ENGR4300 | **Test 2A&B** | **Spring 2004**
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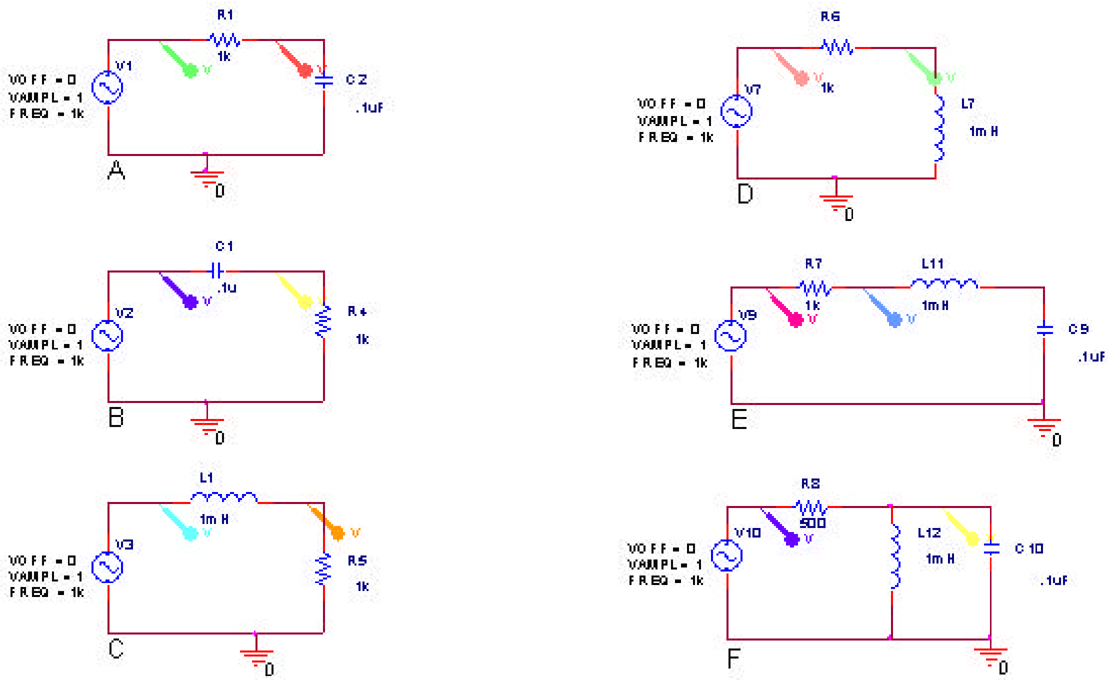
Question 1 – Transfer Functions (32 points)

Consider a variety of filter configurations that can be analyzed with PSpice. All the resistors (except one) shown are 1k, all the inductors are 1mH and all the capacitors are 0.1uF. In general the components can assume any realistic value. Thus, in most of this problem, we will only assume that they have some unknown value. *For each of these circuits, assume that the input and output voltages are measured at the two locations where we have added a voltage marker.*



a) First, identify which type of filter these are: (6 points)

Which is a low pass filter (list all): ***A, C***

Which is a high pass filter (list all): ***B, D***

Which is a band pass filter (list all): ***F***

Which is a band reject filter (list all): ***E***

b) The complex transfer functions for all of these filters are given below. Identify which circuit goes with each transfer function. (6 points)

$$\frac{jwL}{R+jwL} \quad \textbf{\textit{D}} \qquad \frac{1/jwC + jwL}{R + jwL + 1/jwC} \quad \textbf{\textit{E}} \qquad \frac{R}{R + 1/jwC} \quad \textbf{\textit{B}}$$

$$\frac{R}{R+jwL} \quad \textbf{\textit{C}} \qquad \frac{\dfrac{\frac{jwL}{jwC}}{jwL + 1/jwC}}{R + \dfrac{\frac{jwL}{jwC}}{jwL + 1/jwC}} \quad \textbf{\textit{F}} \qquad \frac{1/jwC}{R + 1/jwC} \quad \textbf{\textit{A}}$$

c) The voltage and phase vs frequency for each of these filters is shown on the following two pages. Identify which plot goes with each transfer function. Show your work below for partial credit: (12 points)

The easiest way to distinguish between the plots is as follows:
- *F is the only band pass filter*
- *E is the only band reject filter*
- *A and C are both low pass filters. Find the corner frequency of each to determine which one is which (see below).*
- *B and D are both high pass filters. Find the corner frequency of each to determine which one is which (see below).*

Here is the complete analysis of each circuit. Note how each corresponds to the plots.

A: $$\frac{1/jwC}{R+1/jwC} = \frac{1}{jwRC+1} \quad H_{LO}=1 \quad |H_{LO}|=1 \quad \angle H_{LO}=0 \quad H_{HI}=\frac{1}{jwRC} \quad |H_{HI}|=0 \quad \angle H_{HI}=-\frac{p}{2}$$

$$w_c=\frac{1}{RC}=\frac{1}{(1K)(.1mF)}=10000 \quad f_c=\frac{w_c}{2p}\approx 1600Hz \quad LPF$$

B: $$\frac{R}{R+1/jwC}=\frac{jwRC}{jwRC+1} \quad H_{LO}=jwRC \quad |H_{LO}|=0 \quad \angle H_{LO}=\frac{p}{2} \quad H_{HI}=\frac{jwRC}{jwRC}=1 \quad |H_{HI}|=1 \quad \angle H_{HI}=0$$

$$w_c=\frac{1}{RC}=\frac{1}{(1K)(.1mF)}=10000 \quad f_c=\frac{w_c}{2p}\approx 1600Hz \quad HPF$$

C: $\frac{R}{R+jwL}$ $\quad H_{LO}=\frac{R}{R}=1 \quad |H_{LO}|=1 \quad \angle H_{LO}=0 \quad H_{HI}=\frac{R}{jwL} \quad |H_{HI}|=0 \quad \angle H_{HI}=-\frac{p}{2}$

$$w_c=\frac{R}{L}=\frac{1K}{(1mH)}=1000000 \quad f_c=\frac{w_c}{2p}\approx 160000Hz \quad LPF$$

D: $\frac{jwL}{R+jwL}$ $\quad H_{LO}=\frac{jwL}{R} \quad |H_{LO}|=0 \quad \angle H_{LO}=\frac{p}{2} \quad H_{HI}=\frac{jwL}{jwL}=1 \quad |H_{HI}|=1 \quad \angle H_{HI}=0$

$$w_c=\frac{R}{L}=\frac{1K}{(1mH)}=1000000 \quad f_c=\frac{w_c}{2p}\approx 160000Hz \quad HPF$$

$$\frac{1/jwC+jwL}{R+jwL+1/jwC}=\frac{1-w^2LC}{jwRC+1-w^2LC} \quad H_{LO}=\frac{1}{1} \quad |H_{LO}|=1 \quad \angle H_{LO}=0$$

E: $H_{HI}=\frac{-w^2LC}{-w^2LC}=1 \quad |H_{HI}|=1 \quad \angle H_{HI}=0$

$$w_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(1m)(0.1m)}}=100000 \quad f_0=\frac{w_0}{2p}\approx 16000Hz \quad BRF$$

F:

$$\frac{\dfrac{\frac{jwL}{jwC}}{jwL+1/jwC}}{R+\dfrac{\frac{jwL}{jwC}}{jwL+1/jwC}}=\frac{\dfrac{jwL}{-w^2LC+1}}{R+\dfrac{jwL}{-w^2LC+1}}=\frac{jwL}{R(1-w^2LC)+jwL}$$

$$H_{LO}=\frac{jwL}{R} \quad |H_{LO}|=0 \quad \angle H_{LO}=\frac{p}{2}$$

$$H_{HI}=\frac{jwL}{-w^2RLC}=\frac{-j}{wRC} \quad |H_{HI}|=0 \quad \angle H_{HI}=-\frac{p}{2}$$

$$w_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{(1m)(0.1m)}}=100000 \quad f_0=\frac{w_0}{2p}\approx 16000Hz \quad BPF$$

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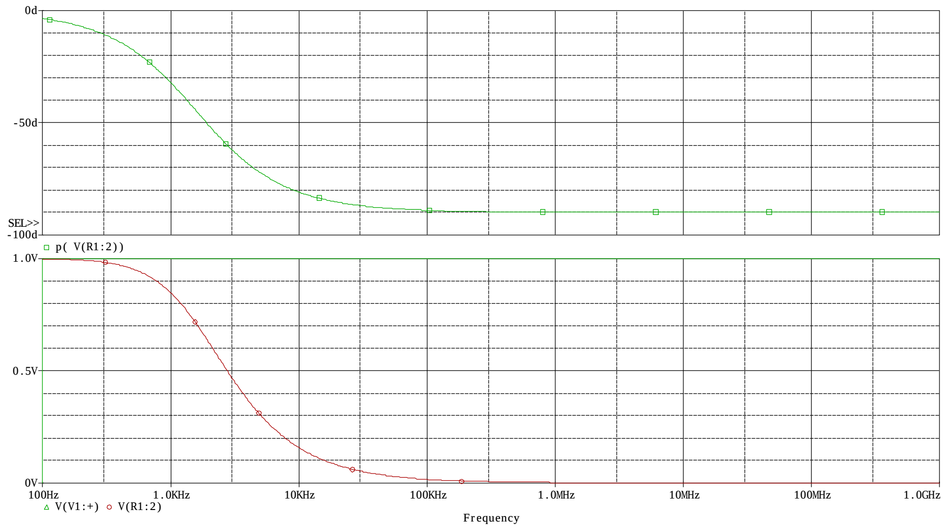
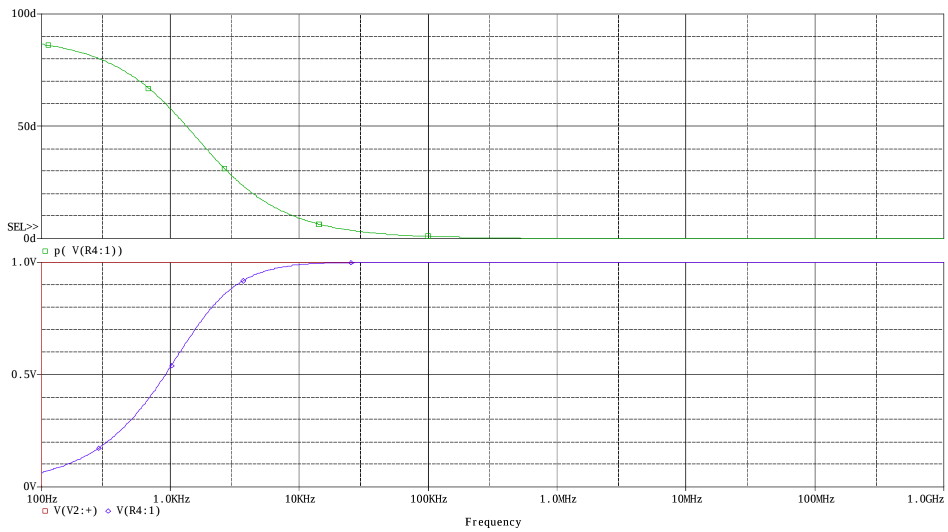
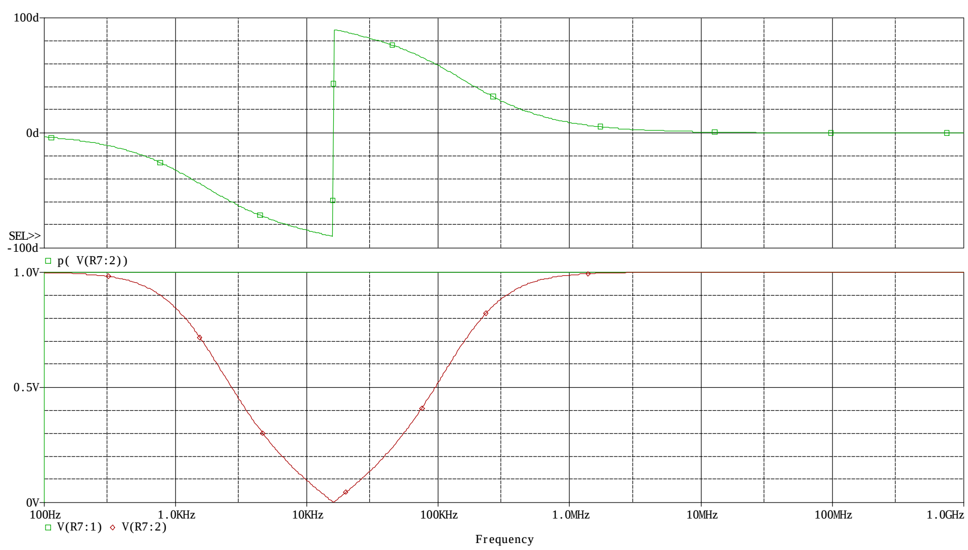


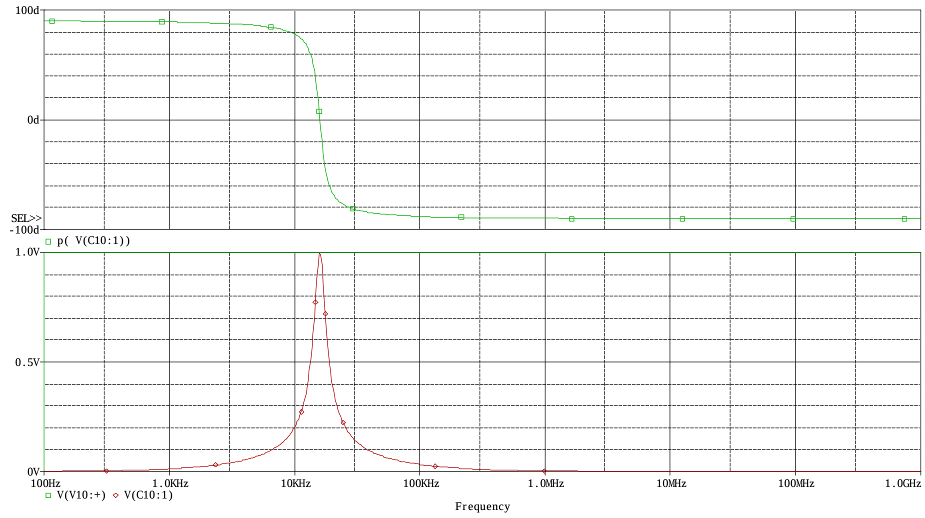
A



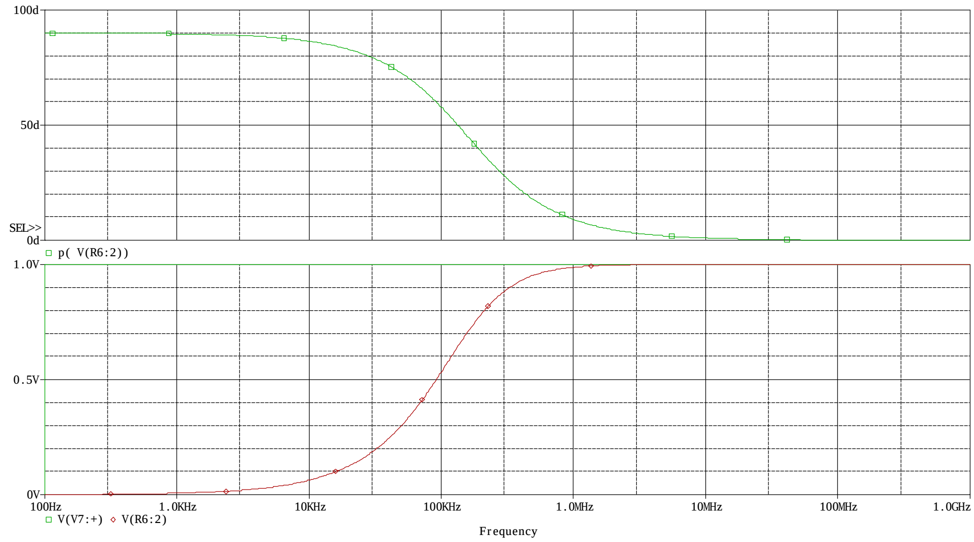
B



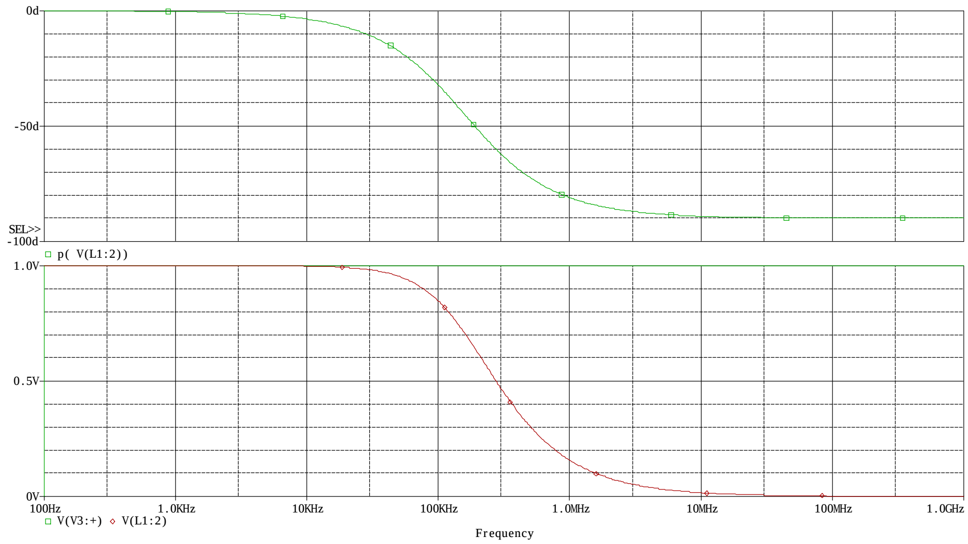
E


100d
0d
SEL>>
-100d
p(V(C10:1))
1.0V
0.5V
0V
100Hz
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
V(V10:+) V(C10:1)
Frequency

F

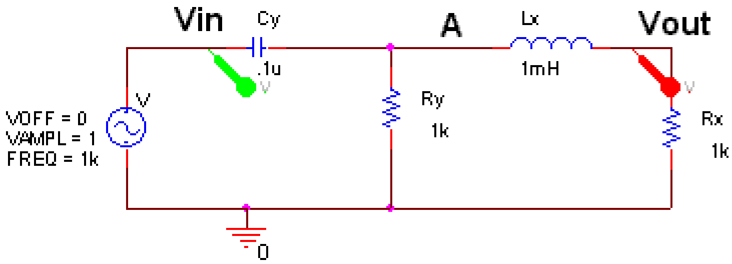

100d
50d
SEL>>
0d
p(V(R6:2))
1.0V
0.5V
0V
100Hz
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
V(V7:+) V(R6:2)
Frequency

D


0d
-50d
SEL>>
-100d
p(V(L1:2))
1.0V
0.5V
0V
100Hz
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
V(V3:+) V(L1:2)
Frequency

C

d) Find the transfer function for the following circuit. Assume there is a buffer at point A which forces the two transfer functions to be separate. (Hint- it is a combination of two of the circuits we have seen above.) (4 points)



Assuming the presence of the buffer, we can assume that the output of one part of the circuit (y) is the input to the other (x). Hence:

$$\frac{V_A}{V_{in}} = H_y \qquad \frac{V_{out}}{V_A} = H_x \qquad \frac{V_{out}}{V_{in}} = H_x H_y$$

$$H_y = \frac{R_y}{1/jwC_y + R_y} = \frac{jwR_yC_y}{1 + jwR_yC_y} \qquad H_x = \frac{R_x}{jwL_x + R_y}$$

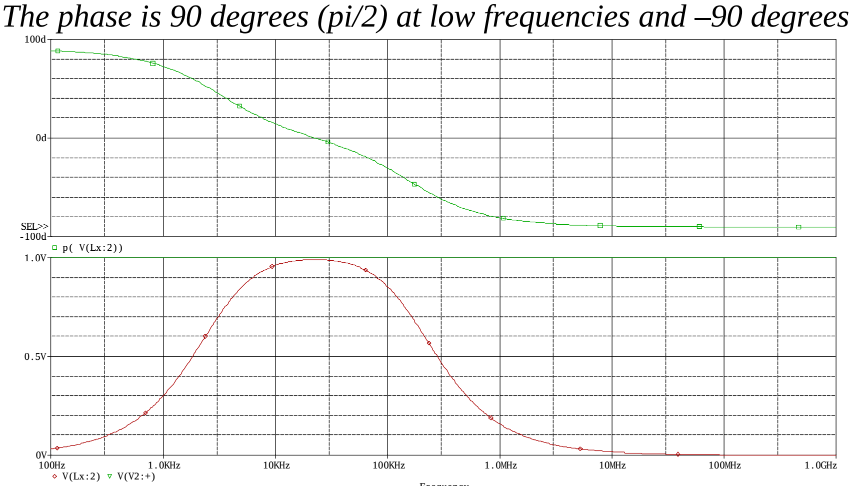
$$H_y H_x = \frac{jwR_xR_yC_y}{(1 + jwR_yC_y)(jwL_x + R_y)} = \frac{jwR_xR_yC_y}{jwL_x + R_y - w^2R_yL_xC_y + jwR_y^2C_y}$$

e) Simplify this transfer function at very low frequencies and very high frequencies and show that your results are consistent with the voltage magnitude and phase plot below. What kind of a filter is this combined circuit? (4 points)

$$H_y H_x = \frac{jwR_xR_yC_y}{jwL_x + R_y - w^2R_yL_xC_y + jwR_y^2C_y}$$

$$H_{LO} = \frac{jwR_xR_yC_y}{R_y} = jwR_xC_y \quad |H_{LO}| = 0 \quad \angle H_{LO} = \frac{p}{2} \quad H_{HI} = \frac{jwR_xR_yC_y}{-w^2R_yL_xC_y} = \frac{-jR_x}{w\ L_x} \quad |H_{HI}| = 0 \quad \angle H_{HI} = -\frac{p}{2}$$

The magnitude is zero at both high and low frequencies, as seen in the plot.
The phase is 90 degrees (pi/2) at low frequencies and –90 degrees (-pi/2) at high frequencies.



f) Extra Credit – Which of the original six filters is also a combination of two other filters? Explain your answer.

The band reject filter, E, is a combination of two other filters. A high pass filter with a corner frequency of 160000 Hertz and a low pass filter with a corner frequency of 1.600 Hertz. This makes it a combination of A and D. You can see this by looking at the plots for both phase and magnitude. When the magnitude of each goes to zero, the other dominates.

Mathematically, this can be shown, but it is not that obvious and you should read the links below if you want to understand it fully....This was not required for the extra point.

$H_E = \frac{1-w^2LC}{jwRC+1-w^2LC}$ *There is a zero at the resonant frequency. To examine the function below resonance, we throw out the terms that dominate at the highest frequencies. This would be* $-w^2LC$*. This means that at frequencies below resonance, the function simplifies to*

$H_{E-LO} = \frac{1}{jwRC+1}$. *This is* H_A*, the filter that determines low frequency behavior. At frequencies above the resonant frequency, we simplify the function by throwing out the terms that dominate at low frequency. Therefore the function becomes*

$H_E = \frac{-w^2LC}{jwRC-w^2LC} = \frac{-wL}{jR-wL} = \frac{-jwL}{-R-jwL} = \frac{jwL}{R+jwL}$. *This is* H_D*, the filter that determines high frequency behavior.*

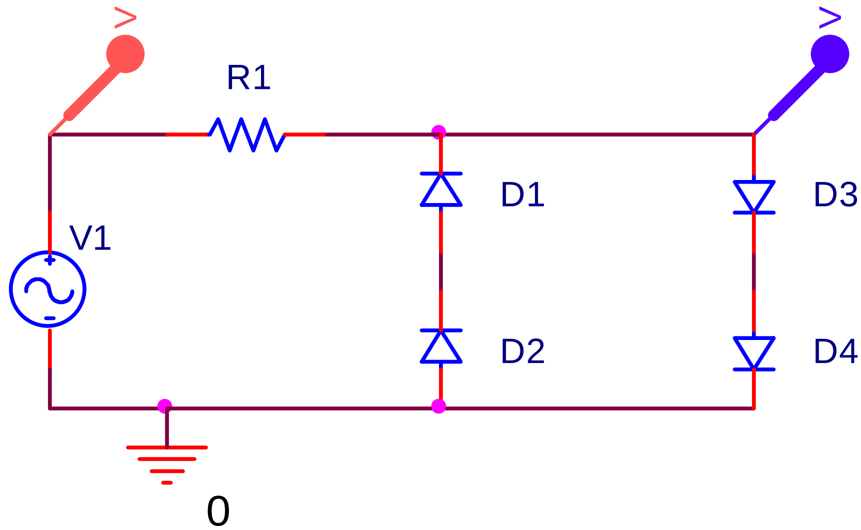
The band pass filter, F, is also a combination of two filters. A high pass filter with a corner frequency of around 160000 Hertz and a low pass filter with about the same corner frequency. These are not any of the filters we have here, however, the question is vague about whether you need to combine two filters that are pictured, or just two filters.

For more details on why this is true, see the Gingrich notes:
http://www.phys.ualberta.ca/%7Egingrich/phys395/notes/node48.html#SECTION00450000000000000000
and
http://www.phys.ualberta.ca/%7Egingrich/phys395/notes/node50.html#SECTION00461000000000000000

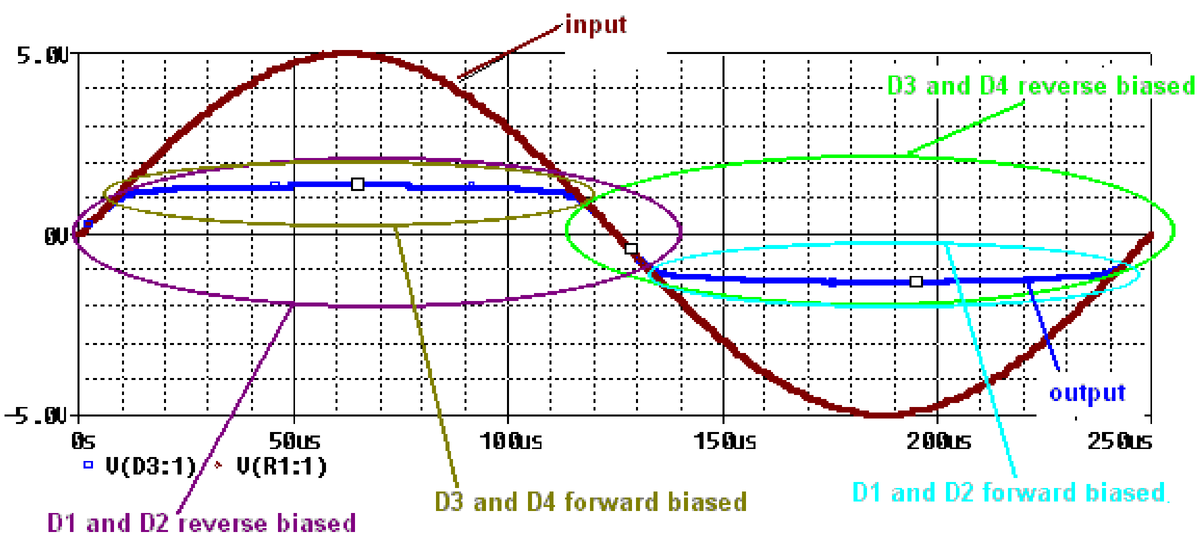
Filter design is not always an exact science. Notice how he uses limits to decide the behavior of his circuit elements instead of the exact transfer functions.

Question 2 – Diodes (23 points)

The figure below is a dual voltage limiter. A: Assume R1=1K. B: Assume R1=3K.



Here is a picture of the output for the above circuit:



a) Indicate the input and output signals on the plot (4 points)

b) The input is in the form v(t)=Asin(ωt). What is the general equation for the input signal to the circuit? Please give the numerical values for A and ω. (4 points)

A=5V T=250us=.25ms=(1/4)ms f= 4K Hz   w=2pf=8Kp rad/sec=25.1Krad/sec

v(t) = 5V sin(25.1K t)

c) Each diode in the circuit has a forward bias region, a reverse bias region and a breakdown region. Answer the following questions with regard to the circuit and PSpice output on the previous page.

i) Which of the three regions does not affect the input signal pictured in the output on the previous page? (1 point)

breakdown region

ii) Circle and label the area on the PSpice output where diode D1 and D2 are in the reverse bias region. (2 points)

iii) Circle and label the area on the PSpice output where diode D3 and D4 are in the reverse bias region. (2 points)

iv) Circle and label the area on the PSpice output where diodes D1 and D2 are in the forward bias region. (2 points)

v) Circle and label the area on the PSPice output where the diodes D3 and D4 are in the forward bias region. (2 points)

d) What is the value of the current through the resistor when the input voltage is at the values listed below. Assume V_{on} for each diode is 0.7 Volts.

i) 4 volts (3 points)

A: R3=1K 4 – 2(0.7) = 2.6V I=2.6/1K ***I = 2.6mA***

B: R3=3K 4 – 2(0.7) = 2.6V I=2.6/3K ***I = 0.87mA***

ii) -4 volts (3 points)

A: -4 – 2(-0.7) = -2.6V I=-2.6/1K ***I = -2.6mA***

A: -4 – 2(-0.7) = -2.6V I=-2.6/3K ***I = -0.87mA***

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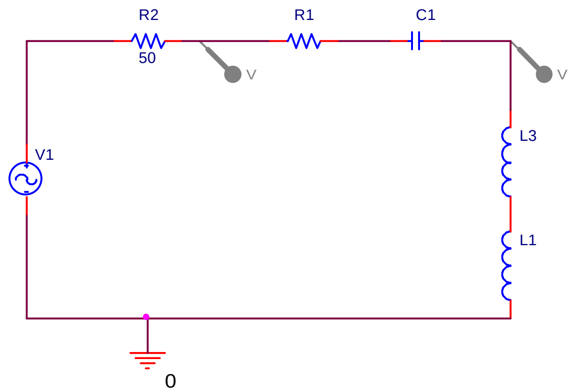
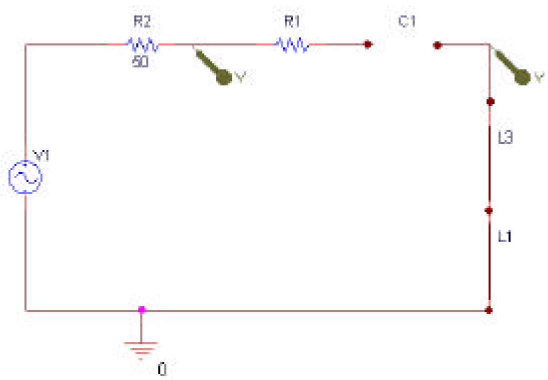
Question 3 – Filters (25 points)

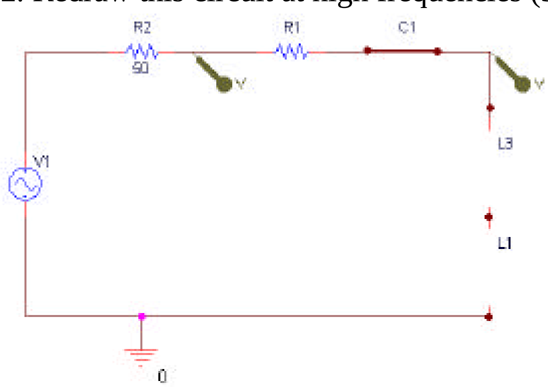


In the circuit above, V1 is a sinusoidal source and R2 is the 50 mohm impedance of the function generator. The remaining four components define some type of filter.

1. Redraw this circuit at low frequencies (3 points)



2. Redraw this circuit at high frequencies (3 points)

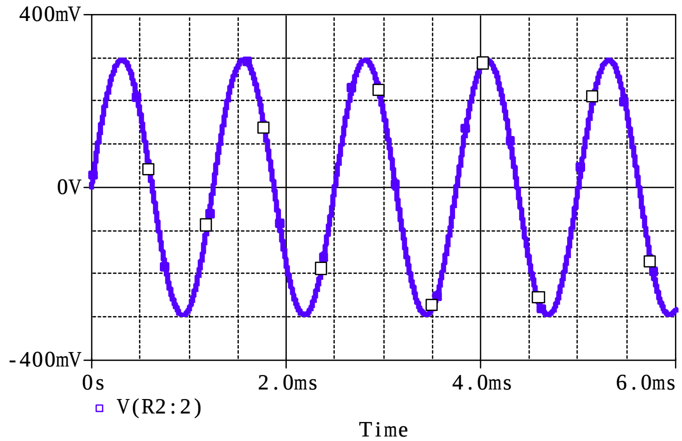


3. Is this a high pass filter, a band pass filter, a low pass filter or a band reject filter? Explain your answer. (4 points)

This is a high pass filter. *At low frequencies, the output point is connected to ground (0) [Vout=0] and at high frequencies the output point is connected to the input voltage and, since there is no current flowing, there is no voltage drop across the resistors [Vout=Vin].*

The following is for test A:

4A. Given the following source input:



Write the mathematical expression for Vin in the form: Vin=Asin(ωt) (4 points)

A=300mV T=2.5ms/2cylces=1.25ms f= 800 Hz w=2pf=1.6Kp rad/sec=5.03Krad/s

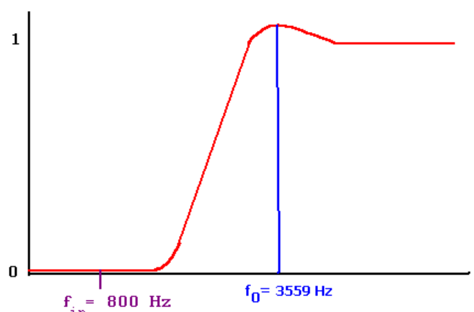
v(t) = 300mV sin(5.03K t)

5A. If L1=2mH, L2=2mH, C1=0.5uF and R1=3K, what is the resonant frequency of this circuit in Hertz? (4 points)

L_{total}=L1+L2=2m+2m=4mH w_0=1/sqrt(LC)=1/sqrt[(4m)(0.5u)]=22361rad/s

f_0=w_0/2p=3559Hz

6A. Create a rough sketch of the magnitude of the transfer function of this circuit as a function of frequency. You need only show the general shape. Indicate on the graph where the resonant frequency and input frequency (from part 4) are located. Please give a numerical value. (4 points)

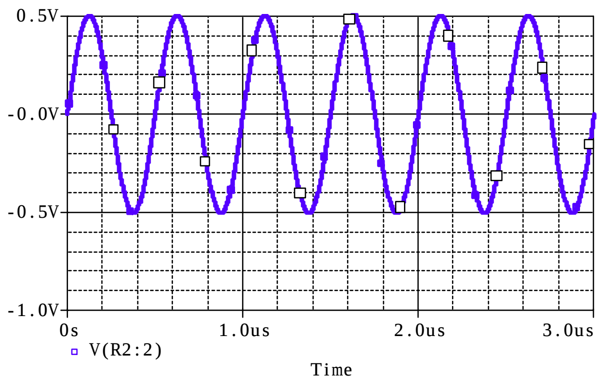


7A. Given your knowledge of corner and resonant frequencies, will Vout have a greater, equal or lesser value than Vin when Vin is equal to the input pictured in part 4? Explain your answer. (3 points)

Vout will be much less than Vin when the input frequency is 800 Hertz. At that value the transfer function is near zero. The output, Vout, is equal to the input, Vin, times the transfer function. Vin times near zero is very small.

The following is for test B:

4B. Given the following source input:



Write the mathematical expression for Vin in the form: Vin=Asin(ωt) (4 points)

A=500mV T=1us/2cylces=0.5us f= 2Meg Hz w=2pf=4Megp rad/sec=12.6Meg rad/s

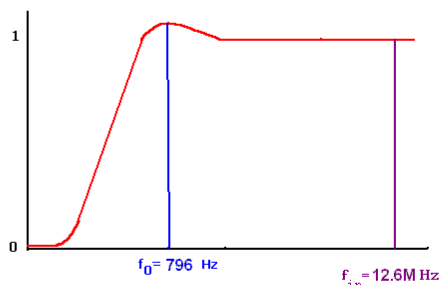
v(t) = 500mV sin(12.6Meg t)

5B. If L1=10mH, L2=10mH, C1=2uF and R1=4K, what is the resonant frequency of this circuit in Hertz? (4 points)

L_{total}=L1+L2=10m+10m=20mH w_0=1/sqrt(LC)=1/sqrt[(20m)(2u)]=5K rad/s

f_0=w_0/2p=796 Hz

6B. Create a rough sketch of the magnitude of the transfer function of this circuit as a function of frequency. You need only show the general shape. Indicate on the graph where the resonant frequency and input frequency (from part 4) are located. Please give a numerical value. (4 points)



7A. Given your knowledge of corner and resonant frequencies, will Vout have a greater, equal or lesser value than Vin when Vin is equal to the input pictured in part 4? Explain your answer. (3 points)

Vout will be about the same as Vin when the input frequency is 12.6M Hertz. At that value the transfer function is near one. The output, Vout, is equal to the input, Vin, times the transfer function. Vin times one is Vin.

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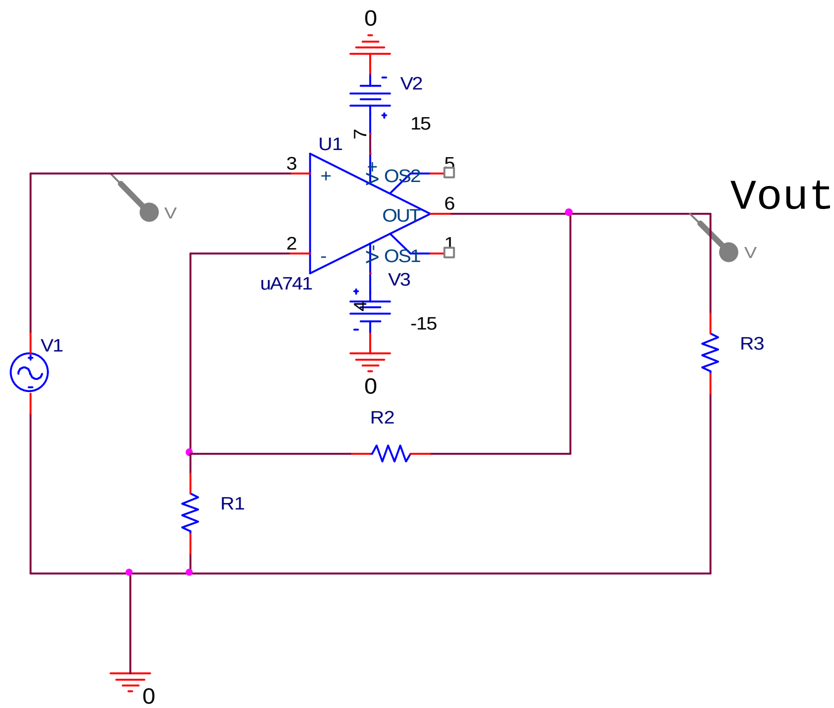
Question 4 – Op Amps (20 points)

Below is a Capture schematic of an op-amp amplifier circuit that you should recognize.



1. What kind of amplifier is it? (2 points)

non-inverting amplifier

2. What are the two "golden rules: of op-amp analysis? (2 points)

1) The current at both of the op-amp inputs is zero. $I_+ = I_- = 0$
2) The op-amp attempts to make the voltages at the two inputs equal. $V_+ = V_-$

3. Use these rules to derive an expression for Vout in terms of R1, R2 and V1. (8 points)

1) Remove the op-amp from the circuit
2) Draw a circuit for each input:
+: V1 o----------o V+
- : Vout o------[R2]-----o-------[R1]-------o GND
V-
3) Simplify and solve for Vout/Vin:
V+=V-=V1
V- = Vout(R1)/(R1+R2)
V1=Vout(R1)/(R1+R2)
Vout/V1 = (R1+R2)/R1

Vout = [1 + (R2/R1)]V1

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The following is for Test A:

4A. If V1=500mV, R1=1K and R2=4K, what is Vout? (4 points)

Vout = [1+(4K/1K)](500mV) = 2500mV ***Vout=2.5V***

5A. Find the current through the load resistor, R3, assuming the component values in part 4 and R3= 2K ohms. (4 points)

I = V/R = 2.5V/2K = 1.25mA ***I=1.25mA***

The following is for Test B:

4B. If V1=300mV, R1=2K and R2=8K, what is Vout? (4 points)

Vout = [1+(8K/2K)](300mV) = 1500mV ***Vout=1.5V***

5B. Find the current through the load resistor, R3, assuming the component values in part 4 and R3= 5K ohms. (4 points)

I = V/R = 1.5V/5K = 0.3mA ***I=0.3mA***