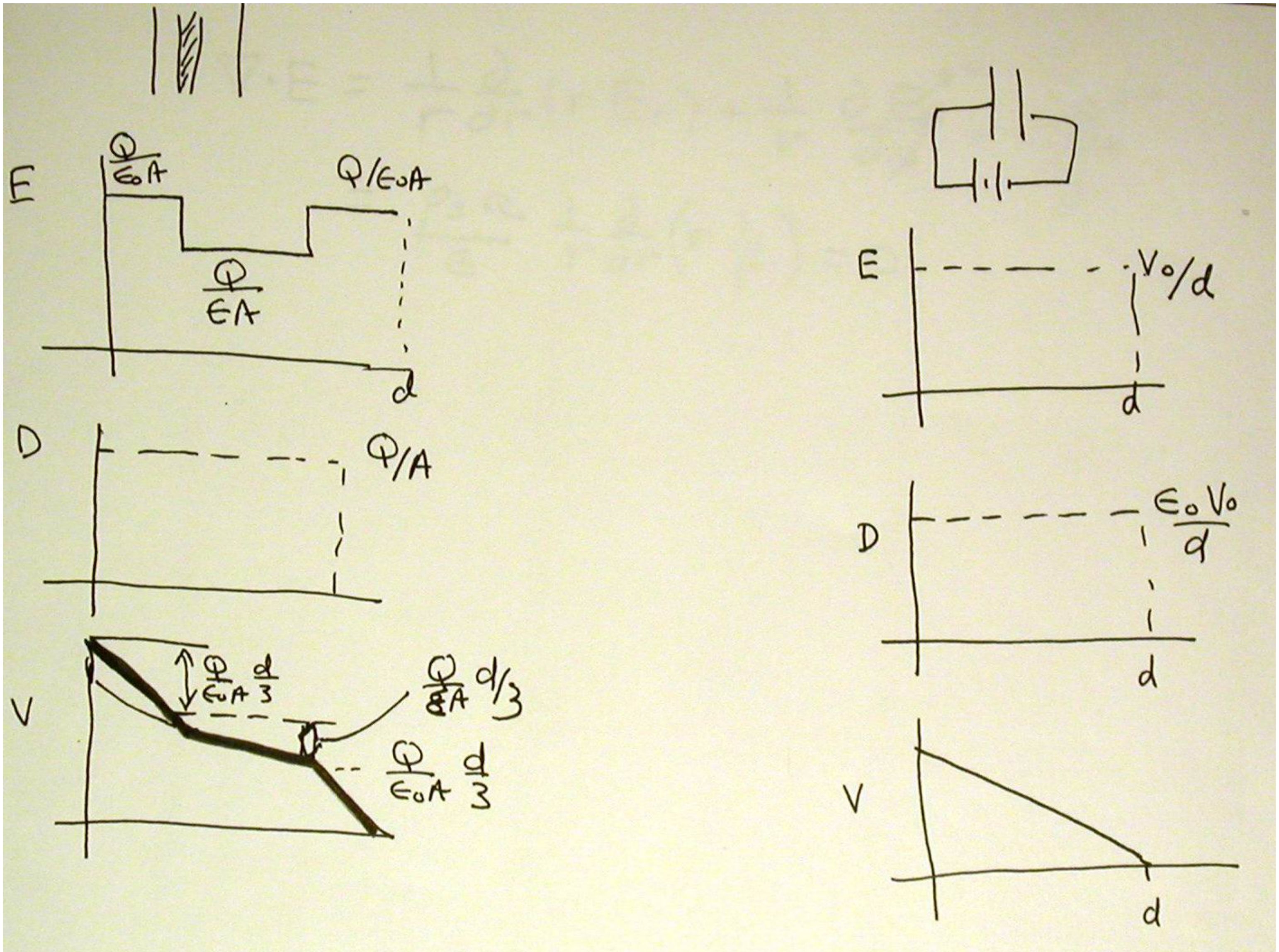
$$\nabla \cdot E = \frac{1}{r}\frac{\partial}{\partial r}(r E_r) + \frac{1}{r}\frac{\partial \cancelto{0}{E_\phi}}{\partial \phi} + \frac{\partial \cancelto{0}{E_z}}{\partial z}$$

$$= \frac{\rho_s a}{\epsilon}\,\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{1}{r}\right) = 0$$

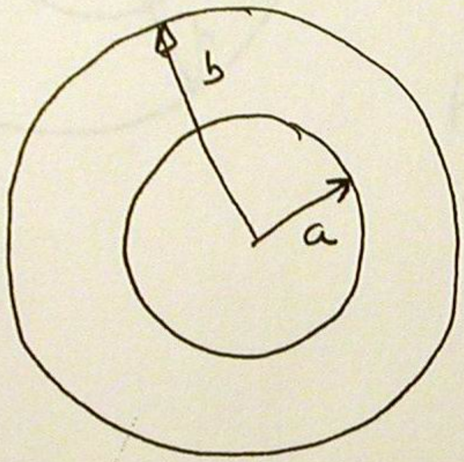


Find E, D, V for the 2 cases.




E
Q/ε₀A
Q/εA
d
D
Q/A
V
Q/ε₀A d/3
Q/εA d/3
Q/ε₀A d/3
E
V₀/d
d
D
ε₀V₀/d
d
V
d

Find the capacitance of a spherical capacitor with inner radius a and outer radius b.



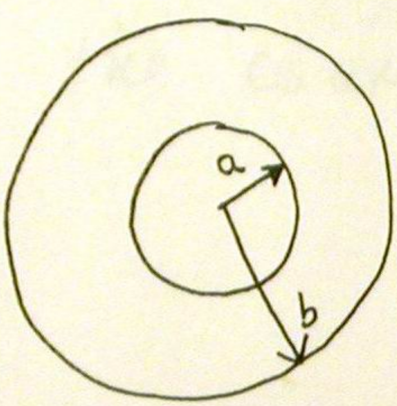
$$E = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

$$V = -\int_b^a E\cdot dl = -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r\cdot\hat{a}_r\,dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$$C = \frac{Q}{V} = 4\pi\epsilon_0\,\frac{ab}{b-a}$$

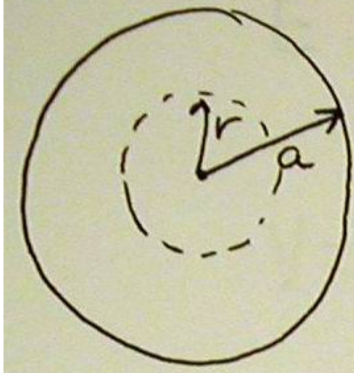


We have a coaxial conductor with uniform current in the inner conductor returning on the outer conductor.

Find B for $r<a$

$r>a,\ r<b$

$r>b$



$$\oint H \cdot dl = I_{enc}$$

$r < a$

$$H \cdot 2\pi r = I_0 \frac{\cancel{\pi} r^2}{\cancel{\pi} a^2}$$

$$H = \frac{I_0}{a^2} \frac{r}{2\pi} \hat{a}_\phi$$

$$B = \frac{\mu_0 I_0 r}{a^2 2\pi} \hat{a}_\phi$$

$$\int H \cdot dl = I_0$$

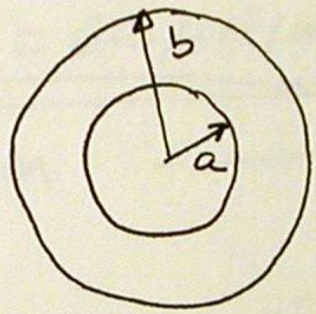
$r > a$

$$H \cdot 2\pi r = I$$

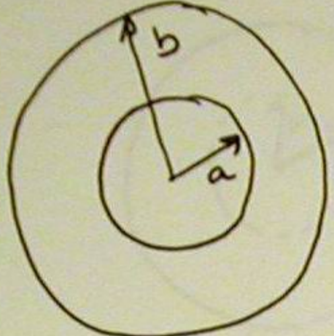
$$H = \frac{I}{2\pi r} \hat{a}_\phi$$

$$B = \frac{\mu_0 I}{2\pi r} \hat{a}_\phi$$

Find the inductance per unit length of the coax.



$a < r < b$



$$\Psi_m = \int B \cdot dS = \int_0^1 dz \int_a^b \frac{\mu_0 I}{2\pi r} dr$$

$$= \frac{\mu_0 I}{2\pi} \ln \frac{b}{a}$$

$r < a$

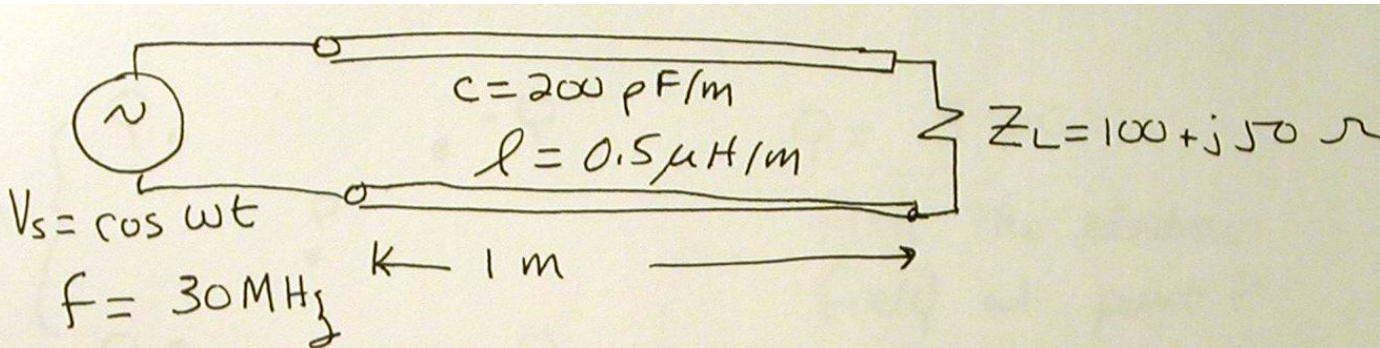
$$\Psi_m = \int B \cdot ds = \int_0^a \frac{\mu_0 I_0 r}{a^2 2\pi}$$

but! only the fraction $\frac{r^2}{a^2}$ is linked.

$$d\lambda = \frac{\mu_0 I}{2\pi a^4} r^3 dr$$

$$\lambda = \frac{\mu_0 I}{2\pi a^4} \int_0^a r^3 dr = \frac{\mu_0 I}{8\pi}$$

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} + \frac{\mu_0}{8\pi}$$



Find Z_0, v_p, electrical length, Γ_L, SWR

$$Z_0 = \sqrt{\ell / c} = 50\,\Omega$$

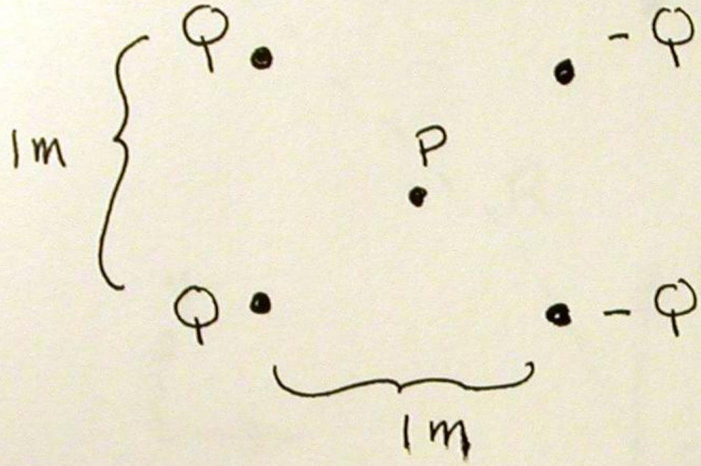
$$v_p = \frac{1}{\sqrt{\ell c}} = 100 \text{ m}/\mu\text{s}$$

electrical length $= 0.3\lambda$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

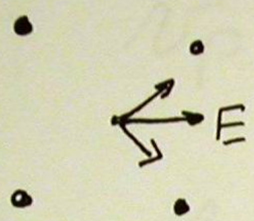
$$= \frac{100 + j50 - 50}{100 + j50 + 50} = .447\angle 26.6^\circ$$

$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + .477}{1 - .477} = 2.62$$



$Q = 3\ pC$

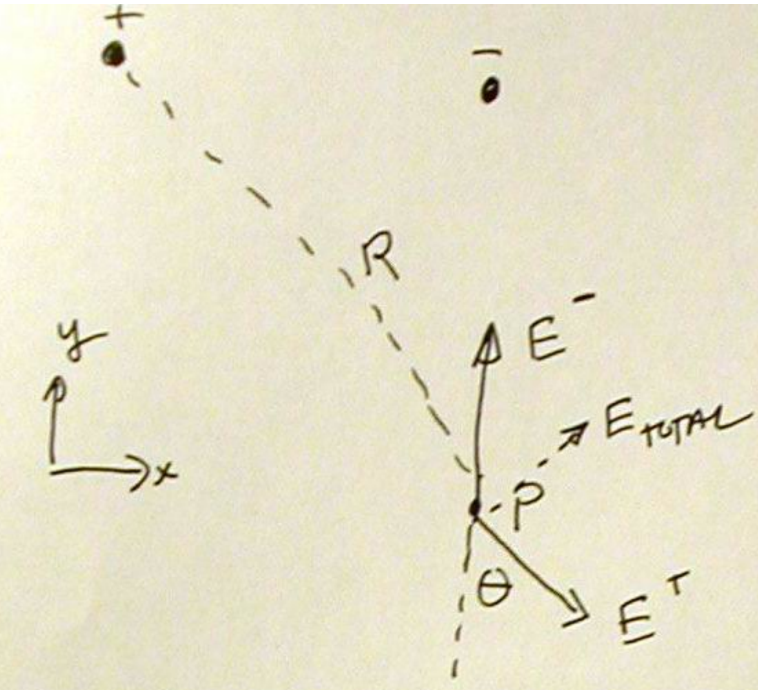
find the electric field at point P



$$E = 4\,\frac{Q}{4\pi\epsilon_0 r^2}\cos\theta$$

$$= \frac{4 \times 3 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12} \times 0.707^2} * 0.707$$

$$= .153\ V/m$$



$$E^- = \frac{\rho_l}{2\pi\epsilon_0}\cdot\frac{1}{4}\,\hat{a}_y$$

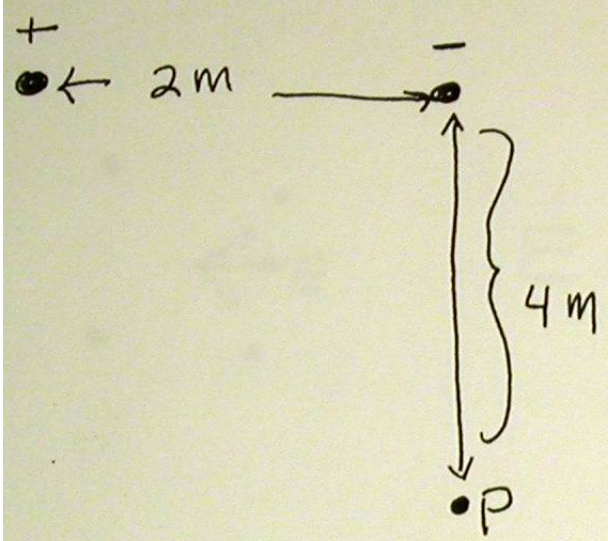
$$E^+ = \frac{\rho_l}{2\pi\epsilon_0}\left(-\frac{\cos\theta}{R}\hat{a}_y + \frac{\sin\theta}{R}\hat{a}_x\right.$$

$$\theta = \tan^{-1}\frac{2}{4} = 26.6^\circ$$

$$R = \sqrt{2^2+4^2} = 4.47$$

$$E_{TOTAL} = \underbrace{\frac{5\times10^{-6}}{2\pi\times8.85\times10^{-12}}}_{9\times10^{4}}\left[0.25\,\hat{a}_y - 0.224\overset{\cos\theta}{\hat{a}_y} + 0.224\overset{\sin\theta}{\hat{a}_x}\right]$$

$$9\times10^4\left[0.05\,\hat{a}_y + 0.1\,\hat{a}_x\right]$$



DC transmission line

$\rho_l = 5\,\mu C/m$

Find the electric field at point P.