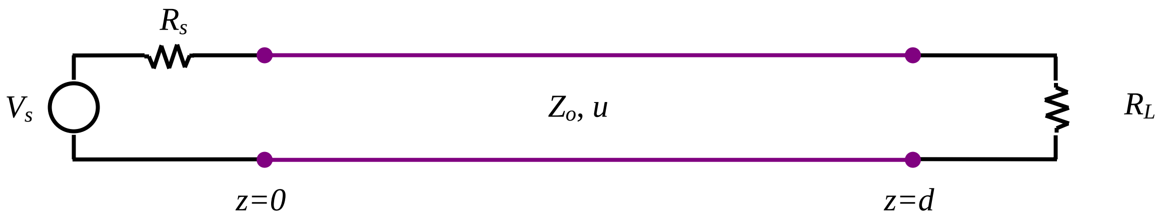
Appendix ?? Spice Models for Lossless Transmission Lines

The basic configuration for a lossless transmission line, driven by some kind of a signal generator and terminated in a resistive load is usually shown as follows.



The length d, the characteristic impedance Z_o and propagation velocity u fully characterize the operation of the line.The general solution for voltages and currents propagating on transmission lines can be written as

$$V(z,t) = V_+\left(t - \frac{z}{u}\right) + V_-\left(t + \frac{z}{u}\right)$$

$$I(z,t) = I_+\left(t - \frac{z}{u}\right) + I_-\left(t + \frac{z}{u}\right)$$

The current equation can be re-written in term of voltage and the characteristic impedance of the line producing a simpler pair of equations

$$V(z,t) = V_+\left(t - \frac{z}{u}\right) + V_-\left(t + \frac{z}{u}\right)$$

$$I(z,t) = \frac{V_+\left(t - \frac{z}{u}\right)}{Z_o} - \frac{V_-\left(t + \frac{z}{u}\right)}{Z_o}$$

The voltage and current at the input end are given by

$$V(0,t) = V_+(t) + V_-(t) \qquad \text{(a)}$$

$$I(0,t) = \frac{V_+(t)}{Z_o} - \frac{V_-(t)}{Z_o} \qquad \text{(b)}$$

while at the load end, they are

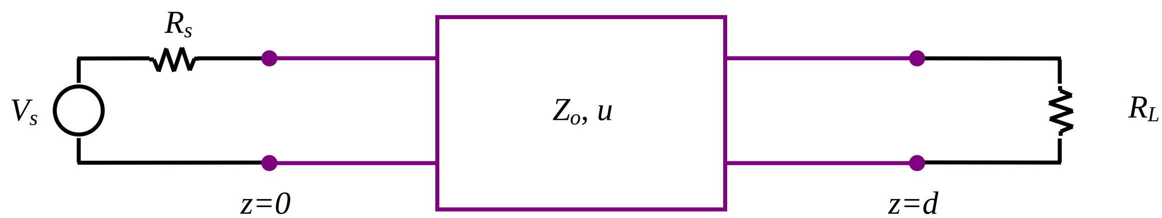
$$V(d,t) = V_+\left(t - \frac{d}{u}\right) + V_-\left(t + \frac{d}{u}\right) = V_+(t-T) + V_-(t+T) \qquad \text{(c)}$$

$$I(d,t) = \frac{V_+\left(t - \frac{d}{u}\right)}{Z_o} - \frac{V_-\left(t + \frac{d}{u}\right)}{Z_o} = \frac{V_+(t-T)}{Z_o} - \frac{V_-(t+T)}{Z_o} \qquad \text{(d)}$$

where $T = \frac{d}{u}$ is the propagation time from one end of the line to the other.

Google two port network

To develop a PSpice model for the transmission line, we want to replace it with a two port network of discrete components.



Our goal is to develop a relationship between the voltages and currents at the input and output ends of the line and find a simple combination of components that can be used to represent their functional dependence. We can approach this many different ways, since there are many useful representations of two port networks. We will only pursue one approach here and leave it to the reader to address some of the others.

[Reference – Paul, Whites, and Nasar, 3rd edition, pages 453-457]

To replace the transmission line with a two-port, we need to eliminate any reference to $V_+(t)$, $V_-(t)$, $V_+(t-T)$, or $V_-(t-T)$ since, in a circuit diagram, only the total voltages and currents are useful. Begin by multiplying equation (b) by Z_o and then subtracting it from (a).

$$V(0,t) - Z_o I(0,t) = 2V_-(t)$$

Multiply (d) by Z_o and add it to (c).

$$V(d,t) + Z_o I(d,t) = 2V_+(t-T)$$

Mutiply (b) by Z_o , add it to (a) and delay it by T.

$$V(0,t-T) + Z_o I(0,t-T) = 2V_+(t-T)$$

Finally, multiply (d) by Z_o, subtract it from (c) and delay it by T.

$$V(d,t-T) - Z_o I(d,t-T) = 2V_-(t+T-T) = 2V_-(t)$$

We now can use two of these four equations to eliminate any reference to V_+ or V_-.

$$V(0,t) - Z_o I(0,t) = V(d,t-T) - Z_o I(d,t-T)$$
$$V(d,t) + Z_o I(d,t) = V(0,t-T) + Z_o I(0,t-T)$$

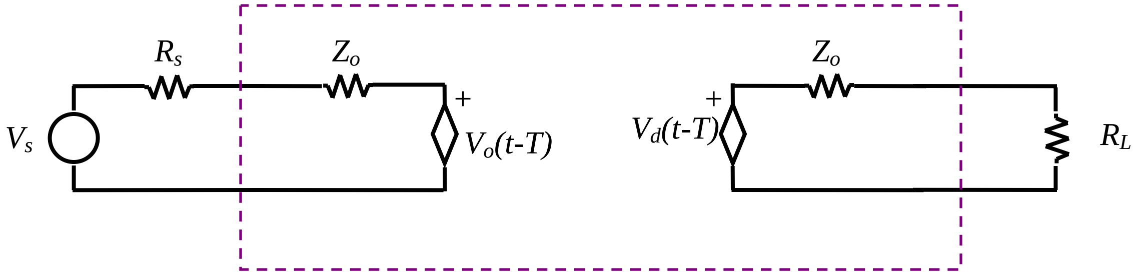
Define the right hand sides in terms of a voltage.

$$V(0,t) - Z_o I(0,t) = V_o(t-T)$$
$$V(d,t) + Z_o I(d,t) = V_d(t-T)$$

where

$$V_o(t-T) = V(d,t-T) - Z_o I(d,t-T)$$
$$V_d(t-T) = V(0,t-T) + Z_o I(0,t-T)$$

These expressions lend themselves to a simple circuit representation involving only resistors and controlled sources.



The transmission line has now been replaced by the four components inside the box shown with the dashed line. Note that the voltage controlled voltage sources take the signal at the other end of the line and delay it by *T*, which reflects the time it takes for the signal at one end to reach the other. There will only be such a signal at the other end if, for the first transit of the line, there is a source voltage or, for other transits of the line, if there is a reflection. If we have done the math above correctly, we should see that this is the case. Consider the case when the load and line impedances are the same, since the result should be very simple and clear. Then,

$$\begin{aligned} V_o(t-T) &= V(d,t-T) - Z_o I(d,t-T) \\ &= R_L I(d,t-T) - Z_o I(d,t-T) \\ &= Z_o I(d,t-T) - Z_o I(d,t-T) = 0 \end{aligned}$$

so nothing will appear back at the input end when there is no reflection at the output end. The same holds for $V_d(t-T)$. Now consider the more general case of a reflection from the load for an arbitrary load resistance. To address this situation, we begin at the beginning when the source voltage is first launched onto the line. For time $t=0$, we know that there is no voltage or current at the output end and, thus

$$V_o(t-T) = 0$$

The voltage that appears at the input end is given by the voltage divider relationship, as expected.

$$V_{in} = V(0,t) = \frac{Z_{in}}{Z_{in}+R_s}V_s = \frac{Z_o}{Z_o+R_s}V_s$$

At a time $t=T$, this appears at the load end through the voltage controlled source.

$$V_d(t-T) = V(0,t-T) + Z_o I(0,t-T)$$

Since $V_o(t-T) = 0$ when the signal is launched on the line,

$$V_d(t-T) = V(0,t-T) + Z_o I(0,t-T) = 2V(0,t-T)$$

The voltage observed across the load is then given by the voltage divider relation

$$V(d,t) = 2V(0,t-T)\frac{R_L}{R_L+Z_o}$$

which agrees with the standard transmission line analysis. When this signal reflects and returns to the source end, we should also find similar agreement. This is left to the reader to confirm.

The conclusion we can come to after this analysis is that, for lossless transmission lines, the standard T model used in PSpice gives completely accurate results. For lossy transmission lines, Tlossy should likewise be as accurate. This is not addressed here.

TLossy transmission line models can also incorporate skin effects (frequency dependent parameters). Google: skin effect spice transmission line model