ENGR-4300
Spring 2007
Test 2A

Name Solution

Section 1

Question I (20 points) ________

Question II (20 points) ________

Question III (15 points) ________

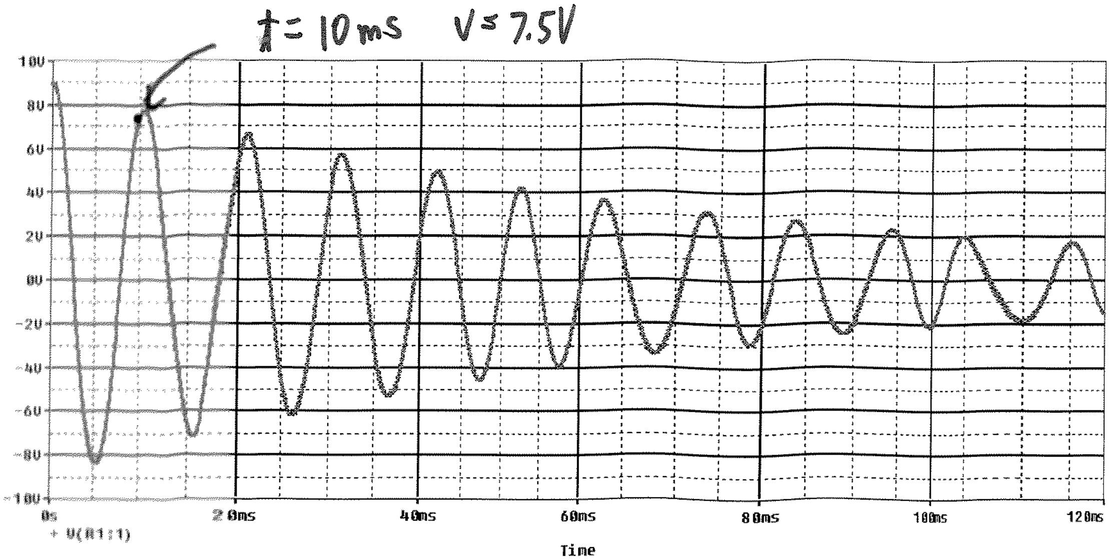
Question IV (20 points) ________

Question V (25 points) ________

Total (100 points): ________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question I – Bridges and Damped Sinusoids (20 points)



1) The figure above is the output signal from an amplified strain gauge bridge similar to the one you are using for experiment 5. The signal has the form of: $v(t) = V_0 e^{-\alpha t}\cos(\omega t)$ and has been calibrated to indicate the displacement of the end of the beam as a function of time. The initial voltage at t = 0 corresponds to a positive displacement of 1.8cm.

For partial credit: show your work and mark and label the graph with the data points you used.

a) (2pt) From the plot determine V_0 = 9V (include units)

b) (2pt) What is the scaling constant for converting volts to cm? $\frac{1.8\text{cm}}{9\text{V}} = 0.2\,\frac{\text{cm}}{\text{V}}$

c) (2pt) What is the displacement of the end of the beam at t = 10ms (include units)?

@ t = 10ms V = 7.5 V ⟹ 1.5cm

d) (3pt) Write an expression for the displacement x(t) in terms of α and ω.

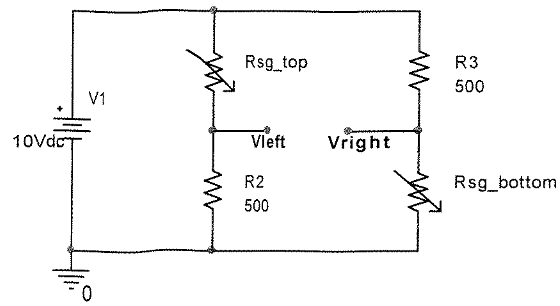
$$x(t) = 1.8 e^{-\alpha t}\cos(\omega t)\ \text{cm}$$

e) (2pt) Write an equation or explain how you could determine the speed s(t) of the end of the beam as a function of time.

$$s(t) = \frac{d}{dt}x(t) = \frac{d}{dt}1.8 e^{-\alpha t}\cos(\omega t)\ \text{cm}$$

Question I – Bridges and Damped Sinusoids (continued)

2) The diagram on the right is a schematic of a bridge circuit on a cantilevered beam with 2 strain gauges. It is a little different from the one used for Experiment 5.



R_{sg_top} represents the strain gauge on the top of the beam, R_{sg_bottom} is the strain gauge on the bottom of the beam.

Let $v_{out} = (v_{left} - v_{right})$ and both strain gauges have a resistance of 500Ω if the beam isn't deflected.

a) (4pt) If the beam is pulled up 1cm, the resistance of the top strain gauge decreases by 2Ω, and the resistance of the bottom stain gauge increases by the same amount. Fill in the table below:

Beam Deflection	v_{out}
1cm up	40μV
0 deflection	0V
2 cm down	0.16 mV

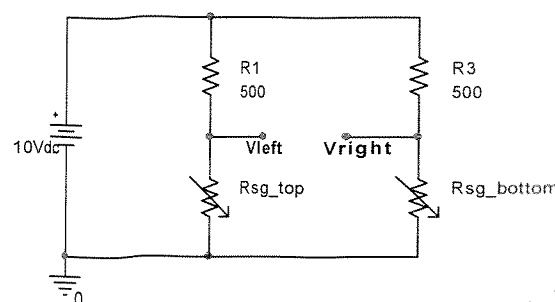
1cm up: $V_{out} = 10\left[\frac{500}{500+498} - \frac{502}{502+500}\right]$

$= 40\mu V$

2 cm down: $V_{out} = 10\left[\frac{500}{500+504} - \frac{496}{496+500}\right]$

$= 0.16\ mV$

b) (4pt) Repeat the calculations but now assume that the circuit configuration has changed to that on the right.



Beam Deflection	v_{out}
1cm up	-20 mV
0 deflection	0V
2 cm down	+40 mV

1cm up: $V_{out} = 10\left[\frac{498}{498+500} - \frac{502}{502+500}\right]$

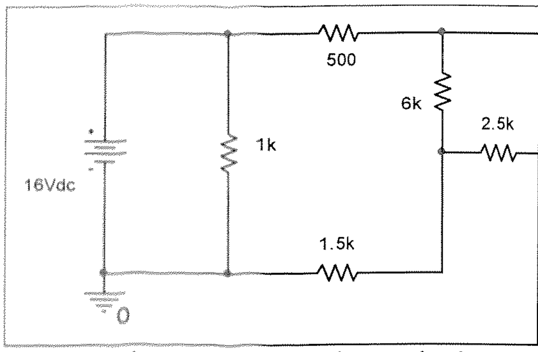
$= -20mV$

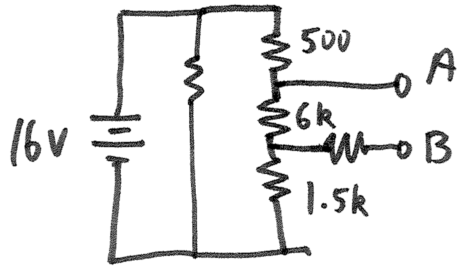
2cm down: $V_{out} = 10\left[\frac{504}{504+500} - \frac{496}{496+500}\right]$

$= +40mV$

c) (1pt) Which circuit's output voltage v_{out} is more sensitive to strain gage resistance change for the same displacement? SECOND, b) CIRCUIT

Question II – Thevenin Equivalents (20 points)

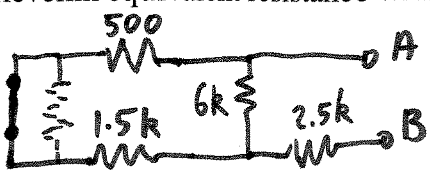




1) (5pt) Find the Thevenin equivalent voltage with respect to A and B for the circuit shown above.

$$V_{AB} = 16 \frac{6k}{500 + 6k + 1.5} = \underline{\underline{12V}} = V_{TH}$$

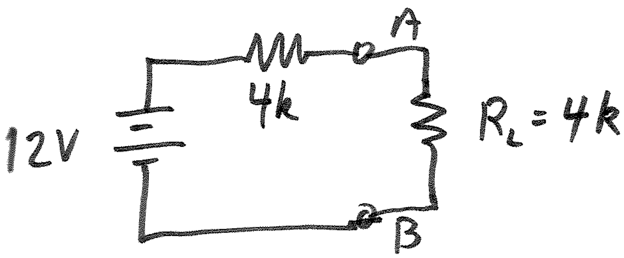
2) (5pt) Find the Thevenin equivalent resistance with respect to A and B for the circuit shown above.



$$(500 + 1.5k) \| 6k = 1.5k$$

$$1.5k + 2.5k = \underline{\underline{4k}} = R_{TH}$$

3) (2pt) Draw the Thevenin equivalent circuit with labeled values and a load resistor of 4k between points A and B.



4) (3pt) What is the voltage across the 4k load resistor in the circuit you drew in part 3)?

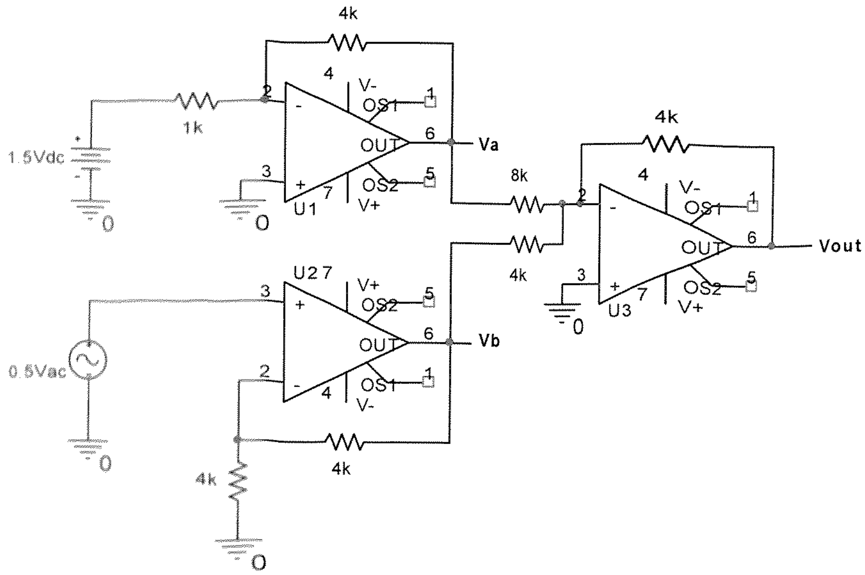
$$V_{R_L} = 12 \frac{4k}{4k + 4k} = \underline{\underline{6V}}$$

5) (5pt) If a 4mH inductor were added to the circuit between A & B (4k load removed), what is the maximum current that will flow through the inductor (as time goes to infinity)?

AS $t \to \infty$ $L \to$ SHORT

$$I_{MAX} = \frac{12V}{4k} = \underline{\underline{3mA}}$$

Question III – Op-Amp Applications (15 points)



Assume that power supplies have been properly connected to all three op-amps in the circuit above. V1 is a dc source with a value of 1.5 Volts, *V1(t) = 1.5 Volts*. (equ. 1) V2 is an ac source which can be represented as: *V2(t) = 0.5sin(2π2000t) Volts*. (equ. 2)

1) (3pt) The circuit has 3 op-amps labeled as U1, U2, and U3. State what the op-amp circuit is for each. Choices are: 1. Follower/Buffer, 2. Inverting Amp, 3. Non-inverting Amp, 4. Differentiator, 5. Integrator, 6. Adding (Mixing) Amp, 7. Difference (Differential) Amp.

U1 Circuit: 2. INV. AMP U2 Circuit: 3. NON-INV. AMP

U3 Circuit: 6. ADDING

2) (6pt) Determine the voltage, relative to ground, at points Va and Vb as a function of time. Write the voltages in a form similar to equ. 1 and 2 above.

a) Voltage at point Va.

$$V_a = -\frac{4k}{1k} V_{in} = -4(1.5)$$

$$\underline{V_a(t) = -6V}$$

b) Voltage at point Vb.

$$V_b = \left(1 + \frac{4k}{4k}\right) V_{in} = 2\left(0.5 \sin(2\pi 2000t)\right)$$

$$\underline{V_b(t) = \sin(2\pi 2000t)\,V}$$

Question III – Op-Amp Applications (continued)

3) (3pt) Determine the output voltage, Vout. Again use the form of equ. 1 and 2 above.

$$V_{out} = -\frac{4k}{8k} V_a - \frac{4k}{4k} V_b = -\frac{1}{2} V_a - V_b$$

$$\underline{\underline{V_{out}(t) = 3V - \sin(2\pi 2000t) V}}$$

4) (3pt) Will adding a 10k resistor between point Vb and ground change Vout? Explain why or why not in 25 words or less.

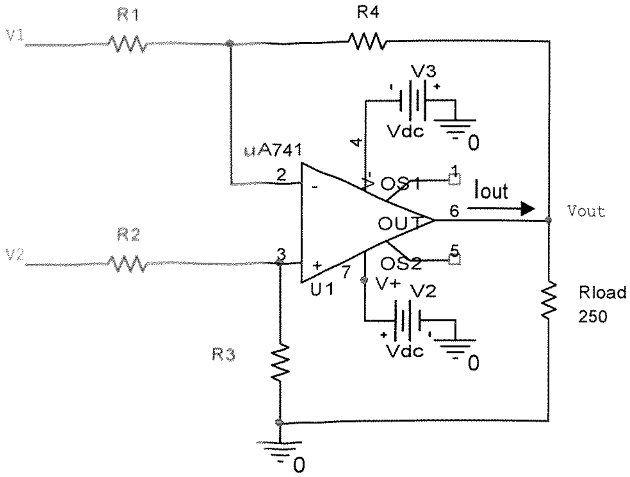
NO

RESISTOR IS JUST A LOAD ON THE OP-AMP OUTPUT U2.

THE LOAD IS SMALL. OP-AMP OUTPUT WILL NOT CHANGE

NOTHING ELSE CHANGES.

Question IV – Op-Amp Analysis (20 points)



You are to use the above circuit to amplify the difference between two inputs. Assume the op-amp is ideal.

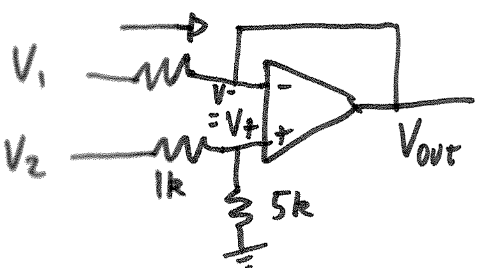
1) (3pt) With resistors R3 = R4 = 5k and R1 = R2 = 1k, find the expression relating Vout to V1 and V2.

DIFFERENCE AMP :

$$V_{out} = \frac{R_f}{R_{in}}(V_2 - V_1) = \frac{5k}{1k}(V_2 - V_1)$$

$$= 5(V_2 - V_1)$$

2) (7pt) When your partner wires up the circuit, an extra wire is added that shorts out R4. Use the golden rules of op-amp analysis to find the new equation relating Vout to V1 and V2.



$$V_- = V_+ = V_2 \frac{5k}{5k+1k} = \frac{5}{6} V_2$$

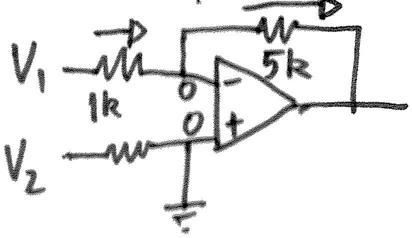
$$V_{out} = V_- = \frac{5}{6} V_2$$

$$V_{out} = \frac{5}{6} V_2$$

V_{out} INDEPENDENT OF V_1

Question IV – Op-Amp Analysis (continued)

3) (4pt) You discover the problem in 2), but in the process of fixing it, you inadvertently short out R3. Find the equation that now relates Vout to V1 and V2



$$I = \frac{V_1 - 0}{1k} = \frac{0 - V_{out}}{5k}$$

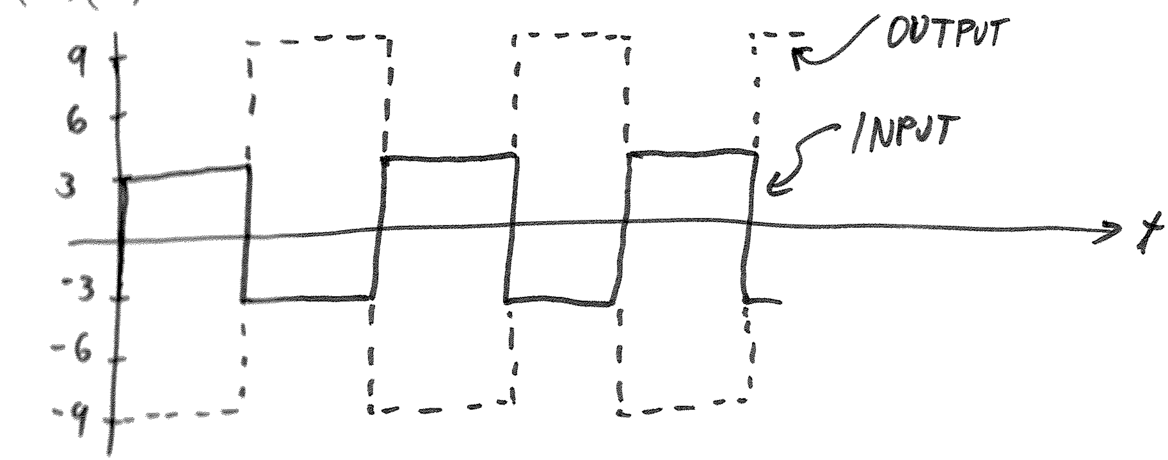
$$\frac{5k}{1k} V_1 = -V_{out}$$

$$\underline{\underline{V_{out} = -5V_1}}$$

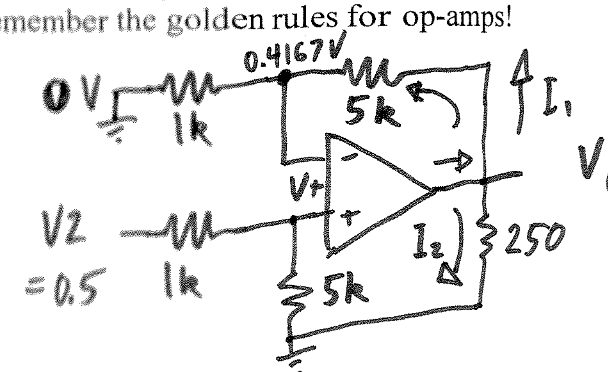
V_{out} INDEPENDENT OF V_2

4) (3pt) The circuit is finally fixed as it should be and the power inputs are correctly set to +9V and -9V. V2 is set to 0V (grounded), and V1 is set to a 3V square wave ($6V_{p\text{-}p}$) with 0V offset. Sketch the input and the output of the op-amp. Be sure to scale the y-axis.

$(-5)(3) = -15V$ > $-9V$ POWER SUPPLY OUTPUT LIMITED TO < ±9V



5) (3pt) If V1 is set to 0V (grounded) and V2 is a sine wave that varies between ±0.5V, what is the maximum magnitude of the current Iout from the op-amp? Note that Iout goes into 2 resistors and remember the golden rules for op-amps!



$$V_{out} = 5(0.5) = 2.5V$$

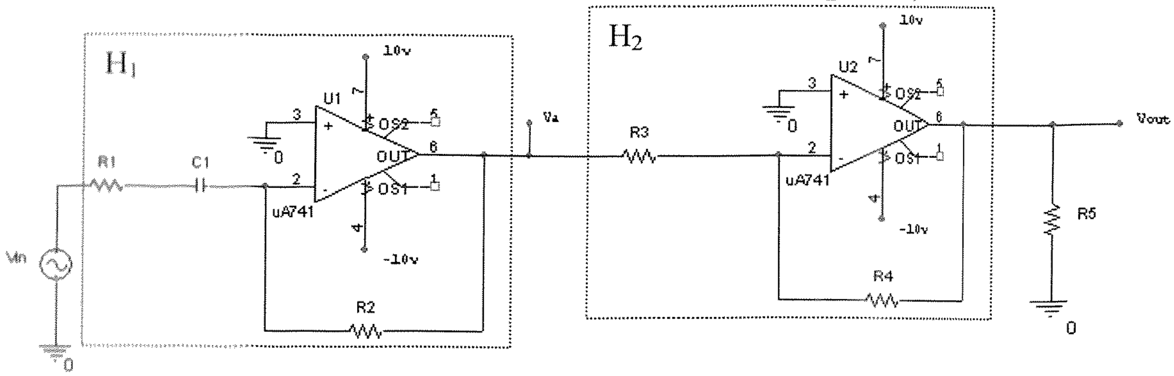
$$I_{out} = I_1 + I_2 = \underline{\underline{10.4167mA}}$$

$$I_1 = \frac{2.5V - 0.4167V}{5k} = 0.4167mA$$

$$I_2 = \frac{2.5V}{250} = 10mA$$

$$V_+ = 0.5 \frac{5k}{5k + 1k} = 0.4167V$$

Question V – Op-Amp Integrators and Differentiators (25 points)



1) Assume the op-amps in the circuit above are ideal and that power supplies are properly connected to pins 4 and 7.

a) (5pt) Write down the transfer function for the first circuit $H_1(j\omega) = Va/Vin$?

$$H_1(j\omega) = -(j\omega R2C1)/(1+j\omega R1C1)$$

b) (4pt) Write down the transfer function for the second circuit $H_2(j\omega) = Vout/Va$?

$$H_2(j\omega) = -R4/R3$$

c) (5pt) Find the total transfer function $H(j\omega) = Vout/Vin$.

$$H(j\omega) = H_1(j\omega) \times H_2(j\omega) = j\omega R2R4C1/(R3+j\omega R1R3C1)$$

Question V – Op-Amp Integrators and Differentiators (continued)

d) (6pt) By calculating the approximate transfer function, show that at frequencies much lower than $\omega_c = 1/(R1C1)$, the circuit acts as a differentiator and at frequencies much higher than ω_c, it acts as an amplifier.

$H_{LO}(j\omega) = j\omega R2R4C1/R3$ This is a differentiator.

$H_{HI}(j\omega) = j\omega R2R4C1/j\omega R1R3C1 = R2R4/R1R3$ This is an amplifier.

2) (5pt) In the range where the circuit works as a differentiator, what is the time domain equation for the whole circuit (i.e. write an expression for Vout(t) as a function of Vin(t) = sin(ωt) and the necessary component values)? For this problem assume that $\frac{Vout(j\omega)}{Vin(j\omega)} = H(j\omega) = -\frac{j\omega 10}{1+j\omega 2}$ and that ω is much less than ½.

The time domain equation for a differentiator is $Vout(t) = -RC(dVin(t)/dt)$ and $H(j\omega) = -j\omega RC$.

Our differentiator has a transfer function of $H(j\omega) = -j\omega 10/1$, therefore the time domain equation must be

$Vout(t) = (-10/1)(dVin(t)/dt) = -10(d\sin(\omega t)/dt) = -10\omega\cos(\omega t)$