Homework 3: Due Friday, 18 June 2008

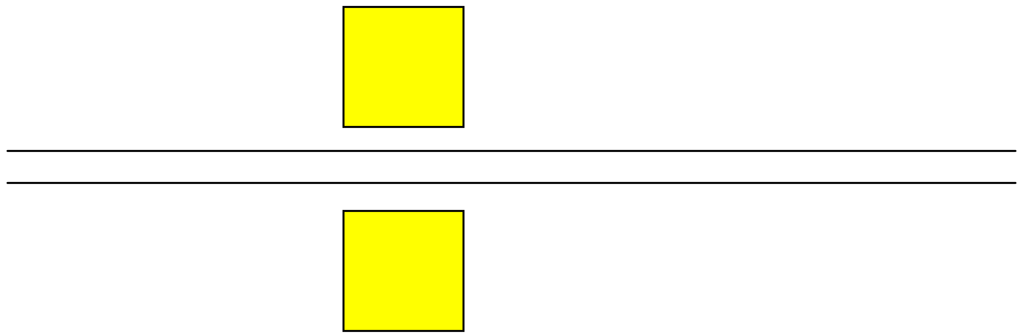
Magnetic Fields, Inductance, Etc.

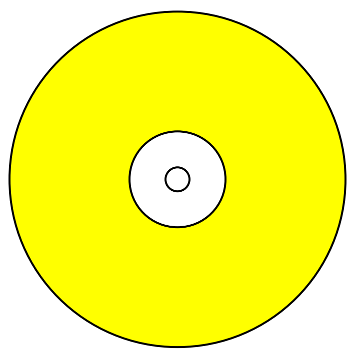
1. One way to reduce noise carried on transmission lines, power lines, etc. is to surround the wires with some kind of a ferrite core. Some examples of products that work this way can be found at:
http://www.radioshack.com/sm-snap-together-ferrite-choke-core--pi-2103222.html
http://www.dxengineering.com/products.asp?ID=182 http://www.howstuffworks.com/question352.htm http://www.mercola.com/forms/ferrite_beads_howto.htm
http://www.ferroxcube.com/
http://www.ukradioamateur.co.uk/full/html/c9-1-3.htm
http://www.amethyst-designs.co.uk/Product_Range/Toroidal_transformers.php
http://www.acma.gov.au/WEB/STANDARD/1001/pc=PC_1264

   As you can see, there are at least two different configurations for such devices. One is a torus that surrounds the wires (like the bumps at the end of computer cables) and the other is a torus around which the wires are wound.



   a. We will first look at the first type. To see how this works, begin by writing down the magnetic field intensity $\vec{H}$ in the region outside a long cylindrical wire of radius $r = a$. Assume that the z axis is aligned with the wire. The current in the wire is I.
   b. Using your answer to part a, find the magnetic flux density $\vec{B}$. For this question, there are no magnetic materials.
   c. Now we assume that we add a toroidally shaped ferrite material around the wire, as shown below. Find $\vec{H}$ and $\vec{B}$ everywhere outside the wire (in the core material and in air).
   d. Using the energy stored in the magnetic field, determine the difference in inductance seen by the wire when the ferrite core is added. *Hint: find the energy stored in the volume where the core is, both before and after the core is added.*

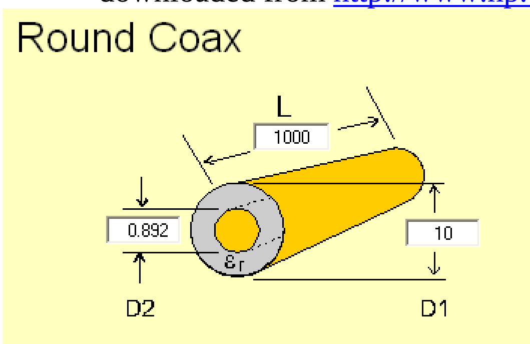


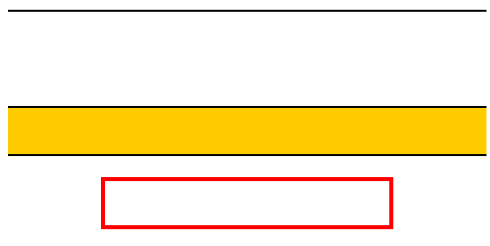


The inner radius of the ferrite core is *b*, its outer radius is *c* and its length is *d*. The permeability of the core is μ.

e. For the second type of core, how many turns are wrapped around the core in the center photo? What is the magnetic field intensity $\vec{H}$ in the core if the wire carries a current *I* and the core permeability is μ?

2. We wish to modify a coaxial cable so that we can monitor the current it carries without directly connecting to the cable. To do this we will add a small coil in the insulating region and use Faraday's Law to find the relationship between a detected signal and the current in the cable. First, assume that we have a standard coaxial type cable except that it is a bit larger than what we have used in class. For this cable, the inner radius is 0.446mm and the outer radius is 5mm. The insulator is Teflon. Use AppCAD to determine the characteristic impedance of the cable. (This is to remind us about transmission lines.) AppCAD can be downloaded from http://www.hp.woodshot.com/





   a. Find the magnetic field in the insulating region if the current carried is 100mA. That is, find the expression for the magnetic field that holds throughout the insulating region.
   b. Now, assume that we have added a 10 turn rectangular coil that extends from $r = 1.5mm$ to $r = 4.5mm$ and is $10mm$ long (in the z direction). Find the voltage induced around the coil if the frequency of the current in the coax is $1MHz$.

3. The inductance of a square cross section torus is given in a classic EE book (Terman's *Radio Engineers Handbook*) as $L_o = 0.0117 n^2 h \log_{10} \frac{d_2}{d_1}$ where n is the number of turns of wire, h is the height of the torus, d_1 is the inner diameter and d_2 is the outer diameter of the torus. Derive this expression from Maxwell's equations. *Hint: You will have to figure out the units used in this expression, since they are not likely to be SI in a classic book. Note also the type of log used.*