Problem 1 - Line integrals & curl
The magnetic field of a straight wire of radius \( a \) which has a constant current density \( J_0 \), is given by:
\[
B = \mu_0 J_0 \frac{r}{2a} \quad \text{inside the wire (} r < a \text{)}
\]
\[
B = \mu_0 J_0 \frac{a^2}{2r} \quad \text{outside the wire (} r > a \text{)}.
\]
where \( \mu_0 \) and \( J_0 \) are constants.

a. Calculate \( \oint \mathbf{B} \cdot d\mathbf{l} \) around the 2 paths shown in the figure below. (The drawing shows a cross-sectional view as if the wire had been cut). The shaded area is perpendicular to the \( z \)-axis.

b. Calculate \( \nabla \times \mathbf{B} \) for both regions.

Problem 2 - Properties of fields with curl
The electric field created by a cylinder of radius \( a \) with constant charge density \( \rho_0 \) is:
\[
E = \rho_0 \frac{r}{2 \varepsilon_0} \mathbf{a}_r \quad \text{inside the cylinder (} r < a \text{)} \quad \text{and}
\]
\[
E = \rho_0 \frac{a^2}{2 \varepsilon_0 r} \mathbf{a}_r \quad \text{outside the cylinder (} r > a \text{)}.
\]
where \( \rho_0 \) and \( \varepsilon_0 \) are constants.

a. Verify that \( \oint \mathbf{E} \cdot d\mathbf{l} = 0 \) on the same paths as above and that \( \nabla \times \mathbf{E} = 0 \) for both regions.

b. An illustration of the \( \mathbf{E} \) and \( \mathbf{B} \) fields can be obtained by running div_curl_example.m using matlab. Fig. 1 is the \( \mathbf{B} \) field while Figure 3 is the \( \mathbf{E} \) field. What are the properties of a field with non-zero curl?


**Problem 3 - Stokes theorem**
Calculate \( \int (\nabla \times \mathbf{B}) \cdot d\mathbf{s} \) over the two surface areas enclosed by each path in Problem 1 (the shaded area). Compare your answer with the results from Problem 1a.

**Problem 4 - Gradient**
Compute the gradient of the following functions.

a. \( f = 8a^2 \cos \phi + 2rz \) (cylindrical)

b. \( f = a \cos 2\theta / r \) (spherical)

You can check your answer with the worksheet associated with Problem 2.8.1 in "Visual Electromagnetics for Mathcad" (you may have to use a specific number instead of the variable \( a \)) or by using Maple.

c. Calculate \( \nabla \times \nabla f \) for each of the functions above.