UNIT XIV

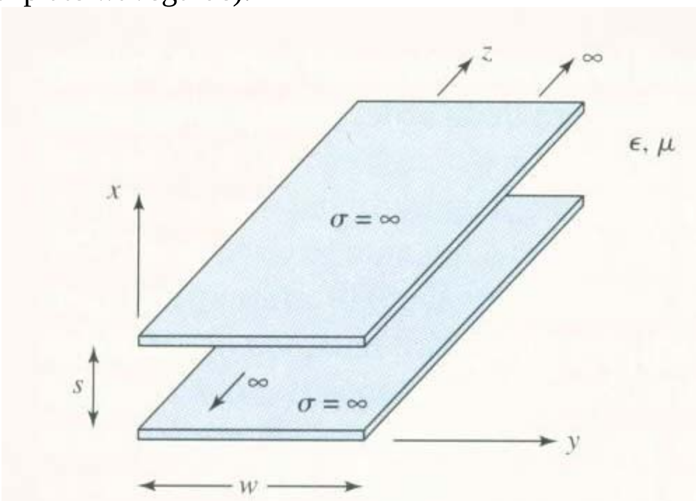
TEM Waves on Transmission Lines

In this unit, we will connect all that has been developed regarding the propagation of uniform plane waves with voltage and current on transmission lines. The general form for uniform plane wave fields propagating in the *z* direction:

$$E_x(z) = E_+ e^{-j\beta z} + E_- e^{+j\beta z}$$

$$H_y(z) = \frac{E_+ e^{-j\beta z} - E_- e^{+j\beta z}}{\eta}$$

Let us assume that these waves are propagating between the parallel conducting plates of a stripline (parallel plate waveguide).



These fields can exist in the region between the conducting plates if the boundary conditions on the plates are reasonably satisfied. First, consider the electric field. Since the electric field has only an *x* component, it is totally normal to the conducting boundaries. This can occur if there is a surface charge on the boundary, which we know there must be for a capacitor. The surface charge density on each plate is equal to

$$\rho_s = \varepsilon E_x(z) = \varepsilon E_+ e^{-j\beta z} + \varepsilon E_- e^{+j\beta z}$$

We will return to this expression below. The magnetic field is totally tangent to the conducting boundary, which can occur if there is a surface current density given by

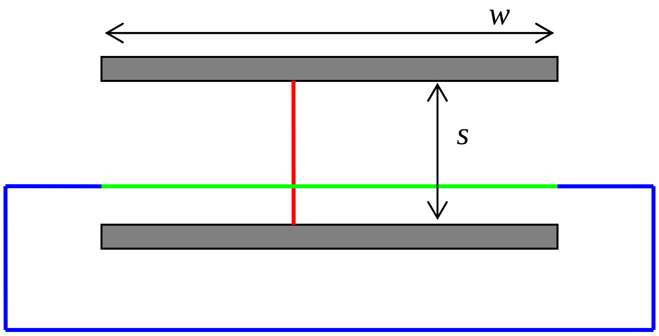
$$J_s = H_y(z) = \frac{E_+ e^{-j\beta z} - E_- e^{+j\beta z}}{\eta}$$

We will return to this expression also. However, at this point, we only need recognize that the fields can indeed exist in the region between the conductors.

Then, assuming that the lower plate is grounded, the voltage on the upper plate will be

$$v(z) = \int_0^s E_x(z)dy = sE_+ e^{-j\beta z} + sE_- e^{+j\beta z} = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$$

where we have integrated the electric field along the vertical (red) path shown.



To connect the magnetic field with the current, we must integrate along a closed path that encloses one of the two conductors. The bottom path shown includes the horizontal (green) path inside the field region and the blue path outside of the field region. (We assume no fringing in this ideal case.) The magnetic field only contributes along the green path. Thus

$$i(z) = \int_0^w H_y(z)dy = \frac{wE_+e^{-j\beta z} - wE_-e^{+j\beta z}}{\eta}$$

$$= \frac{wsE_+e^{-j\beta z} - wsE_-e^{+j\beta z}}{\eta s} = \frac{V_+e^{-j\beta z} - V_-e^{+j\beta z}}{\eta s/w}$$

For a parallel plate waveguide (stripline), the inductance and capacitance per unit length are

$$l = \frac{\mu s}{w} \text{ and } c = \frac{\varepsilon w}{s}$$

thus

$$Z_o = \sqrt{\frac{l}{c}} = \sqrt{\frac{\mu s/w}{\varepsilon w/s}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{s}{w} = \eta\frac{s}{w}$$

so the current expression is

$$i(z) = \frac{V_+e^{-j\beta z} - V_-e^{+j\beta z}}{Z_o}$$

We could have determined this current from the surface current density so we should check to be sure that the two results agree. The total current at any z should be given by

$$i(z) = J_s w = \frac{E_+e^{-j\beta z} - E_-e^{+j\beta z}}{\eta}w = \frac{V_+e^{-j\beta z} - V_-e^{+j\beta z}}{Z_o}$$

as before. Finally, we can check to see if the charge per unit length (as determined from ρ_s) gives us the usual capacitance per unit length.

$$q = \rho_s w = \varepsilon w E_+ e^{-j\beta z} + \varepsilon w E_- e^{+j\beta z} = \frac{\varepsilon w}{s}\left(V_+ e^{-j\beta z} + V_- e^{+j\beta z}\right) = cv(z)$$

as expected.

The same analysis can be done for coaxial cables and two-wire lines. The general results are the same.