**Reading assignment**
Popović and Popović, Appendix 1
Connor and Salon, II-39 → II-44

**Software**
div_curl_example.m

**Problem 1 - Line integrals & curl**
The magnetic field of a straight wire of radius $a$ which has a constant current density $J_0$ is given by:
\[
B = \mu_0 J_0 \frac{r}{2a} \mathbf{a}_\phi \quad \text{inside the wire } (r < a)
\]
\[
B = \mu_0 J_0 \frac{a^2}{2(r)} \mathbf{a}_\phi \quad \text{outside the wire } (r > a).
\]
where $\mu_0$ and $J_0$ are constants.

a. Calculate $\int B \cdot d\mathbf{l}$ around the 2 paths shown in the figure below. (The drawing shows a view as if the wire had been cut).

b. Calculate $\nabla \times B$ for both regions.

**Problem 2 - Properties of fields with curl**
The electric field created by a cylinder of radius $a$ with constant charge density $\rho_0$ is:
\[
E = \rho_0 r / (2\varepsilon_0) \mathbf{a}_r \quad \text{inside the cylinder } (r < a)
\]
\[
E = \rho_0 a^2 / (2\varepsilon_0 r) \mathbf{a}_r \quad \text{outside the cylinder } (r > a).
\]
where $\rho_0$ and $\varepsilon_0$ are constants.

a. Verify that $\int E \cdot d\mathbf{l} = 0$ on the same path as above and that $\nabla \times E = 0$.

b. An illustration of the $E$ and $B$ fields can be obtained by running div_curl_example.m using matlab. Fig. 1 is the $B$ field while Figure 3 is the $E$ field. What are the properties of a field with non-zero curl?

**Problem 3 - Stokes theorem**
Calculate $\int (\nabla \times B) \cdot d\mathbf{s}$ over the surface area enclosed by the 2 paths in Problem 1 (the shaded area). Compare your answer with the result from Problem 1a.
Problem 4 - Gradient
Compute the gradient of the following functions.

a. \[ f = 8 a^2 \cos \phi + 2rz \] (cylindrical)

b. \[ f = a \cos 2\theta / r \] (spherical)

Use the worksheet associated with Problem 2.8.1 in "Visual Electromagnetics for Mathcad" to check your answer. (You may have to use a specific number instead of the variable \( a \)).

c. Calculate \( \nabla \times \nabla f \) for each of the functions above.