Lossless Transmission Lines

Total voltage and current:

$$v(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$
$$i(z) = \frac{V_+ e^{-j\beta z} - V_- e^{+j\beta z}}{Z_o}$$

Phase velocity:

$$u = \pm\frac{\omega}{\beta} = \pm\frac{1}{\sqrt{lc}}$$

Propagation constant:

$$\beta = \omega\sqrt{lc}$$

Characteristic impedance:

$$Z_o = \sqrt{\frac{l}{c}}$$

Generalized reflection coefficient:

$$\Gamma(z) = \frac{V_-}{V_+} e^{+j2\beta z}$$

Total voltage and current again:

$$v(z) = V_+ e^{-j\beta z}\left(1 + \Gamma(z)\right)$$
$$i(z) = \frac{V_+}{Z_o} e^{-j\beta z}\left(1 - \Gamma(z)\right)$$

Line impedance:

$$Z(z) = \frac{v(z)}{i(z)} = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

also:

$$\Gamma(z) = \frac{Z(z) - Z_o}{Z(z) + Z_o}$$

At another location z'

$$\Gamma(z') = \Gamma(z) e^{j2\beta(z'-z)}$$

Continuity of $Z(z)$

$$Z(z-) = Z(z+)$$

Smith Chart normalization:

$$z(z) = \frac{Z(z)}{Z_o} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

Uniform Plane Waves in Lossless Media

Total fields:

$$E_x(z) = E_+ e^{-j\beta z} + E_- e^{+j\beta z}$$

$$H_y(z) = \frac{E_+ e^{-j\beta z} - E_- e^{+j\beta z}}{\eta}$$

Phase velocity:

$$u = \pm \frac{\omega}{\beta} = \pm \frac{1}{\sqrt{\mu\varepsilon}}$$

Propagation constant:

$$\beta = \omega\sqrt{\mu\varepsilon}$$

Intrinsic impedance:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Generalized reflection coefficient:

$$\Gamma(z) = \frac{E_-}{E_+} e^{+j2\beta z}$$

Total fields again:

$$E_x(z) = E_+ e^{-j\beta z}\left(1 + \Gamma(z)\right)$$

$$H_y(z) = \frac{E_+}{\eta} e^{-j\beta z}\left(1 - \Gamma(z)\right)$$

Total wave impedance:

$$Z(z) = \frac{E_x(z)}{H_y(z)} = \eta \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

also:

$$\Gamma(z) = \frac{Z(z) - \eta}{Z(z) + \eta}$$

At another location z'

$$\Gamma(z') = \Gamma(z) e^{j2\beta(z' - z)}$$

Continuity of $Z(z)$

$$Z(z-) = Z(z+)$$

Smith Chart normalization:

$$z(z) = \frac{Z(z)}{Z_o} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

Lossy Transmission Lines

Total voltage and current:

$$v(z) = V_+ e^{-\gamma z} + V_- e^{+\gamma z}$$
$$i(z) = \frac{V_+ e^{-\gamma z} - V_- e^{+\gamma z}}{Z_o}$$

Phase velocity:

$$u = \pm \frac{\omega}{\beta}$$

Propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(g + j\omega c)}$$

Characteristic impedance:

$$Z_o = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

Generalized reflection coefficient:

$$\Gamma(z) = \frac{V_-}{V_+} e^{+2\gamma z}$$

Total voltage and current again:

$$v(z) = V_+ e^{-\gamma z}\left(1 + \Gamma(z)\right)$$
$$i(z) = \frac{V_+}{Z_o} e^{-\gamma z}\left(1 - \Gamma(z)\right)$$

Line impedance:

$$Z(z) = \frac{v(z)}{i(z)} = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

also:

$$\Gamma(z) = \frac{Z(z) - Z_o}{Z(z) + Z_o}$$

At another location z'

$$\Gamma(z') = \Gamma(z) e^{2\gamma(z' - z)}$$

Continuity of $Z(z)$

$$Z(z-) = Z(z+)$$

Smith Chart normalization:

$$z(z) = \frac{Z(z)}{Z_o} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

Uniform Plane Waves in Lossy Media

Total fields:

$$E_x(z) = E_+ e^{-\gamma z} + E_- e^{+\gamma z}$$

$$H_y(z) = \frac{E_+ e^{-\gamma z} - E_- e^{+\gamma z}}{\eta}$$

Phase velocity:

$$u = \pm \frac{\omega}{\beta}$$

Propagation constant:

$$\gamma = j\omega\sqrt{\mu(\varepsilon' - j\varepsilon'')}$$

Intrinsic impedance:

$$\eta = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}}$$

Generalized reflection coefficient:

$$\Gamma(z) = \frac{E_-}{E_+} e^{+2\gamma z}$$

Total fields again:

$$E_x(z) = E_+ e^{-\gamma z}\left(1 + \Gamma(z)\right)$$

$$H_y(z) = \frac{E_+}{\eta} e^{-\gamma z}\left(1 - \Gamma(z)\right)$$

Total wave impedance:

$$Z(z) = \frac{E_x(z)}{H_y(z)} = \eta \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

also:

$$\Gamma(z) = \frac{Z(z) - \eta}{Z(z) + \eta}$$

At another location z'

$$\Gamma(z') = \Gamma(z) e^{2\gamma(z' - z)}$$

Continuity of $Z(z)$

$$Z(z-) = Z(z+)$$

Smith Chart normalization:

$$z(z) = \frac{Z(z)}{Z_o} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$