Preparation Assignments for Homework #8
Due at the start of class.

Reading Assignments
Please see the handouts for each lesson for the reading assignments.

3 December Lesson 5.5

A uniform plane wave is propagating in the z-direction and has an x-directed electric field $E_x$ and a y-directed magnetic field $H_y$. Assume that there is a boundary between two media at $z = 0$. Write the boundary condition for $E_x$ and the boundary condition for $H_y$ at $z = 0$ for the following two cases:
1. Medium 1 ($z < 0$) is a dielectric and medium 2 ($z > 0$) is a conductor
2. Both media are dielectrics

6 December Lesson 5.6

Write out Snell’s Law and draw a diagram explaining the terms in this expression. What is the critical angle? What is Brewster’s angle?

Class time 8 December (Note the date – Wednesday)

Open shop to work on Homework 8. Due at 5 pm on 8 December.
Problem 1 (10 points) Normal Incidence on a Lossless Dielectric and a Conductor

A plane wave with frequency $f = 70$ MHz propagating in an unknown lossless material (call it Material X), is incident normally on a perfectly conducting boundary. Inside Material X, the standing wave pattern shown below is observed. (We won’t worry about how it was actually observed inside the material.)

![Standing Wave Pattern](image)

a. What is the wavelength $\lambda$, the wavenumber $\beta$, and the dielectric constant $\varepsilon_r$ of the unknown material. What do you suppose Material X is?

b. Now the perfect conductor is removed so that the wave is now incident normally on a boundary between Material X and air ($\varepsilon_r = 1$). Which of the two standing wave patterns plotted on the next page will be observed in Material X?

c. Write expressions for the electric and magnetic field (in phasor form) that will be observed in air.
Problem 2 (10 points) Oblique Incidence

A uniform plane wave is incident obliquely in air on the boundary of a dielectric medium as shown below. The electric field vector $\mathbf{E}$ is shown, as is the direction of propagation. On the plot below, indicate the incident, reflected and transmitted angles: $\theta_i$, $\theta_R$, $\theta_t$

![Diagram of oblique incidence with vectors $\mathbf{E}$ and $\mathbf{H}$]

a. Circle the direction of $\mathbf{H}$: (x-direction, y-direction, z-direction). At this point, we are not concerned with the sign. Add a symbol that shows the direction of $\mathbf{H}$ for each of the three waves to create a plot something like in Figure 6.21 of the text. Now, you should be careful about the sign in each case. Is this wave parallel or perpendicularly polarized?

![Diagram with vectors $\mathbf{H}$]

Now assume that $\mathbf{E}$ is replaced by $\mathbf{H}$, as shown above. Note that the reflected $\mathbf{H}$ field is labeled $-\mathbf{H}$ so that the same diagram can be used. By convention, we usually assume that $\mathbf{H}$ changes direction upon reflection. On the plot above, indicate the incident, reflected and transmitted angles: $\theta_i$, $\theta_R$, $\theta_t$
b. Circle the general direction of \( \mathbf{E} \): (x-direction, y-direction, z-direction). At this point, we are not concerned with the sign. Add a symbol that shows the direction of \( \mathbf{E} \) for each of the three waves to create a plot something like in Figure 6.21 of the text. Now, you should be careful about the sign in each case. Is this wave parallel or perpendicularly polarized?

c. Assume that we have measured the reflected wave power for the first type of polarization and found that there is no reflected wave at the incident angle \( \theta_i = 63^\circ \). What is the dielectric constant \( \varepsilon \) of the medium in the region \( z > 0 \)? What is the angle of transmission \( \theta_t \) for this angle of incidence? What simple relationship exists between \( \theta_i \) and \( \theta_t \)?

d. Determine the reflection coefficient \( \Gamma \) for the first polarization for all angles of incidence \( \theta_i = \theta_1 \) from 0 to 90\(^\circ\). Use Matlab or Maple to plot this. Be sure you input the angles in radians. Your result should look like one of the plots on the next page (when plotted vs. angle in degrees). Which one is it? To generate this plot, the following expression was used. This is a little more convenient than the expressions in the text (194a or 188a).

\[
\Gamma_1 = -\frac{\varepsilon_{r2} \cos \theta_i + \sqrt{\varepsilon_{r2} - \sin^2 \theta_i}}{\varepsilon_{r1} \cos \theta_i + \sqrt{\varepsilon_{r1} - \sin^2 \theta_i}}
\]

e. Now assume that the wave has the other polarization. Determine the reflection coefficient \( \Gamma \) for all angles of incidence \( \theta_i = \theta_1 \) from 0 to 90\(^\circ\). Use Matlab or Maple to plot this. Be sure that you input the angles in radians. Your results should look like one of the plots on the next page. Which one is it? To generate this plot, the following expression was used. This is a little more convenient than the expressions in the text (194a or 188a).

\[
\Gamma_2 = \frac{\cos \theta_i - \sqrt{\varepsilon_{r2} - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_{r1} - \sin^2 \theta_i}}
\]

f. Assume that you have only a parallel polarized plane wave incident at an angle of 63\(^\circ\). Write the incident, reflected and transmitted electric and magnetic field vectors in phasor form. Assume that the incident electric field amplitude is \( E_0 \). Also assume that the wave frequency is 30 MHz. Note that the general form of these expressions are found in equation 190. Use the information you are given or have calculated to write out the field expressions for the specific case considered here.

g. Assume that the incident power density is 10 kW/m\(^2\). Determine \( E_0 \) and the transmitted power density. Is the power density conserved in this case? Why or why not?
Reflection Coefficient vs. Angle of Incidence

Angle of Incidence

Reflection Coefficients

0 10 20 30 40 50 60 70 80 90
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1

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