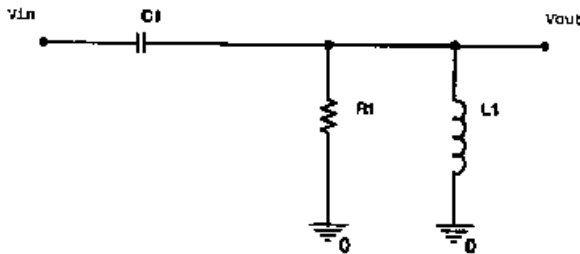


1. RLC Circuits (25 points)



a) Find the complex transfer function for the above circuit

$$Z_{RL} = \frac{Z_R Z_L}{Z_R + Z_L} = \frac{R(j\omega L)}{R + j\omega L} = \frac{j\omega RL}{R + j\omega L}$$

$$H(j\omega) = \frac{Z_{RL}}{Z_C + Z_{RL}} = \frac{\frac{j\omega RL}{R + j\omega L}}{\frac{1}{j\omega C} + \frac{j\omega RL}{R + j\omega L}} = \frac{(j\omega RL)(j\omega C)}{R + j\omega L + (j\omega RL)(j\omega C)}$$

$$= \frac{-\omega^2 RLC}{R - \omega^2 RLC + j\omega L}$$

b) Give an expression for the resonance frequency  $f_0$ .

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

c) Find the magnitude and phase of the transfer function for the following three cases. Make sure that your answer makes sense with the basic knowledge you have about capacitors and inductors.

i.  $f=0 \Rightarrow \omega=0$

$$\omega \rightarrow 0 \Rightarrow H(j\omega) \approx \frac{-\omega^2 RLC}{R} = -\omega^2 LC$$

$$\Rightarrow \left. \begin{aligned} |H(j\omega)| &= 0 \\ \angle H(j\omega) &= \pm \pi \end{aligned} \right\}$$

ii.  $f = f_0 \Rightarrow \omega = \omega_0$

$$\Rightarrow H(j\omega) = \frac{-\omega_0^2 RLC}{j\omega_0 L} = j \frac{\frac{1}{\sqrt{LC}} RC}{1} = jR \frac{\sqrt{C}}{\sqrt{L}}$$

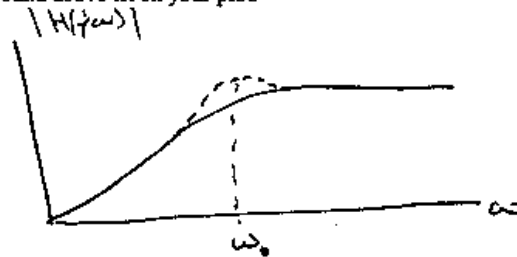
$$\Rightarrow \boxed{\begin{aligned} |H(j\omega)| &= R \frac{\sqrt{C}}{\sqrt{L}} \\ \angle H(j\omega) &= \frac{\pi}{2} \end{aligned}}$$

iii.  $f = \text{Infinity}$

$$\omega \rightarrow \infty \Rightarrow H(j\omega) \approx \frac{-\omega^2 RLC}{-\omega^2 RLC} = 1$$

$$\Rightarrow \boxed{\begin{aligned} |H(j\omega)| &= 1 \\ \angle H(j\omega) &= 0 \end{aligned}}$$

d) Plot the magnitude of the transfer function versus frequency. Make sure that the three points you found above fit on your plot.



e) Determine if this circuit is a

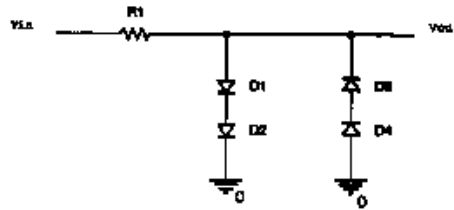
- a. Low-Pass Filter
- b. High-Pass Filter
- c. Band-Pass Filter
- d. Band Reject-Filter

Why?

Since it passes high freq. and does not pass low freq.

2. Diode Circuits (25 points)

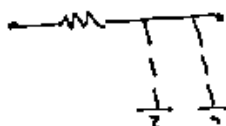
In figure below the diode is not ideal (turns on at  $V_D = 0.7v$ ), and  $R = 1k\Omega$ .



a) Determine  $V_{out}$  for

i.  $V_{in} = -1v$

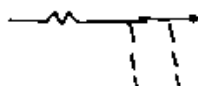
$D_1, D_2, D_3, D_4 : OFF$



$\Rightarrow V_{out} = V_{in} \Rightarrow \boxed{V_{out} = -1}$

ii.  $V_{in} = 0v$

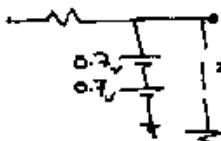
$D_1, D_2, D_3, D_4 : OFF$



$\Rightarrow V_{out} = V_{in} \Rightarrow \boxed{V_{out} = 0}$

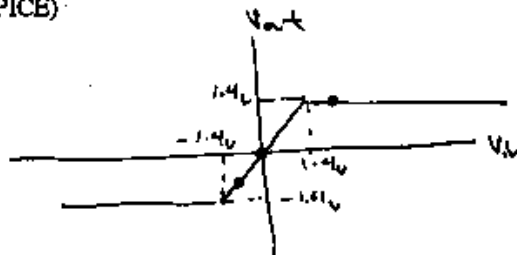
iii.  $V_{in} = 2v$

$D_1, D_2 : ON \quad D_3, D_4 : OFF$



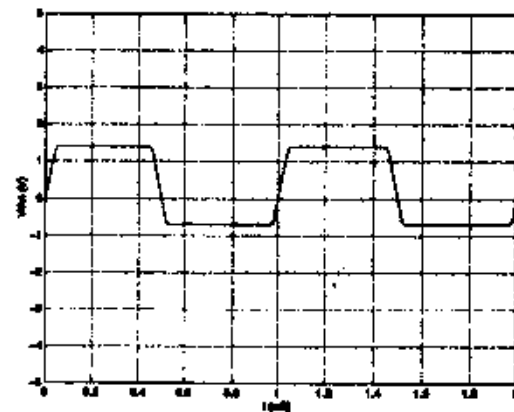
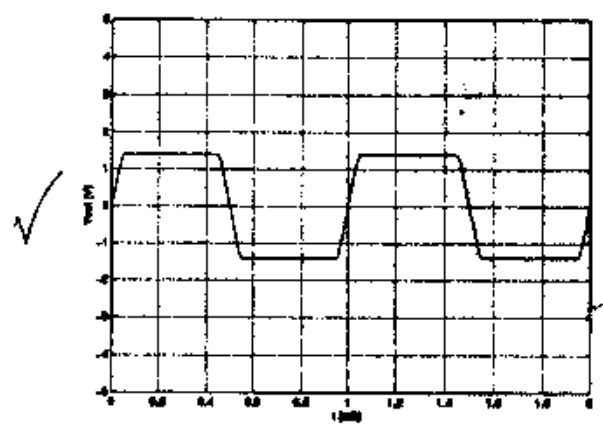
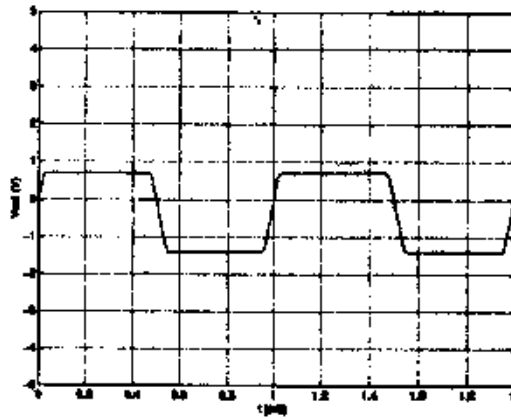
$\Rightarrow V_{out} = V_{D1} + V_{D2} = 0.7v + 0.7v = 1.4v$   
 $\Rightarrow \boxed{V_{out} = 1.4v}$

b) Plot  $V_{out}$  versus  $V_{in}$  for the range  $-5v \leq V_{in} \leq 5v$ . Make sure that the three points you calculated in part a fits on your plot. (This is equivalent to DC Sweep that you have performed in PSPICE)



c) If  $V_{in} = A\sin(2\pi f t)$ , where  $A = 5v$  and  $f = 1kHz$ , which of the following plots represents  $V_{out}$ ? Why?

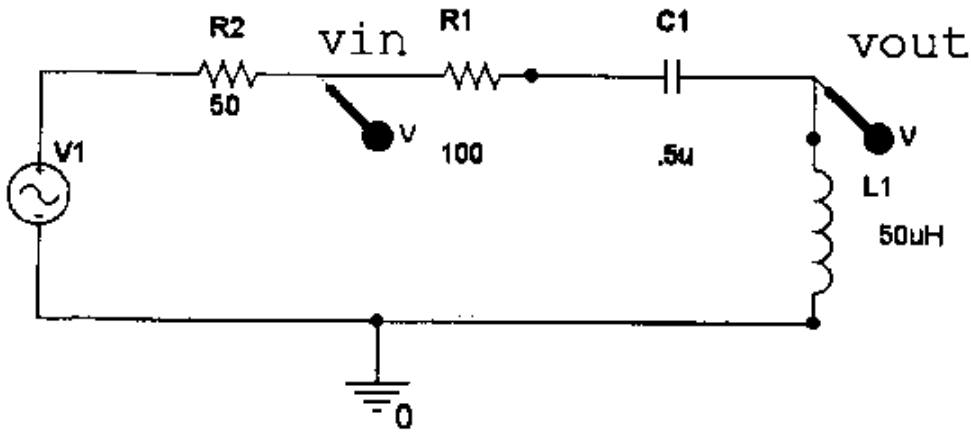
Since it is clipped at  $\pm 1.4v$



4

3. Filters (25 points)

The following circuit consists of a sinusoidal source, an inductor, a capacitor and a resistor.



If  $v_{in}$  is the sinusoidal source (including the 50 ohm internal impedance) and  $v_{out}$  is the voltage across the inductor, is this configuration a high pass filter, a low pass filter, a band pass filter or a band reject filter? Explain your answer.

Low  $f$ :  $C \rightarrow$  open  $L \rightarrow$  short  $\Rightarrow V_{out} = 0$   
 High  $f$ :  $C \rightarrow$  short  $L \rightarrow$  open  $\Rightarrow V_{out} = V_{in}$   
High-pass Filter

The source is a sinusoidal voltage with some amplitude and frequency. The source voltage, as a function of time, is shown on the next page. Write out the mathematical expression for this voltage function in the form  $v_{in} = v_0 \sin(\omega t + \phi_0)$ . Be sure that you give values for  $v_0$ ,  $\omega$ , and  $\phi_0$ .

$v_0 = 100 \text{ mV}$

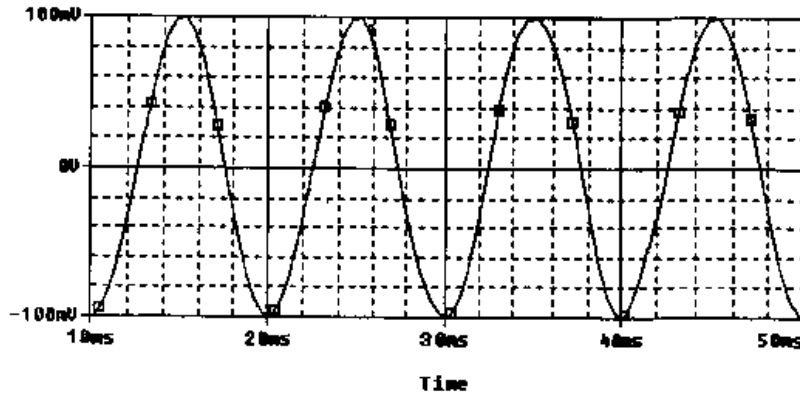
$T = 10 \text{ ms} \Rightarrow f = \frac{1}{T} = 100 \text{ Hz}$

$\omega = 2\pi f = 2\pi \times 100 \text{ rad/sec}$

$\phi_0 = -\frac{\pi}{2}$

$t_0 = 2.5 \text{ ms} \Rightarrow \phi_0 = -\frac{\pi}{2}$

$v_{in} = 100 \text{ mV} \sin(2\pi \times 100 t - \frac{\pi}{2})$



Now that you have determined the magnitude, frequency and phase of the input voltage, you should have some idea of what will happen at the output. From your knowledge of the corner or resonant frequency of this circuit, will the output voltage be about the same as the input, substantially smaller or substantially larger than the input? Explain your answer.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \mu\text{H} \times 0.5 \mu\text{F}}} = \frac{1}{\sqrt{50 \times 10^{-6} \times 0.5 \times 10^{-6}}} = \frac{1}{5 \times 10^{-6}}$$

$$= 200,000 \text{ rad/sec}$$

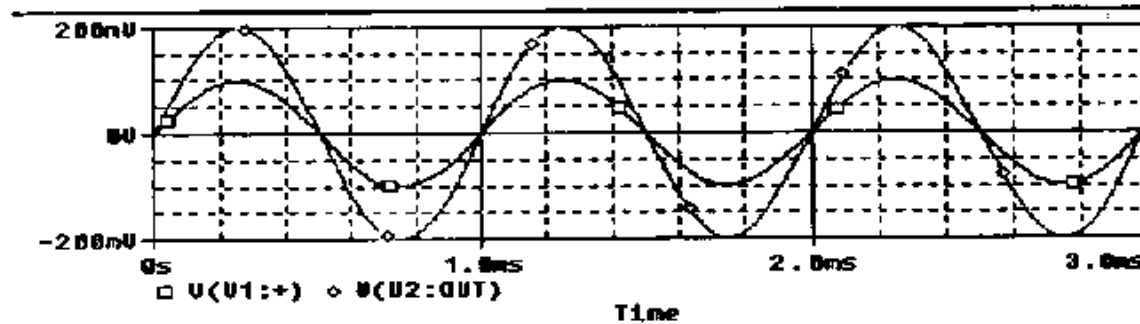
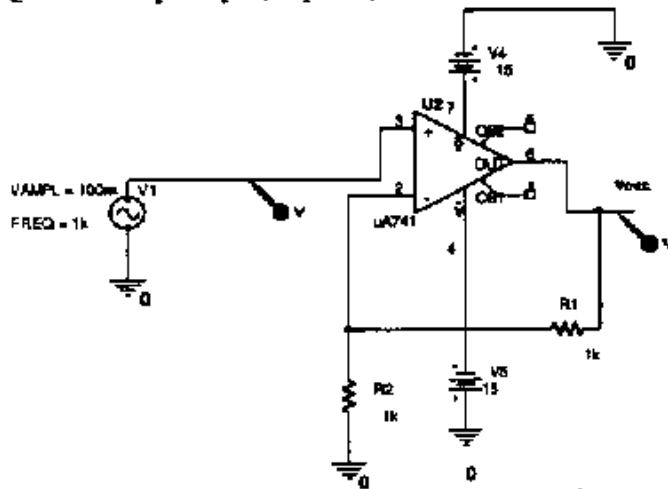
$\omega \ll \omega_0 \Rightarrow$  Substantially smaller

Would you say that, for this circuit, the frequency of the source is high, low or neither? Very roughly sketch the magnitude of the transfer function for this circuit as a function of frequency. You only need to show the general shape of the magnitude, not the phase.

Low:  $\omega \ll \omega_0$



Question 4 Op-Amps (25 points)



Above is a figure of an Op Amp Circuit and its input and output voltage as seen in Pspice.

a) Is this an Inverting, Non inverting, or Differential Op Amp Circuit?

*Non - inverting*

b) If the Gain shown on the graph is 2, Calculate the resistance needed for R2 to give the Op Amp Circuit a Gain of 10.

$$A_v = \frac{V_{out}}{V_{in}} = \left(1 + \frac{R_1}{R_2}\right) = 10 \Rightarrow \frac{R_1}{R_2} = 9 \Rightarrow R_2 = \frac{R_1}{9} \Rightarrow \boxed{R_2 = \frac{1}{9} k\Omega}$$

c) What is the Maximum amount of voltage that can ever be read at the Output of the Circuit?

$$V_{max} = V_{cc} = 15V$$