ENGR4300
Fall 2005
Test 2

Name____solution_____________

Section________________________

Question 1 (25 points)___________

Question 2 (25 points) ___________

Question 3 (25 points)___________

Question 4 (25 points)___________

Total (100 points): ______________

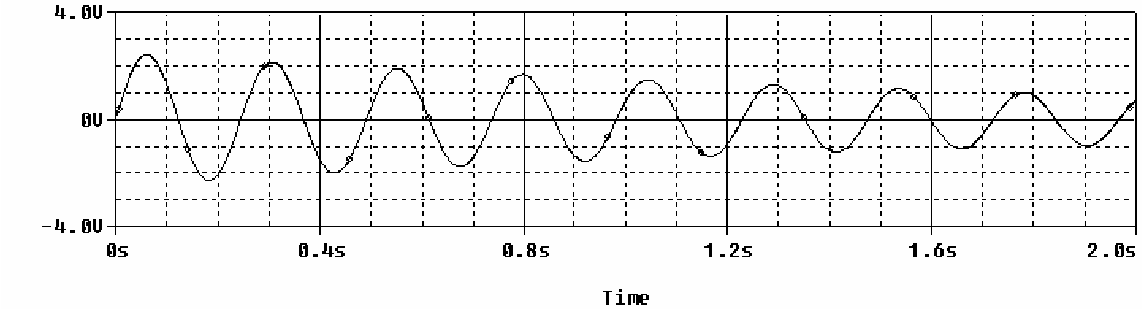
Please do not write on the crib sheets.

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

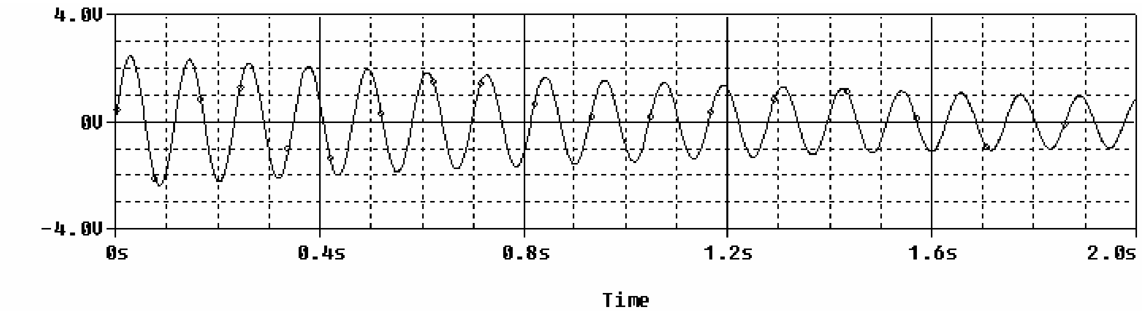
Question 1 (A, B and C) – Damped Sinusoids and Strain Gauge Bridge (25 points)

You are given a cantilever beam similar to the one you used in experiment 4. You place two weights on the end of the beam (0.1 kg and 0.5 kg) and you get the following two plots.

Plot 1



Plot 2



1) What is the frequency of plot 1? (Use at least 2 significant figures) (2 points)

f1 = 8 cycles/1.97s = ***4.1 Hz***

2) What is the frequency of plot 2? (Use at least 2 significant figures) (2 points)

f2 = 17 cycles/1.98s = ***8.6 Hz***

3) What is the damping constant for plot 1? (Use at least 2 significant figures) (6 points)

(t0,v0) = (0.07,2.4) (t1,v1)=(1.78,1.0)

$1.0 = 2.4\, e^{-\alpha(1.78-0.07)}$ $\quad 0.875=\alpha(1.71)$ $\quad$ $\boldsymbol{\alpha = 0.5/s}$

4) Given the following formula, $k = (m + m_n)(2\pi f_n)^2$, and assuming that the two data points that you found are ideal, find values for k and m. (6 points)

$k = (m+0.1)(2\pi(8.6))$^^2 $\quad$ $k=(m+0.5)(2\pi(4.1))$^^2

k = (2920)m + (292) k = (663.6)m + 331.8

(2920)m + (292.0) = (663.6)m + 331.8

m = 0.017 kg

*k = (0.017+0.1)(2920)=****343 kg/s^^2***

5) What is the mass of the beam? (3 points)

$$(0.017) = 0.23\ (m_b)$$
$$m_b = .074\ kg = \mathbf{74\ grams}$$

6) Using the chart for Young's Modulus, determine the probable material that the beam is made out of given that the dimensions of the beam are: width = 1.5 cm, length = 20 cm, and thickness = 2 mm. (6 points)

$$k=Ewt^3/4l^3$$
$$343 = E\ (0.015)(.002\wedge3)/4(0.2)\wedge3$$
$$E=9.2 \times 10^{10}\ N/m\wedge2$$

The beam is brass.

TABLE 9.1

Young's Modulus Table of Values

Metal	Elastic modulus (N/m2)	Metal	Elastic modulus (N/m2)
aluminum, 99.3%, rolled	6.96×10^{10}	lead, rolled	1.57×10^{10}
brass	9.02×10^{10}	platinum, pure, drawn	16.7×10^{10}
copper, wire, hard drawn	11.6×10^{10}	silver, hard drawn	7.75×10^{10}
gold, pure, hard drawn	7.85×10^{10}	steel, 0.38% C, annealed	20.0×10^{10}
iron, wrought	19.3×10^{10}	tungsten, drawn	35.5×10^{10}

For Test B:

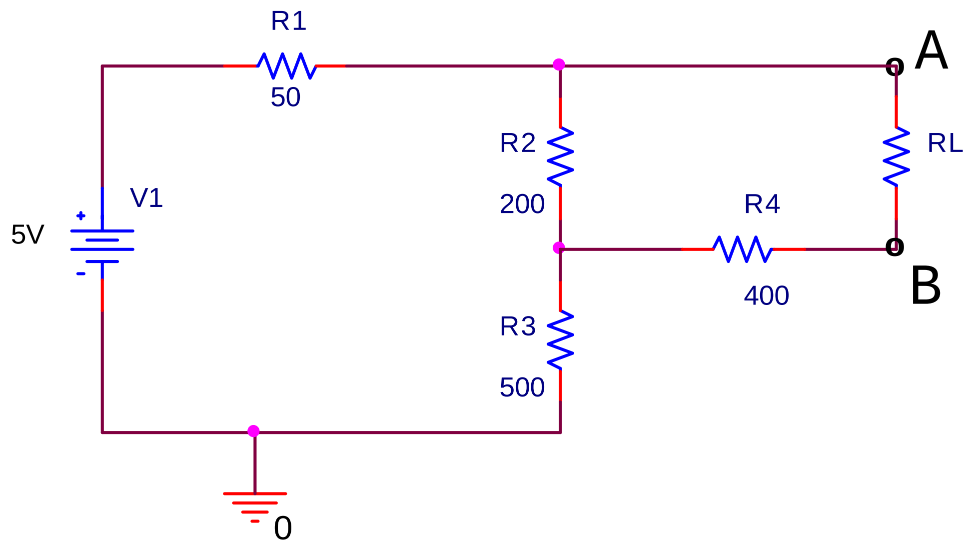
The method is the same.
1) f1=3.0 Hz
2) f2=6.5 Hz
3) $\alpha = 0.8/s$
4) k=181 kg/s^2 m=0.0083 kg
5) m_b ***= 36 grams***
6) E = 6.9 $\times 10^{10}$ ***N/m^2 The beam is aluminum.***

For Test C:

The method is the same.
1) f1=10.3 Hz
2) f2=4.85 Hz
3) $\alpha = 0.6/s$
4) k=475 kg/s^2 m=0.0134 kg
5) m_b ***= 58 grams***
6) E = 11.5 $\times 10^{10}$ ***N/m^2 The beam is copper.***

Question 2 (A and B) – Thevenin Equivalent Sources (25 points)

You are told to wire the following circuit in PSpice. All of the parts in this question refer to this circuit.



Part A: Finding the Thevenin Equivalent Source with respect to the load resistor, RL, between points A and B.

1) Determine the Thevenin Equivalent Voltage, V_{th}, with respect to RL. (6 points)

First I must remove RL. Then I realize that R4 has no current, so the voltage between R2 and R3 is the same as B. Therefore, I can use a voltage divider to find V_{R2} and that is the voltage between A and B.

$V_{R2} = 5V[200/(50+200+500)] = 1.33\ V$

Vth = 1.33V

2) Determine the Thevenin Equivalent resistance, Rth, with respect to RL. (6 points)

When I short out the voltage source, R1 and R3 end up in series. When I combine them, I get R13 = 50+500 = 550. This resistor ends up in parallel with R2. R123 = (550)(200)/(550+200)=146.7. This resistor is in series with R4. Therefore, the total resistance between A and B is 146.7 + 400 = 546.7 ohms.

Rth = 546.7 ohms

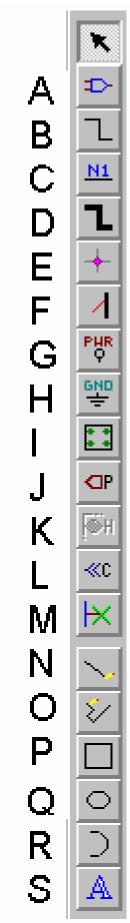
3) If RL is 2K, what is the voltage between A and B? (3 points)

VL = Vth[RL/(RL+Rth)] = 1.33[2000/(2000+546.7)] = ***1.04 V***

4) What is the current through the 2K resistor, RL? (2 points)

VL = I (RL) ***I = 0.52mA***

Part B: Wiring the circuit in PSpice



1) Of the icons shown at left, which would you use to add the DC source to the circuit? (2 points)

A

3) Of the icons shown at left, which would you use to add the ground to the circuit? (2 points)

H

2) Of the icons shown at left, which would you use to add a wire to the circuit? (2 points)

B

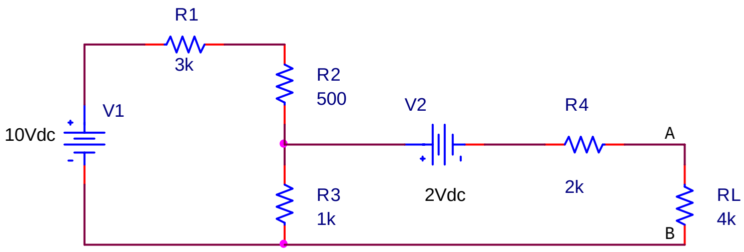
4) Which PSpice library contains the model for the resistors shown in the circuit above (circle one)? (2 points)

SOURCE EVAL ***ANALOG*** BREAKOUT

For Test B: The method is the same
A: 1) Vth = 4.2V
2) Rth = 437 ohms
3) Vab = 3.7V
4) I = 1.2mA
B: same answers as Test A

Question 2 (C) – Thevenin Equivalent Sources (20 points)

For the following circuit, show all work using resistor variable names (e.g. R1, R2, R3, R4, and RL) before substituting resistance values:



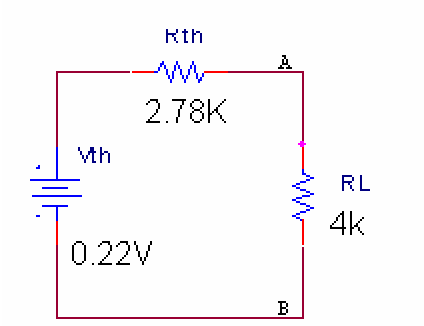
a) Find the Thevenin voltage between points A and B: (7 pts)

Need Voc across AB, so voltage divider to find voltage across R3: VR3 = V1*(R3/(R1+R2+R3) = 2.222V, then subtract V2 = 0.222V, no drop in R4 as open circuit, so Va-b = 0.222V

b) Find the Thevenin resistance between points A and B: (7 pts)

Short sources, find combined resistance looking in with load removed: R4 + R3||(R1+R2) = 2.7778k

c) Draw the Thevenin equivalent of the circuit shown, showing RL in the equivalent circuit: (3 pts)

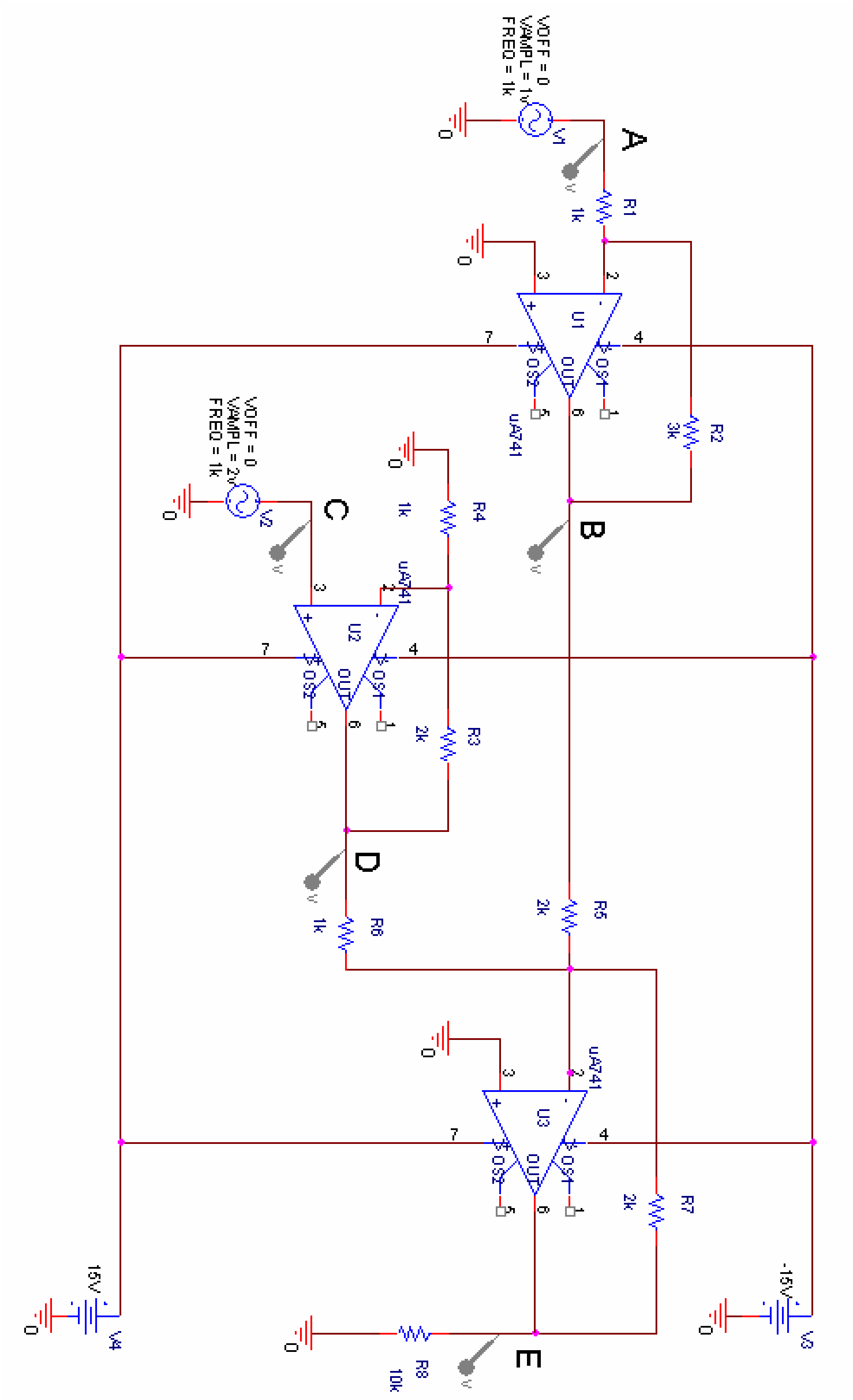


Use Vth, Rth, calculated above, RL from original circuit

d) What is the voltage across A-B with the load resistor shown: (3 pts)

Use the circuit from C, by voltage divider, Vrl = Vth * (RL/(Rth+RL))= 131.1mV

Question 3 – Op Amp Applications (25 points)



The following questions refer to the circuit on the previous page.

1) What type of op-amp circuit is between points A and B? (1 point)

Inverting Amplifier

2) What type of op-amp circuit is between points C and D? (1 point)

Non-inverting amplifier

3) What type of op-amp circuit is between (B and D) and E? (1 point)

(Weighted) Adder

4) Write an expression for the voltage at point B, V_B, in terms of the voltage at A, V_A. Please substitute values. (3 points)

$V_B = -(Rf/Rin)V_A = -(3k/1k)\ V_A$ $\mathbf{V_B = -3\ V_A}$

5) Write an expression for the voltage at point D, V_D, in terms of the voltage at C, V_C. Please substitute values. (3 points)

$V_D = (1+Rf/Rin)V_C = (1+2k/1k)\ V_C$ $\mathbf{V_D = 3\ V_C}$

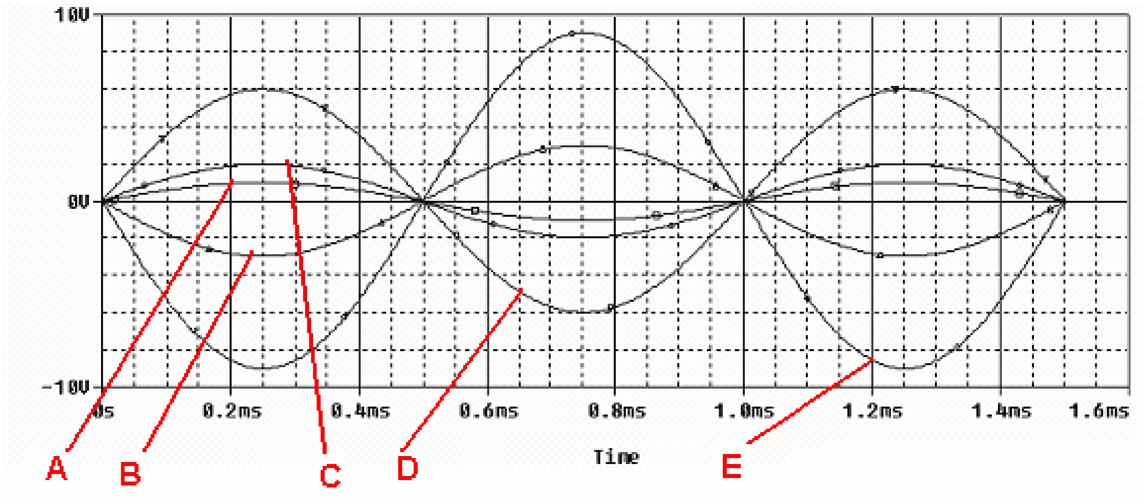
6) Write an expression for the voltage at point E, V_E, in terms of the voltage at B, V_B, and the voltage at D, V_D. Please substitute values. (3 points)

$V_E = -Rf(V1/R1+V2/R2)$ $V_E = -2k(V_B/2k + V_D/1k)$ $\mathbf{V_E = -V_B - 2V_D}$

7) Write an expression for the output voltage at E, V_E, in terms of the two input voltages in the circuit, V_A and V_C. (3 points)

$V_E = -V_B + 2V_D$ $V_E = -(-3V_A) - 2(3V_C)$ $\mathbf{V_E = 3V_A - 6V_C}$

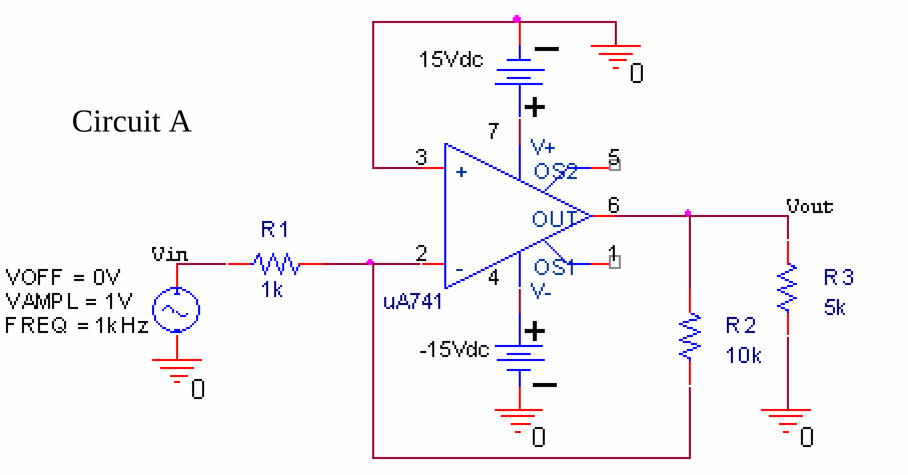
8) Identify the signals at points A, B, C, D and E in the PSpice plot below. (10 points)

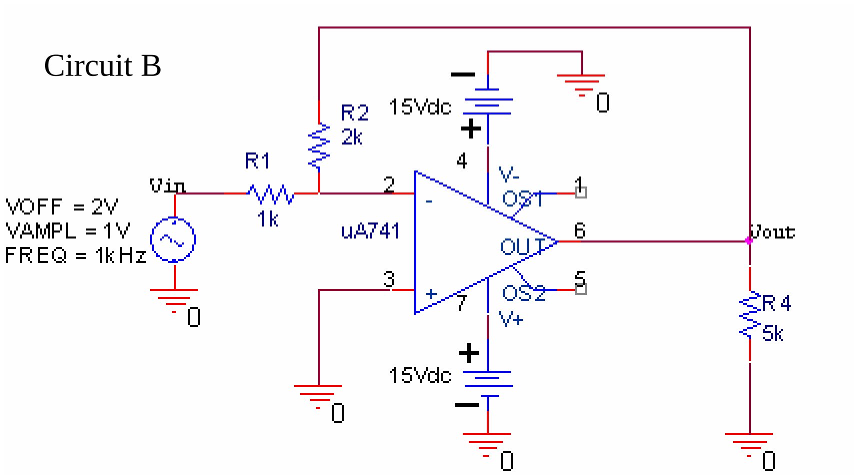


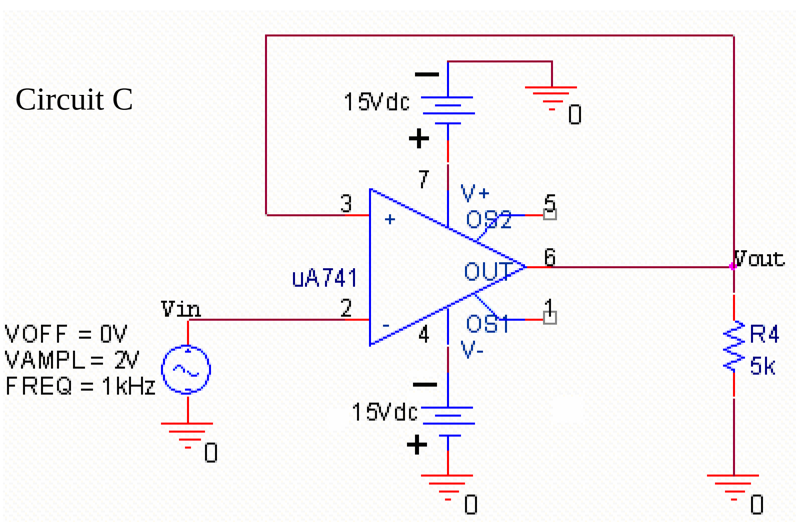
B and C the same.

Question 4 (A and B)– Op Amp Analysis (25 points)

Part A: Below are three op amp circuits you have wired in PSpice. One will function. The other two will not.







1) Which two of the three circuits will not function? For each circuit explain why you know it will not function. (2 points each = 4 points)

Test A: 1] ***circuit B – positive voltage at negative power terminal (pin 4)***
2] ***circuit C – positive feedback is unstable***

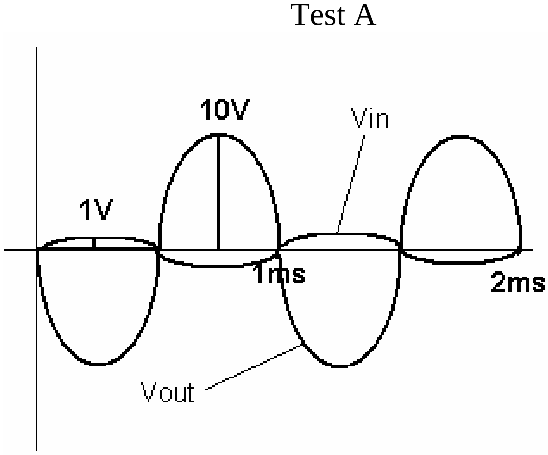
Test B: 1] ***circuit A – positive voltage at negative power terminal (pin 4)***
(the extra resistor looks odd, but does not keep the ckt from functioning)
2] ***circuit B – positive feedback is unstable***

2) Which of the circuits does function? What type of amplifier is it? (2 points)

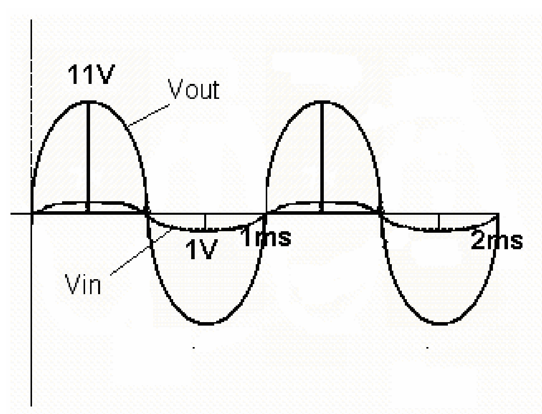
Test A: circuit A is an inverting amplifier
Test B: circuit C is a non-inverting amplifier

3) Draw two cycles of the input and corresponding output waveforms for the functioning circuit you identified in part 2. Specifically identify the amplitude for Vin and Vout. Label the time scale of the plot. (3 points) *(A: gain = -10 B: gain=+11)*

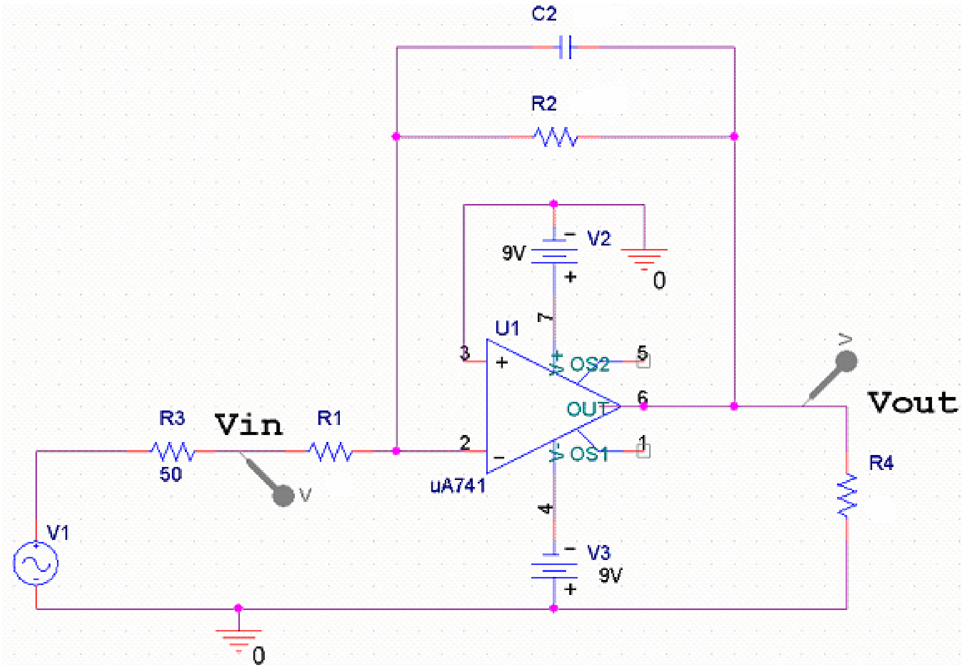
Test A



Test B



Part B In the circuit below, R1 = 2K ohms, R2 = 47K ohms, C2=0.01μF, R4=1k ohms



1) What is the transfer function of this circuit in the form: $H(j\omega) = \dfrac{x_1 + jy_1}{x_2 + jy_2}$. Please substitute values. (3 points)

$$\textbf{Test A: } H(j\omega) = \frac{-Rf}{Rin + j(\omega Rf\,Rin Cf)} = \frac{-47k}{2k + j(\omega(47k)(2k)(0.01u)} = \frac{-47k}{2k + j\omega(0.94)}$$

$$\textbf{Test B: } H(j\omega) = \frac{-Rf}{Rin + j(\omega Rf\,Rin Cf)} = \frac{-4.7k}{3k + j(\omega(4.7k)(3k)(0.01u)} = \frac{-4.7k}{3k + j\omega(0.14)}$$

2) Find an expression (in terms of ω) for the phase of this circuit for any frequency. The phase is given by: $\phi = \tan^{-1}\left(\dfrac{y_1}{x_1}\right) - \tan^{-1}\left(\dfrac{y_2}{x_2}\right)$. (2 points)

$$\textbf{Test A: } \phi = \tan^{-1}\left(\frac{0}{-47k}\right) - \tan^{-1}\left(\frac{0.94\omega}{2000}\right) = \pi - \tan^{-1}\left(\frac{0.94\omega}{2000}\right)$$

$$\textbf{Test B}: \phi = \tan^{-1}\left(\frac{0}{-4.7k}\right) - \tan^{-1}\left(\frac{0.14\omega}{3000}\right) = \pi - \tan^{-1}\left(\frac{0.14\omega}{3000}\right)$$

Note that π is the phase of a negative real number.

GRADING: *You should not have been penalized more than 1 point for forgetting the π.*

3) What is the corner frequency of this circuit in Hz? (2 points)

Test A: fc = 1/(2π Rf Cf) = 1/(2π)(47k)(0.01u) = ***339 Hz***
Test B: *fc = 1/(2π Rf Cf) = 1/(2π)(4.7k)(0.01u) =* ***3390 Hz***

4) For each of the following input frequencies, calculate the phase shift of the circuit AND use the phase to justify whether the circuit is behaving approximately as an inverting amplifier, an ideal integrator, or neither. (3 points each = 9 points)

Test A:

5 Hz: $\phi = \pi - \tan^{-1}\left(\frac{0.94\omega}{2000}\right) = 3.14 - \tan^{-1}(.94(2\pi)(5)/2000) = 3.14 - 0.01 = 3.13 rad$

This is about pi. An inverting amplifier should invert (180 degree phase shift). ***At 5Hz, this circuit acts like an inverting amplifier.***

GRADING: *If you forgot the π in part 2) and you get the correct answer for that mistake (ϕ=-0.01 rad), you should get 1 point for doing the math correctly. If you identify it as an inverting amplifier, you should get one point for that. Since the inverting amplifier does not have a phase shift of near zero, you cannot correctly use the phase you calculated to justify your conclusion so you should not get that third point.*

345 Hz: $\phi = \pi - \tan^{-1}\left(\frac{0.94\omega}{2000}\right) = 3.14 - \tan^{-1}(.94(2\pi)(345)/2000) = 3.14 - 0.79 = 2.35 rad$

This is neither pi nor pi/2. An inverting amplifier should invert (180 degree phase shift) and an integrator should integrate and invert (90 degree phase shift). This is neither. ***At 345Hz, this circuit acts like neither.***

GRADING: *If you forgot the π in part 2) and you get the correct answer for that mistake (ϕ=-0.79 rad), you should get 1 point for doing the math correctly. If you identify it as neither, you should get one point for that. Since the phase you found is neither an inverting amplifier nor an inverter, you should also get the third point. The phase you found justifies the conclusion.*

34500 Hz: $\phi = \pi - \tan^{-1}\left(\frac{0.94\omega}{2000}\right) = 3.14 - \tan^{-1}(.94(2\pi)(34500)/2000)$

$= 3.14 - 1.56 = 1.57 rad$

This is pi/2. An integrator should integrate and invert (90 degree phase shift). This is an integrator. ***At 34500Hz, this circuit acts like an ideal integrator.***

GRADING: *If you forgot the π in part 2) and you get the correct answer for that mistake (ϕ=-1.56 rad), you should get 1 point for doing the math correctly. If you identify it as an ideal integrator, you should get one point for that. Since the magnitude of the phase you found is π/2, but the sign is wrong, you should get only half of the third point. The* *magnitude of the phase you found justifies the conclusion, however, the sign is not correct.*

Test B:

34 Hz: $\phi = \pi - \tan^{-1}\left(\frac{0.14\omega}{3000}\right) = 3.14 - \tan^{-1}(.14(2\pi)(34)/3000) = 3.14 - 0.01 = 3.13 rad$

This is about pi. An inverting amplifier should invert (180 degree phase shift). ***At 34Hz, this circuit acts like an inverting amplifier.***

GRADING: *see notes for test A*

3400Hz:

$\phi = \pi - \tan^{-1}\left(\frac{0.14\omega}{3000}\right) = 3.14 - \tan^{-1}(.14(2\pi)(3400)/3000) = 3.14 - 0.78 = 2.36 rad$

This is neither pi nor pi/2. An inverting amplifier should invert (180 degree phase shift) and an integrator should integrate and invert (90 degree phase shift). This is neither. ***At 3400Hz, this circuit acts like neither.***

GRADING: *see notes for test A*

340,000 Hz: $\phi = \pi - \tan^{-1}\left(\frac{0.14\omega}{3000}\right) = 3.14 - \tan^{-1}(.14(2\pi)(340000)/3000)$

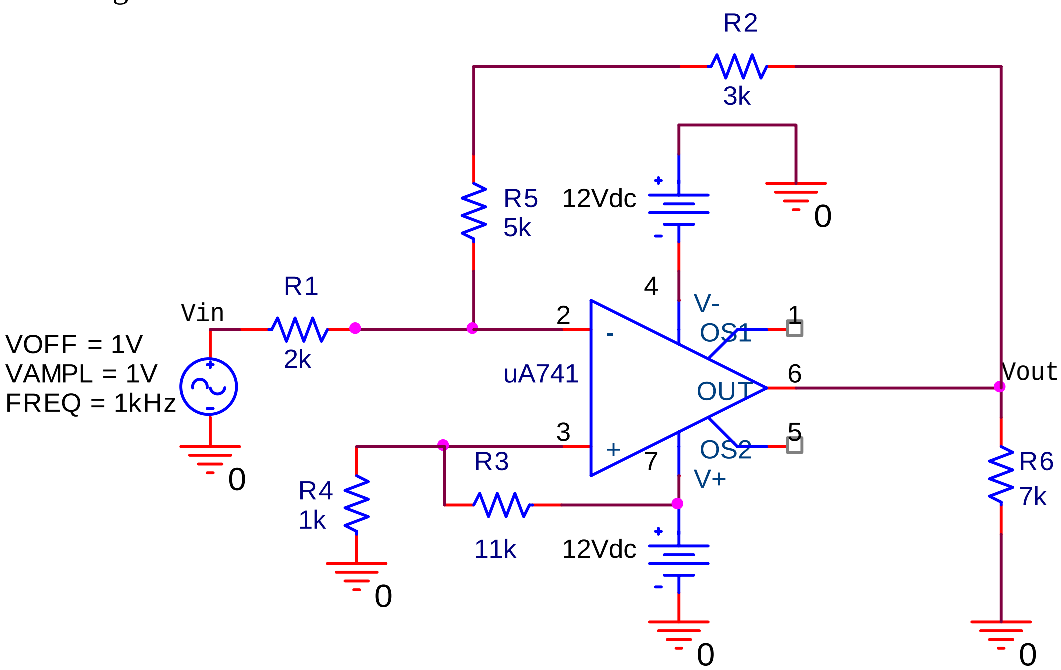
$= 3.14 - 1.56 = 1.57 rad$

This is pi/2. An integrator should integrate and invert (90 degree phase shift). This is an integrator. ***At 340,000Hz, this circuit acts like an ideal integrator.***

GRADING: *see notes for test A*

Question 4 (C) – Op Amp Analysis (30 points)

You are given the circuit below:



1) What are the golden rules of op amp analysis (2 points):

1. ***I+ = I- = 0***

2. ***V+ = V-***

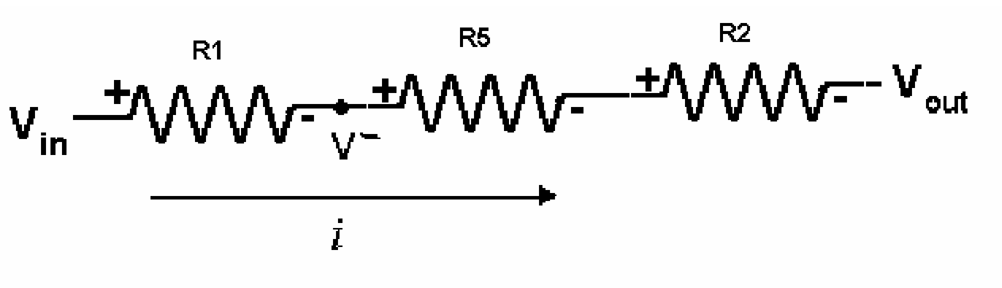
2) Without a feedback connection, can either of these rules be violated for an ideal op amp? If so, which one(s) and why (2 points)?

The terminals of the op amp are very high impedance – ideally an open circuit – so it is certainly possible to set the voltages at the two terminals to be different, consequently, V+ = V- can be violated and the device can still be useful as a comparator. The high intrinsic gain of the device will drive it to saturation at +Vdc if V+ > V- or at –Vdc if V+ < V-. I+=+-=0 cannot be violated if the input terminals are open circuit internal to the device.

Grading:1 pt for correct rule by itself (not if listed both rules), 1pt for any sensible explanation.

3) Use the golden rules of op amp analysis to derive an expression for Vout in terms of Vin and the resistors in the circuit as labeled. Do NOT substitute resistor values until instructed to do so.

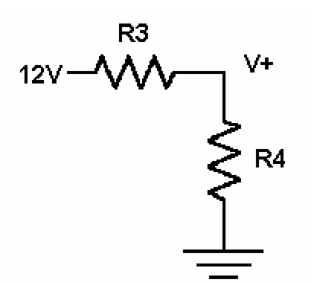
a) Draw the circuit corresponding to the inverting terminal of the op-amp (3 points)



b) Based on the circuit in a) write the equation at the inverting terminal of the op amp (Do not substitute values) (3 points)

$$\frac{V_{in} - V^-}{R1} = \frac{V^- - V_{out}}{R5 + R2}$$

c) Draw the circuit corresponding to the non-inverting terminal of the op-amp (3 points)



d) Based on the circuit in c) write the equation at the non-inverting terminal of the op amp (Do not substitute values) (3 points)

$$V^+ = \frac{R4}{R3 + R4} 12V$$

e) Use these equations to solve for Vout in terms of Vin and resistor values R1-R6. (Do not substitute values.) (7 points)

$$\frac{V_{in} - V^-}{R1} = \frac{V^- - V_{out}}{R5 + R2} \quad V^+ = \frac{R4}{R3 + R4}12V \quad V^+ = V^-$$

$$\frac{V_{in} - \frac{R4}{R3 + R4}12V}{R1} = \frac{\frac{R4}{R3 + R4}12V - V_{out}}{R5 + R2}$$

$$V_{in}(R5 + R2) - 12R4\frac{R5 + R2}{R3 + R4} = 12R1\frac{R4}{R3 + R4} - V_{out}(R1)$$

$$-(R1)V_{out} = V_{in}(R5 + R2) - 12R4\frac{R5 + R2}{R3 + R4} - 12R1\frac{R4}{R3 + R4}$$

$$\boxed{V_{out} = -V_{in}\frac{R5 + R2}{R1} + \frac{12R4}{R1(R3 + R4)}(R5 + R2 + R1)}$$

f) Substitute resistor values into your equation from part e to get Vout in terms of Vin (3 points)

$$V_{out} = -V_{in}\frac{5k + 3k}{2k} + \frac{12(1k)}{2k(11k + 1k)}(5k + 3k + 2k)$$

$$\boxed{V_{out} = -4V_{in} + 5}$$

g) For the input signal shown in the circuit schematic, what is the minimum and maximum output voltages generated at Vout? Show your work. (4 points)

if Vin = 1Vsin(ωt) + 1V then Vout = -4(1V sin(ωt) + 1V) + 5V
Vout = -4V sin(ωt) +1V

Minimum: *Negative Amplitude is -4V, Offset is 1V -4+1 is* ***-3V***

Maximum: *Amplitude is 4V, Offset is 1V 4+1 is* ***5V***