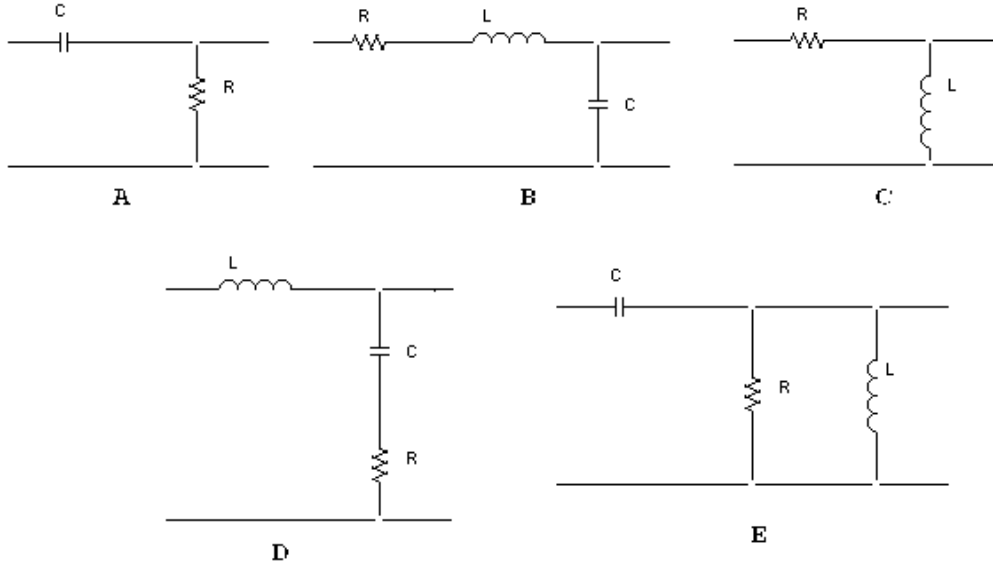


Question 1 -- RLC, RL and RC Circuits (30 points)

Shown below are 5 circuits. Assume the input voltage (V_{in}) is applied across the left-most terminals and the output voltage (V_{out}) is measured across the rightmost terminals.



Given below are several possible expressions for generic transfer functions for such circuits. Indicate which circuit goes with which function. (1 point each)

$\frac{j\omega L}{R + j\omega L + \frac{1}{j\omega C}}$	$\frac{R}{R + \frac{1}{j\omega C}} \quad \mathbf{A}$	$\frac{R + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \quad \mathbf{D}$	$\frac{j\omega C + \frac{1}{j\omega L}}{R + j\omega C + \frac{1}{j\omega L}}$
$\frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} \quad \mathbf{B}$	$\frac{R}{R + j\omega L}$	$\frac{R + j\omega L}{R + j\omega L + \frac{1}{j\omega C}}$	$\frac{R}{j\omega RC + 1}$
$\frac{R}{R + j\omega L + \frac{1}{j\omega C}}$	$\frac{j\omega L}{R + j\omega L} \quad \mathbf{C}$	$j\omega L + \frac{1}{j\omega C}$	$j\omega L + \frac{R}{j\omega RC + 1}$
$\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$	$\frac{j\omega LR}{R + j\omega L} \quad \mathbf{E}$	$R + j\omega L + \frac{1}{j\omega C}$	$\frac{1}{j\omega C} + \frac{j\omega LR}{R + j\omega L}$

Note: Test B has D and E switched

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Find the approximate resonant frequency ω_0 for the RLC circuits and the corner frequency ω_c for the other circuits. That is, write the general expression for each frequency. (1 point each)

A. $\omega_c = 1/RC$

B. $\omega_0 = 1/(LC)^{1/2}$

C. $\omega_c = R/L$

D. $\omega_0 = 1/(LC)^{1/2}$

E. $\omega_0 = 1/(LC)^{1/2}$

Determine the complex transfer function for two of the five circuits (A and B) at the resonant frequency or corner frequency. Be sure your answer is given in terms of R, L, and/or C and does not contain ω . This may seem like an obvious comment, but we want to make sure you have the simplest possible expression. Identify the magnitude and the phase of the transfer function at this frequency.

$$A. (4 \text{ points}) V_{\text{out}}/V_{\text{in}} = H(j\omega_c) = j \omega_c RC / (j \omega_c RC + 1) = j(RC/RC) / (j(RC/RC) + 1) = j/(j+1)$$

$$H(j\omega_c) = j/(j+1)$$

$$|H(j\omega_c)| = 1/(1+1)^{1/2}$$

$$|H(j\omega_c)| = 1/(2)^{1/2}$$

$$\angle H(j\omega_c) = \pi/2 + \pi/4$$

$$\angle H(j\omega_c) = 3\pi/4$$

$$B. (4 \text{ points}) V_{\text{out}}/V_{\text{in}} = H(j\omega_0) = 1/(j\omega_0 RC + 1 - \omega_0^2 LC) = 1/(j(RC/(LC))^{1/2}) = -j(LC)^{1/2}/RC$$

$$H(j\omega_0) = -j(LC)^{1/2}/RC$$

$$|H(j\omega_0)| = (LC)^{1/2}/RC$$

$$\angle H(j\omega_0) = -\pi/2$$

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Determine the transfer function, magnitude of the transfer function, and phase of the transfer function at low frequencies for C and D. These should be simplified and expressed these in terms of ω where appropriate.

C. (4 points) $V_{\text{out}}/V_{\text{in}} = H(j\omega_o) = j\omega_o L/R$

$$|H(j\omega_o)| = 0$$

$$\angle H(j\omega_o) = \mathbf{p/2}$$

D. (4 points)

$$\text{Test A: } V_{\text{out}}/V_{\text{in}} = H(j\omega_o) = (j\omega_o RC + 1)/(j\omega_o RC + 1 - \omega_o^2 LC) = 1/1 = 1$$

$$\mathbf{H(j\omega_o) = 1}$$

$$|H(j\omega_o)| = \mathbf{1}$$

$$\angle H(j\omega_o) = \mathbf{0}$$

$$\text{Test B: } V_{\text{out}}/V_{\text{in}} = H(j\omega_o) = (-\omega_o^2 LRC)/[R(1 - \omega_o^2 LC) + j\omega_o L] = -\omega_o^2 LRC/R = -\omega_o^2 LC$$

$$\mathbf{H(j\omega_o) = -\omega_o^2 LC}$$

$$|H(j\omega_o)| = \mathbf{0}$$

$$\angle H(j\omega_o) = \mathbf{p \text{ or } -p}$$

Finally, find the transfer function, magnitude of the transfer function, and phase of the transfer function as ω approaches infinity for E. These should be simplified and expressed in terms of ω where appropriate.

E. (4 points)

$$\text{Test A: } V_{\text{out}}/V_{\text{in}} = H(j\omega_{hi}) = (-\omega_{hi}^2 LRC)/[R(1 - \omega_{hi}^2 LC) + j\omega_{hi} L] = -\omega_{hi}^2 LRC / -\omega_{hi}^2 LRC = 1$$

$$\mathbf{H(j\omega_{hi}) = 1}$$

$$|H(j\omega_{hi})| = \mathbf{1}$$

$$\angle H(j\omega_{hi}) = \mathbf{0}$$

$$\text{Test B: } V_{\text{out}}/V_{\text{in}} = H(j\omega_{hi}) = (j\omega_{hi} RC + 1)/(j\omega_{hi} RC + 1 - \omega_{hi}^2 LC) = j\omega_{hi} RC / -\omega_{hi}^2 LC = -jR/\omega_{hi} L$$

$$\mathbf{H(j\omega_{hi}) = -jR/\omega_{hi} L}$$

$$|H(j\omega_{hi})| = \mathbf{0}$$

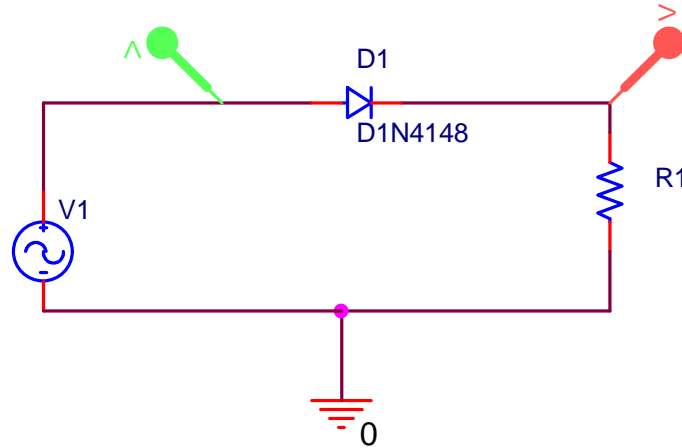
$$\angle H(j\omega_{hi}) = \mathbf{-p/2}$$

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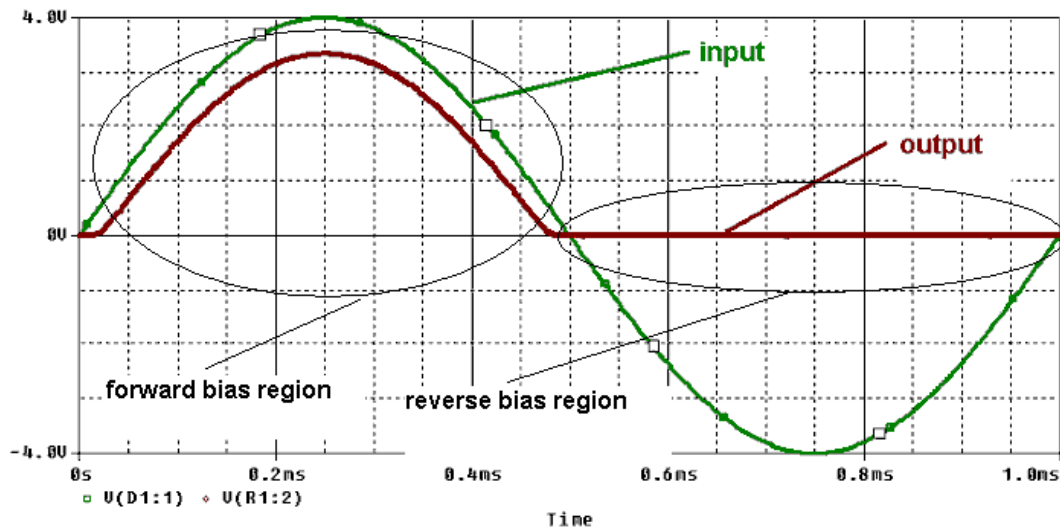
Section _____

Question 2 -- Diodes (25 points)

The figure below is a half wave rectifier.



Let $R1 = 2K$. Here is the Capture/Pspice output for the above circuit:



- a) Indicate the input and output signals on the plot. (4 points)
- b) The input is in the form $v(t) = A \sin(\omega t)$. What is the general equation for the input signal to this circuit? Please give numerical values for A and ω . (4 points)

$$A = 4V \quad \omega = 2\pi/T = 2\pi/1ms = 2K\pi \text{ rad/sec}$$

$$v(t) = 4V \sin(2K\pi t) = 4V \sin(6.28Kt)$$

- c) How would you fill in the following Capture parameters to get this output plot?
Note: Any reasonable step size will be accepted. (3 points)

Run to time: seconds (TSTOP)

Start saving data after: seconds

Transient options

Maximum step size: seconds

Step sizes between 100m and 0.1u are reasonable.

- d) A diode has three active regions: forward bias region, reverse bias region, and breakdown region. Answer the following two questions in regard to the circuit and PSpice output on the previous page.

i. Which of the three regions does not affect the input signal pictured in the PSpice output on the previous page? (2 points)

breakdown region

ii. Circle and label the areas on the output signal when the diode is in the other two regions. (4 points)

- e) What is the value of the *current* through the resistor when the input voltage is at the values listed below. Assume V_{on} for the diode is 0.7V.

i. 4 volts (4 points)

$$\text{Test A: } V=4-0.7=3.3V \quad I=V/R=3.3/2K=1.65mA \quad \mathbf{I=1.65mA}$$

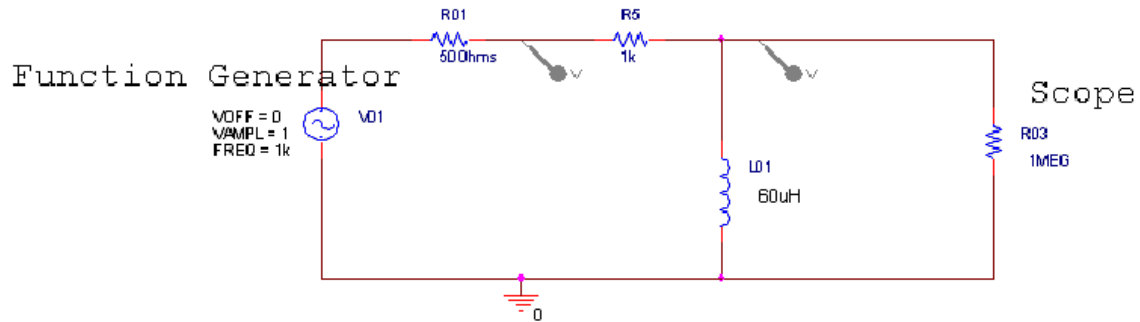
$$\text{Test B: } V=4-0.7=3.3V \quad I=V/R=3.3/1K=3.3mA \quad \mathbf{I=3.3mA}$$

ii. -4 volts (4 points)

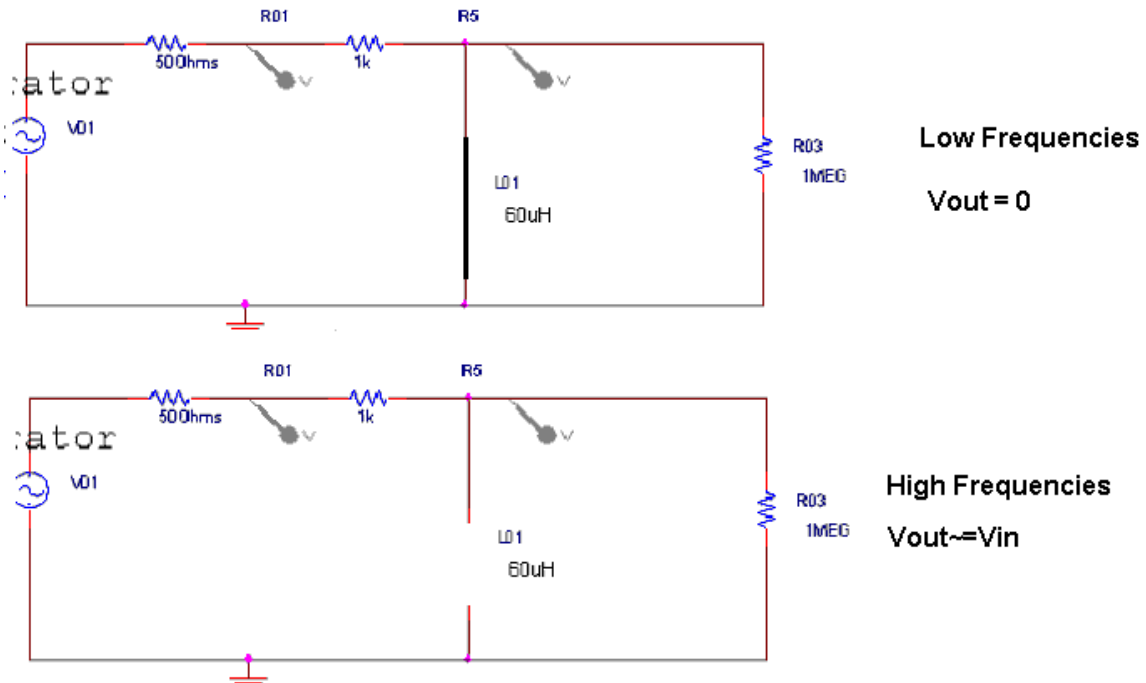
$$V = 0V \quad \mathbf{I=0mA} \text{ (a very small } I_s \text{ is also ok)}$$

Question 3 - Filters (25 points)

The following circuit consists of a sinusoidal source, an inductor and a resistor.



If V_{in} is the sinusoidal source and V_{out} is the voltage across the inductor, is this configuration a high-pass filter, a low-pass filter, a band-pass filter or a band-reject filter? Explain your answer. (8 points)



This is a **HIGH PASS FILTER** because it rejects low frequencies (top figure) and passes high frequencies (bottom figure).

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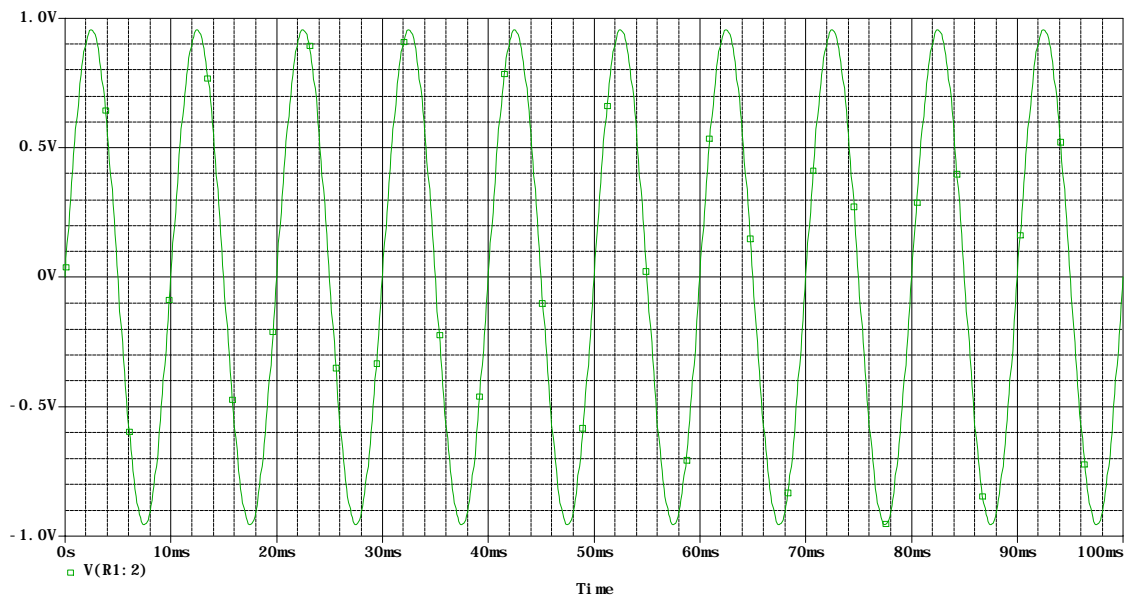
The source is a sinusoidal voltage with some amplitude and frequency. The source voltage, as a function of time, is shown on the next page. Write out the mathematical expression for this voltage function in the form $V_{in} = V_o \sin(\omega t + \phi_o)$. Be sure that you give values for V_o , ω , and ϕ_o . (5 points)

$$V_o = 0.94 \text{ V}$$

$$\omega = 2\pi / T = 2\pi / 10\text{ms} = 2\pi \text{ K} = 628 \text{ rad/sec}$$

$$\phi_o = 0 \text{ rad}$$

$$V_{in} = 0.94\text{V} \sin(628 t)$$



Now that you have determined the magnitude, frequency and phase of the input voltage, you should have some idea of what will happen at the output. From your knowledge of the corner frequency for this circuit, will the output voltage be about the same as the input, substantially smaller or substantially larger than the input? Explain your answer. (5 points)

$$\text{Test A: } \omega c = R/L = 1\text{K}/60\text{EE-6} = 1.67\text{EE+7} \quad \omega c = 17\text{Meg rad/sec} \quad \omega_{in} = 628\text{rad/sec}$$

$$\text{Test B: } \omega c = R/L = 1\text{K}/50\text{EE-6} = 2\text{EE+7} \quad \omega c = 20\text{Meg rad/sec} \quad \omega_{in} = 628\text{rad/sec}$$

Since $\omega c \gg 628\text{rad/sec}$ and this is a high pass filter, the output at ω_{in} will be substantially smaller than the input.

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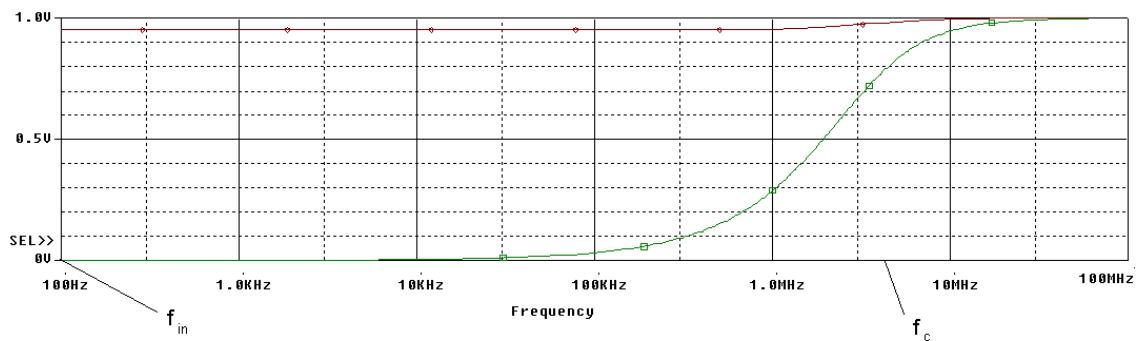
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Would you say that, for this circuit, the frequency of the source is high or low? *Very roughly* sketch the magnitude of the transfer function for this circuit as a function of frequency. You only need to show the general shape of the magnitude, not the phase. Indicate on the graph where the corner frequency is and give the numerical value. Your frequency should cover the entire range that can be measured using the 'scope in the classroom – 1Hz to 60MHz. (6 points)

$$\text{Test A: } f_c = \omega c / 2p = 17\text{Meg} / 2p = 2.7\text{MegHz. } f_{in} = \omega_{in} / 2p = 628 / 2p = 100\text{Hz}$$

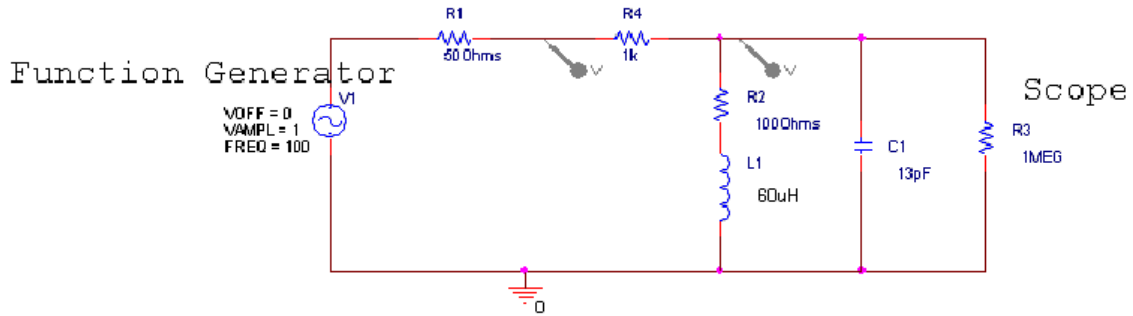
$$\text{Test B: } f_c = \omega c / 2p = 20\text{Meg} / 2p = 3.2\text{MegHz. } f_{in} = 100\text{Hz}$$

The frequency of the source is low.



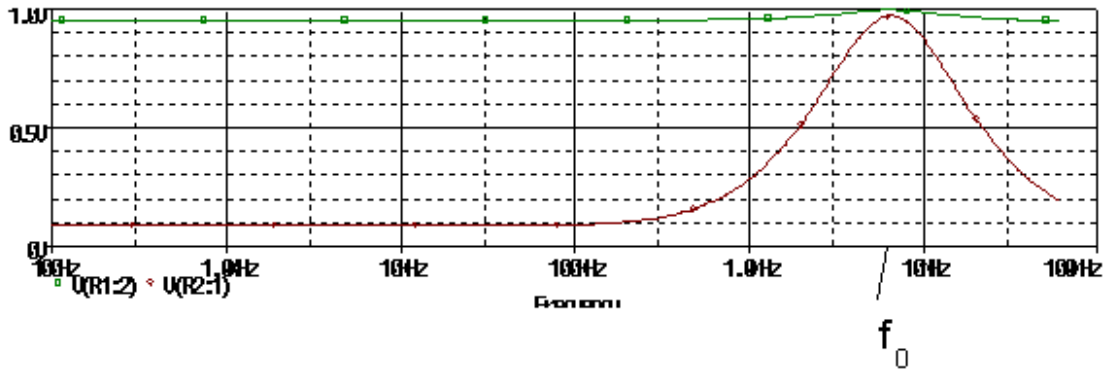
Extra Credit: (2 points)

Realistically, the inductor will have finite resistance and the 'scope has an input capacitance. Answer the last question again for the following circuit, where the inductor now has a resistance of 100 Ohms and the 'scope input capacitance is 13 pF.

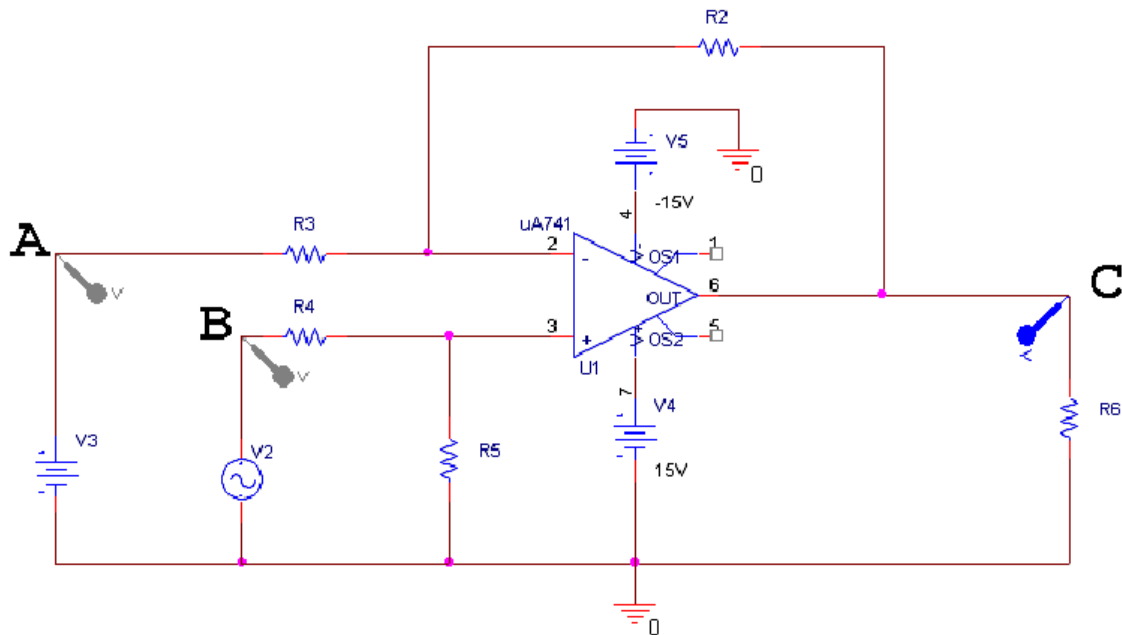


Test A: $\omega_0 = 1/(LC)^{1/2} = 1/(60EE-6 \times 13EE-12)^{1/2} = 3.6EE+7$ $\omega_0 = 36\text{Meg rad/sec}$
 $f_0 = \omega_0/2\pi = 36\text{Meg}/2\pi = 5.7\text{MegHz}$. $f_{in} = \omega_{in}/2\pi = 628/2\pi = 100\text{Hz}$
 Test B: $\omega_0 = 1/(LC)^{1/2} = 1/(50EE-6 \times 13EE-12)^{1/2} = 3.9EE+7$ $\omega_0 = 39\text{Meg rad/sec}$
 $f_0 = \omega_0/2\pi = 39\text{Meg}/2\pi = 6.2\text{MegHz}$. $f_{in} = \omega_{in}/2\pi = 628/2\pi = 100\text{Hz}$

The input frequency is still low. These additions do not change the transition frequencies very much. (They shouldn't if the devices are well designed.) Below is a picture:



Question 4 -- Op-Amps (20 points)



Assume the following about the components in the above circuit:

V2: $V_{OFF}=2V, V_{AMPL}=2V, FREQ=1K$.

V3: $VDC=2V$

$R2=16K, R3=2K, R4=2K, R5=16K, R6=1K$

- a. Above is a picture of a type of amplifier you have seen. What type of amplifier is it? (1 point)

differential (or difference) amplifier

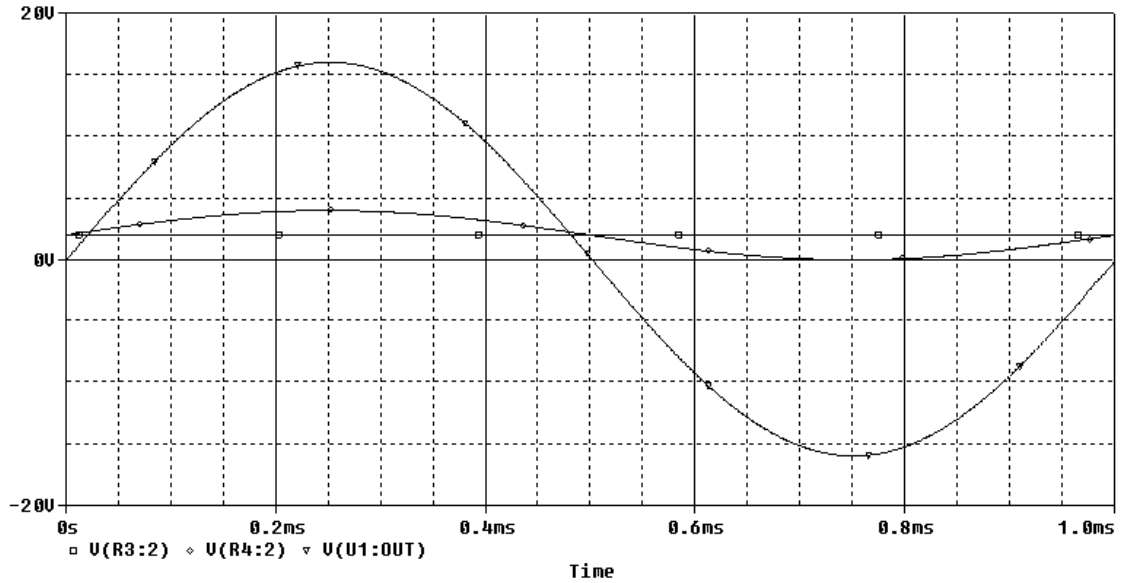
- b. Write an equation for the output at C (V_C) in terms of the input voltages V2 and V3. Simplify. (3 points)

Test A: $V_{out}=(R_f/R_{in})(V_+-V_-)=(16K/2K)(V2-V3)=8(V2-V3)$ **$V_{out}=8(V2-V3)$**

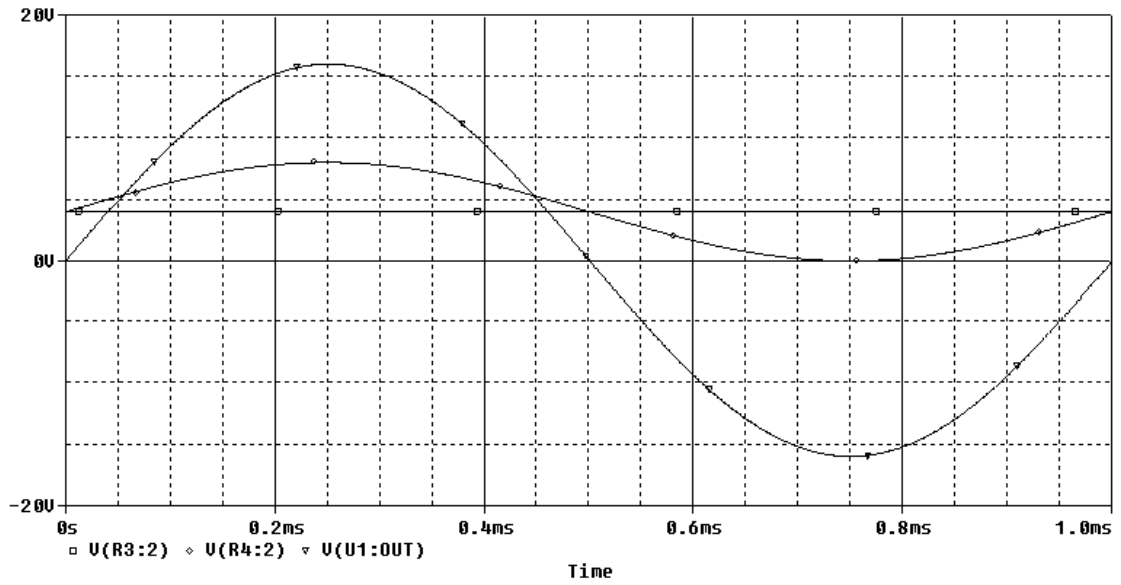
Test B: $V_{out}=(R_f/R_{in})(V_+-V_-)=(12K/3K)(V2-V3)=4(V2-V3)$ **$V_{out}=4(V2-V3)$**

c. Sketch and label one cycle of the input at V2 (point B), the input at V3 (point A) and the output at C (V_C) on the plot below. (16 points)

Test A:



Test B:



More correct for both would cut off at 15 volts like this:

