

## Electromagnetic Plane Waves in Lossless Media

**Reading assignment**

Paul, Whites, and Nasar, 5.5, 5.6, 5.8->6.2

**Problem 1 - Intro to electromagnetic waves**

The electric field of a plane wave is given by  $\mathbf{E} = E_m \cos(\omega t - kz) \mathbf{a}_x$ .

- Write  $\mathbf{E}$  in phasor form.
- Under what conditions does  $\mathbf{E}$  satisfy the wave equation (in phasor form),  $\nabla^2 \mathbf{E}_x / z^2 = -\mu \omega^2 \mathbf{E}_x$ ?
- What is the velocity of a plane wave in free space?
- What are typical values of  $f$ ,  $\lambda$ ,  $T (=1/f)$ ,  $\omega$ , and  $k$  for X-rays, visible light, microwaves, and FM radio in free space? (Check the walls of the studio).
- Find  $\mathbf{H}$  using the phasor form of the  $\nabla \times \mathbf{E}$  equation. Assume the  $\mathbf{E}$  and  $\mathbf{H}$  phasors are only a function of  $z$ .
- Evaluate the amplitude ratio,  $\alpha = |\mathbf{E}| / |\mathbf{H}|$ . Express  $\alpha$  in terms of material properties.
- If  $\mathbf{E}$  was in the  $\mathbf{a}_y$  direction, what direction would  $\mathbf{H}$  be in?
- Run the Java applet linked through the course homepage.
- How many independent parameters are there in the following set?  
 $\epsilon, \mu, \omega, k, T$

**Problem 2 - Waves in lossless media**

WRPI broadcasts at 91.5 MHz. The amplitude of  $\mathbf{E}$  on campus is roughly 0.08 V/m. Assume a coordinate system in which the wave is polarized in the  $\mathbf{a}_y$  direction and propagating in the  $\mathbf{a}_z$  direction. Assume the phase = 0 at  $z = 0$ .

- What are  $\omega$ ,  $\lambda$ , and  $f$  for this wave?
- Write the electric and magnetic fields in phasor form.
- Write the electric field in time domain form.

**Problem 3 - Energy & Power - lossless media**

- What is the average energy density of the electric and magnetic fields for the WRPI signal on campus? (Use Prob. 2 results).
- What is the time average Poynting vector of the wave,  $\mathbf{S}_{av}$ ? Divide its magnitude by the speed of light and compare with your answer from part a.
- The transmitter is about 10 km from campus. What transmitter power is required to radiate the same power density into a sphere of radius 10 km?