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## Question I - Transfer functions of RLC, RL and RC Circuits (52 points)

Circuit A: Answer 1-8 for the RC circuit below


1. Find the transfer function for the above circuit. (2 points)

$$
\mathrm{H}(\mathrm{j} \omega)=[1 / \mathrm{j} \omega \mathrm{C}] /[1 / \mathrm{j} \omega \mathrm{C}+\mathrm{R}]=[1] /[1+\mathrm{j} \omega \mathrm{RC}]
$$

2. Find the function to describe the behavior of the circuit at very low frequencies. Also determine the magnitude and phase of this circuit at very low frequencies. (3 points)

$$
\begin{array}{ll}
H_{L O}(j \omega)=1 / 1=1 & \\
\left|H_{L O}(j \omega)\right|=1 & \angle H_{L O}(j \omega)=0
\end{array}
$$

3. Find the function to describe the behavior of the circuit at very high frequencies. Also determine the magnitude and phase of this circuit at very high frequencies. (3 points)
$H_{H I}(j \omega)=1 / j \omega R C=-j / \omega R C$

$$
\left|\mathrm{H}_{\mathrm{HI}}(\mathrm{j} \omega)\right|=0 \quad \angle \mathrm{H}_{\mathrm{HI}}(\mathrm{j} \omega)=-\pi / 2
$$

4. What is the expression for the corner frequency of the circuit? (1 point)

$$
\omega_{\mathrm{c}}=1 / \mathrm{RC}
$$

5. Find the transfer function which governs the behavior of the circuit at the corner frequency. Also find the magnitude and phase of the function at the corner frequency. (3 points)
$\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)=[1] /[1+\mathrm{j}(1 / \mathrm{RC}) \mathrm{RC}]=[1] /[1+\mathrm{j}]$

$$
\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)\right|=1 / \sqrt{ } 2 \quad \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{c}}\right)=0-\pi / 4=-\pi / 4
$$

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6. If $\mathrm{C}=0.022 \mu \mathrm{~F}$ and $\mathrm{R}=800 \mathrm{ohms}$, find the numerical values for the following. Indicate the units. (8 points)

$$
\omega_{\mathrm{c}}=1 /[(0.022 \mu)(800)]=56818 \mathrm{rad} / \mathrm{s} \quad \mathrm{f}_{\mathrm{c}}=56818 / 2 \pi=9043 \mathrm{~Hz}
$$

|  | Magnitude | Phase |
| :--- | :--- | :--- |
| Very low frequencies | 1 | 0 |
| Very high frequencies | 0 | $-\pi / 2$ |
| Corner frequency | $1 / \sqrt{2}$ | $-\pi / 4$ |

7. Sketch a plot of the magnitude of the transfer function vs frequency (in Hertz). Clearly indicate the value at very high frequencies, very low frequencies and the corner frequency. Mark the corner frequency on the sketch. (3 points)

8. Sketch a plot of the phase of the transfer function (in degrees or radians). Clearly indicate the value at very high frequencies, very low frequencies and the corner frequency. Mark the corner frequency on the sketch. (3 points)

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Circuit B: Answer 9-16 for the RLC circuit below

9. Find the transfer function for the above circuit. (2 points)

$$
H(j \omega)=[j \omega L] /[R+1 / j \omega C+j \omega L]=\left[-\omega^{2} L C\right] /\left[j \omega R C+1-\omega^{2} L C\right]
$$

10. Find the function to describe the behavior of the circuit at very low frequencies. Also determine the magnitude and phase of this circuit at very low frequencies. (3 points)

$$
\begin{aligned}
& \mathrm{H}_{\mathrm{LO}}(\mathrm{j} \omega)=\left[-\omega^{2} \mathrm{LC}\right] /[1]=-\omega^{2} \mathrm{LC} \\
& \left|\mathrm{H}_{\mathrm{LO}}(\mathrm{j} \omega)\right|=0 \quad \angle \mathrm{H}_{\mathrm{LO}}(\mathrm{j} \omega)=+\pi
\end{aligned}
$$

11. Find the function to describe the behavior of this circuit at very high frequencies. Also determine the magnitude and phase of this circuit at very high frequencies. (3 points)

$$
\begin{aligned}
& H_{H I}(j \omega)=\left[-\omega^{2} L C\right] /\left[-\omega^{2} L C\right]=1 \\
& \left|H_{H I}(j \omega)\right|=1 \quad \angle H_{H I}(j \omega)=0
\end{aligned}
$$

12. What is the expression for the resonant frequency of the circuit? (1 point)

$$
\omega_{0}=1 /(\sqrt{ } \mathrm{LC})
$$

13. Find the transfer function which governs the behavior of the circuit at the resonant frequency. Also find the magnitude and phase of the function at the resonant frequency. (3 points)

$$
\begin{aligned}
& \mathrm{H}\left(\mathrm{j} \omega_{0}\right)=[-(1 / \mathrm{LC})(\mathrm{LC})] /[\mathrm{j}(1 / \sqrt{ } \mathrm{LC}) \mathrm{RC}-(1 / \mathrm{LC})(\mathrm{LC})+1]=[+\mathrm{j} \sqrt{ } \mathrm{LC}] /[\mathrm{RC}] \\
& \left|\mathrm{H}\left(\mathrm{j} \omega_{0}\right)\right|=[\sqrt{ } \mathrm{LC}] /[\mathrm{RC}] \quad \angle \mathrm{H}\left(\mathrm{j} \omega_{0}\right)=\pi / 2
\end{aligned}
$$

14. If $\mathrm{C}=0.047 \mu \mathrm{~F}, \mathrm{~L}=10 \mathrm{mH}$ and $\mathrm{R}=500$ ohms, find the numerical values for the following. Indicate the units. (8 points)

$$
\omega_{0}=[1] /[\sqrt{ }(0.047 \mu)(10 \mathrm{~m})]=46127 \mathrm{rad} / \mathrm{s} \quad \mathrm{f}_{0}=46127 / 2 \pi=7341 \mathrm{~Hz}
$$

$\left|\mathrm{H}\left(\mathrm{j} \omega_{0}\right)\right|=[\sqrt{ } \mathrm{LC}] /[\mathrm{RC}]=[\sqrt{ }(0.047 \mu)(10 \mathrm{~m})] /[(500)(0.047 \mu)]=0.92$

|  | Magnitude | Phase |
| :--- | :--- | :--- |
| Very low frequencies | 0 | $+\pi$ |
| Very high frequencies | 1 | 0 |
| Resonant frequency | 0.92 | $\pi / 2$ |

15. Sketch a plot of the magnitude of the transfer function vs frequency in Hertz. Clearly indicate the value at very high frequencies, very low frequencies and the resonant frequency. Mark the resonant frequency on the sketch. (3 points)

16. Sketch a plot of the phase of the transfer function (in degrees or radians). Clearly indicate the value at very high frequencies, very low frequencies and the corner frequency. Mark the resonant frequency on the sketch. (3 points)

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## Question II - Filters (16 points)

Consider the following three input signals:
Signal 1:


Signal 2:


Signal 3:


1. What is the frequency of each of the signals above (in Hertz)? (6 points)

Signal 1: $T=5 \mu \mathrm{~s} \quad \mathrm{f}=200,000 \mathrm{~Hz}$

Signal 2: $T=50 \mu \mathrm{f}=20,000 \mathrm{~Hz}$

Signal 3: $T=0.5 \mathrm{~ms} \quad \mathrm{f}=2000 \mathrm{~Hz}$

## Name

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2. What type of filter is circuit A from question I ? (2 point)

Low pass filter
3. What type of filter is circuit B from question I ? (2 point)

High pass filter
4. Fill out the following chart. Enter "lower" if the amplitude of the output of the given circuit will be substantially lower than the input amplitude. Enter "higher" if the amplitude of the output of the given circuit will be substantially higher than the input amplitude. Enter "same" if the amplitude of the output of the given circuit will be about the same as the input amplitude. Note that the circuits are the ones you analyzed in question I (with the component values we gave you) and the signals are those pictured on the previous page. You can assume the filter transitions are close to ideal. (6 points)

| 200K |  | 20K | Signal 2 |
| :--- | :--- | :--- | :--- |
| Circuit A | Signal 1 | lower | samer |
| Circuit B | same | same | lower |
|  |  |  |  |

(see dots marked on plots in answers to question I)
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## Name

Question III - Diodes (22 points)
The figure below is a dual voltage limiter. Assume R1=1K


Here is a picture of the output of the above circuit when simulated with an input signal of amplitude equals to 0.5 V


1. Explain the reason why both the input and the output are the same (2 points)

In this circuit, diodes D2 and D3 will turn on when the voltage tries to go above 1.4 volts and diodes D1 and D4 will turn on when the voltage tries to go below -1.4 volts. Between 1.4 and -1.4 volts, both pairs of diodes will be off. The circuit looks like the one pictured on the following page. In this circuit Vout $=$ Vin. Since the amplitude of the input pictured above is only 0.5 V , it never tries to go above 1.4 volts or below -1.4 volts. The diodes never turn on and the output remains equal to the input.


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2. Each diode in the circuit can either be forward bias, reverse bias, or in the breakdown region, from the above PSPICE plot, what is the region of operation of: (4 points)

D1: reverse bias (off)
D2: reverse bias (off)
D3: reverse bias (off)
D4: reverse bias (off)
3. Here is a picture of the output of the above circuit when simulated with an input signal of amplitude equals to 5 V

4.In the marked area, what is the region of operation of the 4 diodes (4 points)

D1: reverse bias (off)
D2: forward bias (on)
D3: forward bias (on)
D4: reverse bias (off)
5. What is the value of current through the resistor R1 when the input voltage is at the values listed below. Assume Von for each diode is 0.7 V

3 volts ( 2 points) $\mathrm{I}=(3-1.4) / 1 \mathrm{~K}=1.6 \mathrm{~mA}$
-3 volts (2 points) $\mathrm{I}=(-3-(-1.4))=-1.6 \mathrm{~mA}$
0.2 V (2 points) $\mathrm{I}=0 \mathrm{~mA}$
-0.2 V (2 points) $\mathrm{I}=0 \mathrm{~mA}$

If the 4 diodes (D1-D4) are replaced with one zener diode. The input and output will be as shown in the figure below

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I am assuming that region 1 is the upper region which cuts off at about 4.7 volts and region 2 is the lower region which cuts off at about -0.7 volts.
6. What is the current in the two marked regions ( 4 points)

Method I: Assume that we are at the extremes where Vin $=10 \mathrm{~V}$ or -10 V

1. $\mathrm{I}=[10-4.7] / 1 \mathrm{~K}=5.3 \mathrm{~mA}$
2. $\mathrm{I}=[-10-(-0.7)] / 1 \mathrm{~K}=-9.3 \mathrm{~mA}$

Method II: Assume the general case, where Vout is a function of Vin.

1. $\mathrm{I}=[\operatorname{Vin}-4.7] / 1 \mathrm{~K}$
2. $\mathrm{I}=[\operatorname{Vin}-(-0.7)] / 1 \mathrm{~K}=[\operatorname{Vin}+0.7] / 1 \mathrm{~K}$
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## Question IV -- Op-Amps (10 points)



Assume the following about the components in the above circuit:
V2: $\mathrm{VOFF}=600 \mathrm{mV}, \mathrm{VAMPL}=200 \mathrm{mV}, \mathrm{FREQ}=1 \mathrm{~K}$.
V3: VDC $=200 \mathrm{mV}$
$\mathrm{R} 2=10 \mathrm{~K}, \mathrm{R} 3=2 \mathrm{~K}, \mathrm{R} 4=2 \mathrm{~K}, \mathrm{R} 5=10 \mathrm{~K}, \mathrm{R} 6=20 \mathrm{~K}$

1. Above is a picture of a type of amplifier you have seen. What type of amplifier is it? (1 point)

Difference (or differential) amplifier
2. Write an expression for the input signal at $B$ in the form $v(t)=A \sin (\omega t)+V_{D C}$.
(3 points)

$$
\mathrm{v}(\mathrm{t})=200 \mathrm{mV} \sin (2 \mathrm{~K} \pi \mathrm{t})+600 \mathrm{mV}
$$

3. Write an equation for the output at $\mathrm{C}\left(\mathrm{V}_{\mathrm{C}}\right)$ in terms of the input voltages V 2 and V 3 . Simplify. Do not substitute for V2 and V3. (2 points)

$$
\mathrm{Vc}=[10 \mathrm{~K} / 2 \mathrm{~K}](\mathrm{V} 2-\mathrm{V} 3)=5(\mathrm{~V} 2-\mathrm{V} 3)
$$

4. Write an expression for the output signal at $C$ in the form $v(t)=A \sin (\omega t)+V_{D C}$.
(4 points)

$$
\begin{aligned}
& \mathrm{Vc}=5[200 \mathrm{mV} \sin (2 \mathrm{~K} \pi \mathrm{t})+600 \mathrm{mV}-200 \mathrm{mV}] \\
& \mathrm{Vc}=1000 \mathrm{mV} \sin (2 \mathrm{~K} \pi \mathrm{t})+2000 \mathrm{mV} \\
& \mathrm{Vc}=1 \mathrm{~V} \sin (2 \mathrm{~K} \pi \mathrm{t})+2 \mathrm{~V}
\end{aligned}
$$

