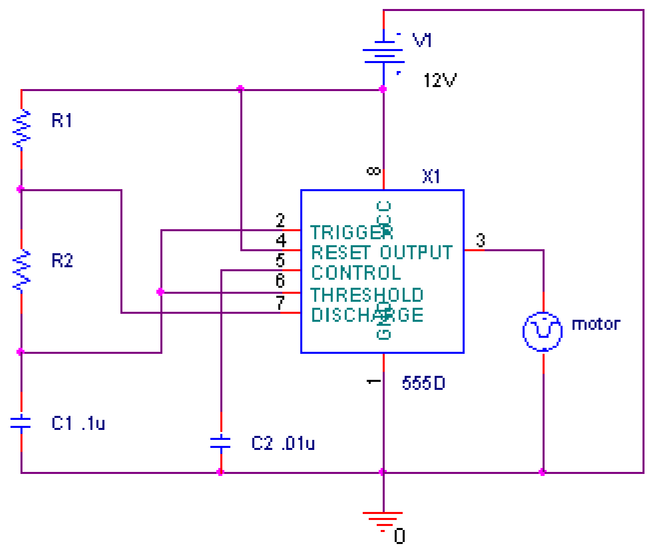
ENGR 4300 **Test 3 Solutions** **Spring 2003**

Name______________________________ **Section_______**

1) 555-Timer (20 pts)

You create the following circuit to control a motor with pulse width modulation:



a) If R1=1K ohms and R2=3K ohms, what will be the duty cycle of the output at pin 3 (8 pts)?

$$DC = T1/(T1+T2) = 0.693(R1+R2)(C1)/0.693(R1+2R2)(C1)$$
$$= (R1+R2)/(R1+2R2)$$
$$= [(R1/R2)+1] / [(R1/R2)+2]$$

(R1/R2) = 1K/3K=1/3 DC = 1.333/2.333 = .57   ***Duty Cycle = 57%***

b) If R1=3K ohms and R2=1K ohms, what will be the duty cycle of the output at pin 3 (8 pts)?

(R1/R2) = 3K/1K = 3 DC = 4/5 = .8   ***Duty Cycle = 80%***

c) Which of the scenarios above (a or b) will cause the motor to spin faster? Why? (4 pts)

b will spin faster because the time it is on relative to the time it is off is greater.

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2) Inductance Calculation and Measurement (30 points)

You have found an inductor and wish to determine its inductance. Here is a picture:

You find that it has a wire gauge diameter of 0.51 mm (24 gauge), a length of 14.5 mm, a core diameter of 5.0 mm and 27 turns. You assume that it has an air core ($\mu = 1.257 \times 10^{-6}$ Henries/meter).

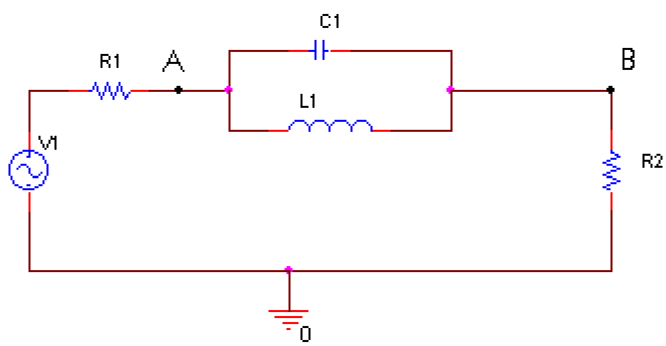
a) Calculate the theoretical inductance of the inductor. (5 pts)

$L = [mN^2pr^2/d]$ $\quad d=14.5x10^{-3}$ m $\; r=2.5x10^{-3}$ m

$L = [1.257x10^{-6}$ x 729 x p x $6.25x10^{-6}]/(14.5x10^{-3}) = 1.24x10^{-6}$ H

L = 1.24 mH

b) You wish to verify that the core is indeed air, so you place the inductor into the circuit you used in experiment 9.
[Note: R1 = 50 ohms, R2 = 50 ohms, C1 = 0.68uF, and L1 is your inductor.]

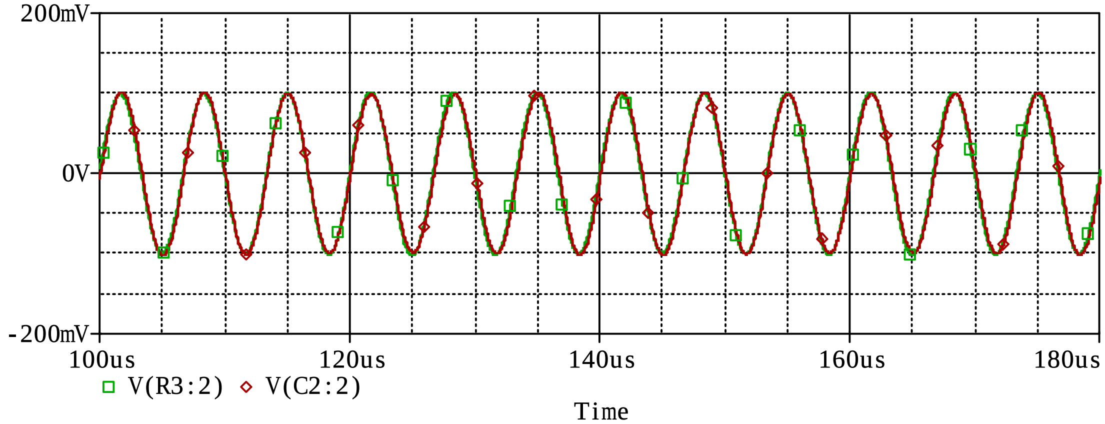


You generate the three plots on the following page.

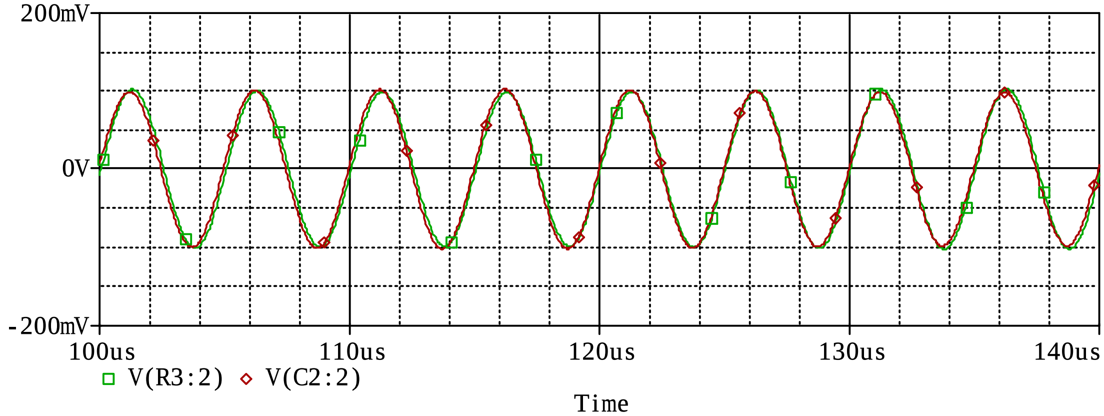
i) Label the three circuits. Which one is at the resonance frequency? below the resonance frequency? above the resonance frequency? (6 pts)

ii) Label the input (point A) and the output (point B) on the plot at resonance on the following page. (4 pts)

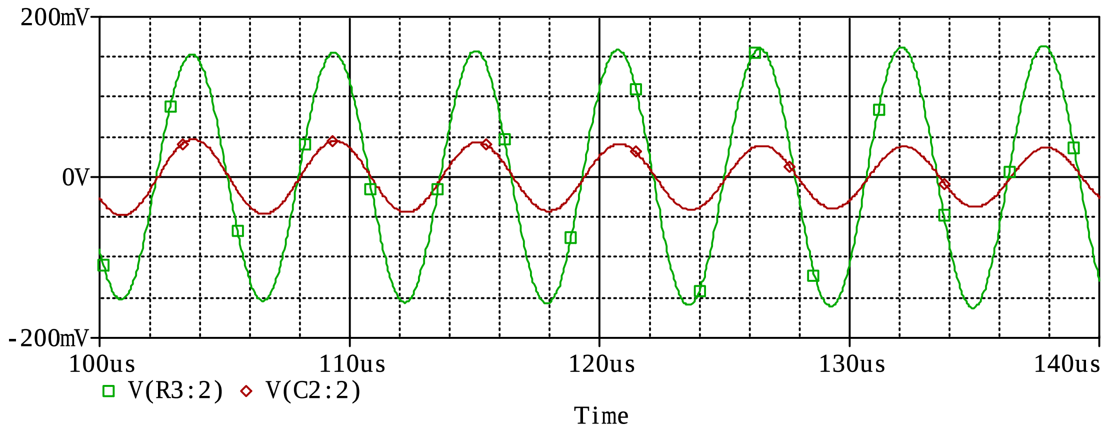
Show work here:



f = 12cycles/80ms =0.15 Meg Hz = 1500K Hz ***BELOW RESONANCE***



f = 8cycles/40ms =0.2 Meg Hz = 2000K Hz ***ABOVE RESONANCE***



f = 5 cycles/(136.3-108)*ms = .1767Meg Hz = 1767K Hz* ***AT RESONANCE***

iii) Given the above plots, calculate the resonance frequency, ωo, of your circuit.(3 pts)

wo = 2pfo = 2 x p x 1767K= 1110.1K rad/sec ***wo=1.11 Meg rad/sec***

iv) According to the frequency in iii), what is your inductance? (3 pts)

wo=1/sqrt(LC) $L = 1/C(wo)^2 = 1/(1110K)^2(0.68x10^{-6}) = 1.193x10^{-6}\ H$

L = 1.193 mH

iv) Given your calculations in part a), calculate the theoretical resonance frequency, ωo, of your circuit. (3 pts)

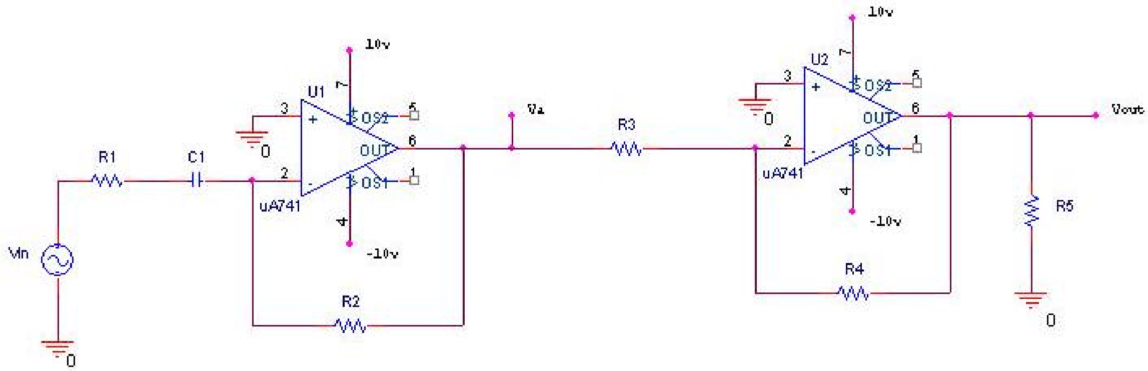
wo=1/sqrt(LC) $wo=1/sqrt(1.24x10^{-6}x0.68x10^{-6})$ ***wo = 1.089 Meg rad/sec***

c) Does the inductance equation overestimate or underestimate inductance? Is your guess that the core is probably made of air correct? Why or why not? (6 pts)

The equation should overestimate the inductance, as stated in experiment 9. From the two inductances we found here, we have verified that this is true. 1.24mH>1.19mH

The values 1.24mH and 1.19mH are not exactly alike, but they are very close. The core must be air. If we had used something besides air (like iron, for example) they would be different by several orders of magnitude.

3. Integrator/Differentiator (30 pts)



a) Write down the transfer function for the first circuit $H_1(j\omega)$=Va/Vin? (4 pts)

$H_1(jw)$ = -(jwR2C1)/(1+jwR1C1)

b) Write down the transfer function for the second circuit $H_2(j\omega)$=Vout/Va? (4 pts)

$H_2(jw)$ = -R4/R3

c) Find the total transfer function $H(j\omega)$=Vout/Vin. (4 pts)

H(jw) = $H_1(jw)$ x $H_2(jw)$ = jwR2R4C1/(R3+jwR1R3C1)

d) By calculating the approximate transfer function, show that at frequencies much lower than ω_c=1/(R1C1), the circuit acts as a differentiator and at frequencies much higher than ω_c, it acts as an amplifier. (6 pts)

$H_{LO}(jw)$ = jwR2R4C1/R3 This is a differentiator.

$H_{HI}(jw)$ = jwR2R4C1/jwR1R3C1=R2R4/R1R3 This is an amplifier.

e) In the range where the circuit works as a differentiator, what is the time domain equation for the whole circuit? i.e. write an expression for Vout(t) as a function of Vin(t) (and the necessary component values). (5 pts)

The time domain equation for a differentiator is Vout(t)=–RC(dVin(t)/dt) and H(jw)=-jwRC.

Our integrator has a transfer function of H(jw)=+jwR2R4C1/R3, therefore the time domain equation must be

Vout(t) = (R2R4C1/R3)(dVin(t)/dt) = 0.01(dVin(t)/dt)

(If we substitute the values, we get (R2R4C1/R3)=1Kx10Kx1m/1K=.01)

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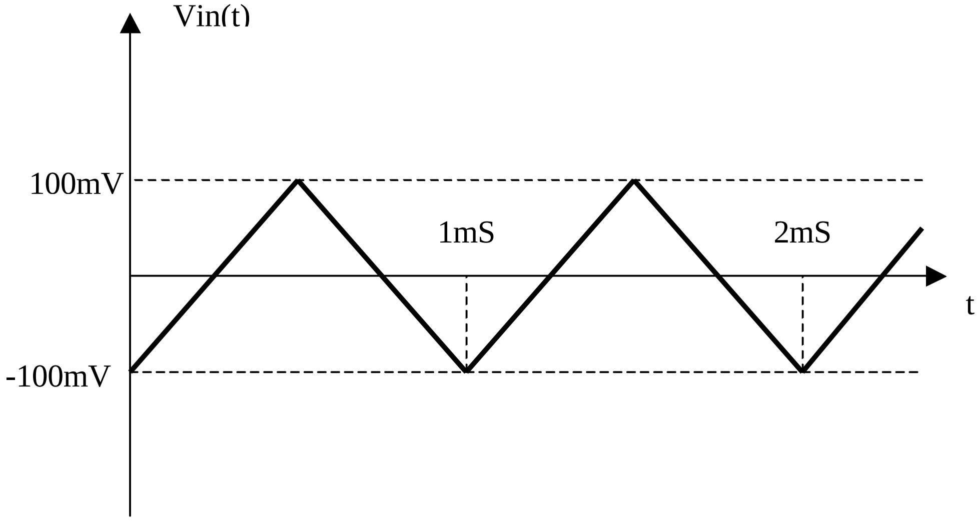
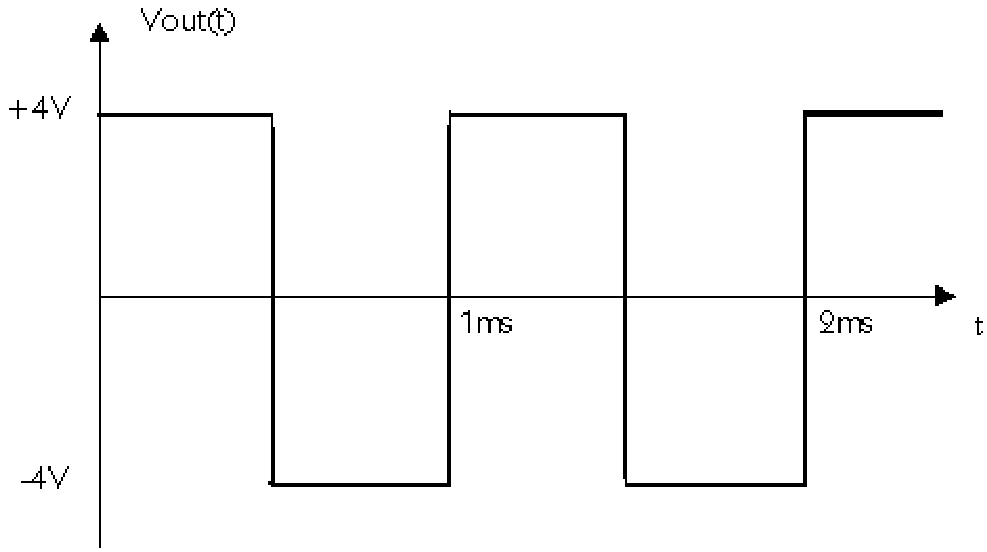
Assume R1 = 100Ω, R2 = 1KΩ, R3 = 1KΩ, R4 = 10KΩ, R5 = 1MΩ and C1 = 1µF.
f) Assume that the input voltage is the triangular wave shown below, draw the waveform of the output signal Vout(t). <u>Make sure that you clearly label all important times and voltages on the plot.</u> (7 pts) *(Hint: Compare the frequency of the signal with w_c and decide how does the circuit act at this frequency.)*



wc=1/R1C1 = 1/(100x1ì)=0.01x10^{6}=10,000 rad/sec
fc = wc/(2p) = 10000/(2p)=1600Hz
T=1ms f=1000Hz 1000<<1600 Therefore, we have a differentiator.
slope=Dy/Dx = 200mV/0.5ms = 400mV/ms = 400 V/s
Vout(t)=0.01(dVin(t)/dt)=0.01x100=4V.



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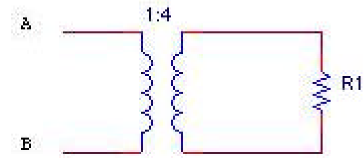
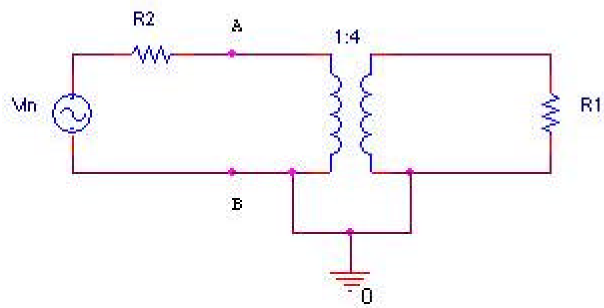
4. Transformer (20 pts)



a) In the circuit above, the transformer is ideal. If R1=1KΩ, find the equivalent impedance, Z_{AB}, seen from points A and B. (6 pts)

$Z_{AB} = R1/a^2$ $a=N2/N1=4$ $Z_{AB} = 1K/16$ $\boldsymbol{Z_{AB}=62.5ohms}$

b) We have connected the above circuit to an AC source with a resistor of R2 = 1KΩ.



If the input voltage has an amplitude of 10V, find the voltage at point A. (8 pts)

$V_A = [(62.5)/(1062.5)]10V = .588V$ $\boldsymbol{V_A = 588mV}$

c) What is the value of the voltage across R1? (6 pts)

$V2/V1 = a$ $V_A = V1$ $V2 = .588(4) = 2.35V$ $\boldsymbol{V_{R1} = 2.35V}$