

Experiment 3 Introduction to AC Circuits

Purpose: In the following exercises, we will investigate the properties of the other two passive circuit components - inductors and capacitors.

Equipment Required:

- **Oscilloscope** (HP 54603B 2 Channel 60 MHz Oscilloscope)
- **Function Generator** (HP 33120A 15 MHz Function/Arbitrary Waveform Generator)
- **Instrumented Beam**
- **Parts Kits**

Background on the Simple Pendulum – An Example of an Harmonic Oscillator

Before we address capacitance and inductance, we will review some of the properties of the simple pendulum. The instrumented beam is a very good example of a simple pendulum, even though it looks more like a small diving board. Let us assume that the end of the beam moves in the x-direction. Obviously, this is a simplification, since it really travels along the arc of a circle. When the beam is stationary, we will assume that it is horizontal and at $x=0$. Again, this is an approximation because the beam must bend downward slightly due to its own weight. When the beam is bent, it experiences a restoring force like a spring

$$F = -kx$$

where k is the spring constant. From Newton's Law

$$F = ma = m \, dv/dt = m \, d^2x/dt^2$$

which, when combined with the previous expression, gives us

$$m \, d^2x/dt^2 + kx = 0$$

the harmonic oscillator equation. In standard form, the harmonic oscillator equation is

$$d^2x/dt^2 + \omega^2 x = 0$$

where ω is the frequency of oscillation. Thus, the beam will oscillate at

$$\omega = (k/m)^{1/2} = 2\pi f$$

The solution to this equation is

$$x = x_o \cos(\omega t + \phi_o)$$

where x_o is the initial deflection of the beam and ϕ_o is the initial phase. For simplicity, there is no need to include ϕ_o . We have also not included any damping in this model. Thus, we get an oscillation that will go on forever, rather than decaying slowly away. (If you do not recall that this is the solution, plug the expression for x into the differential equation and you will see that it works.)

The kinetic energy of the beam is

$$KE = (1/2) m v^2$$

while the potential energy is

$$PE = (1/2) k x^2$$

Upon initial deflection, the energy of the beam is

$$W = PE = (1/2) k x_0^2$$

Since we have assumed no dissipation (no friction or other damping force), this total energy will be conserved and

$$W = (1/2) m v^2 + (1/2) k x^2 = \text{constant}$$

To summarize, the pendulum or any harmonic oscillator works by exchanging energy between two different forms. Not all forms of energy can be easily converted to another state and then back again, but we know this is trivial with the kinetic and potential energy of a mass.

We can start at this expression of energy conservation to determine the equations of motion of the beam or any other simple pendulum. Since the total energy is a constant, we can take the time derivative of the entire expression and set it equal to zero.

$$dW/dt = (1/2) m 2 v dv/dt + (1/2) k 2 x dx/dt = 0$$

Since $v = dx/dt$, we can write

$$(m dv/dt) v + (k x) v = 0$$

or

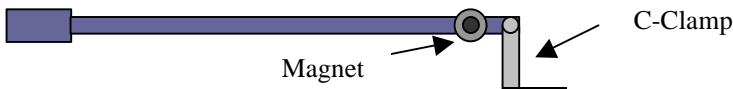
$$m dv/dt = -k x$$

which is the original equation of motion. Thus, once we have a conservation law, we can use it even to find out how things change with time.

Part A The Instrumented Beam as a Harmonic Oscillator

In the last experiment, you should have measured the oscillation frequency of the unloaded beam using the strain gauge and the bridge circuit. This time we will use the output from the coil at the moveable end of the beam as the magnet clamped to the end of the beam moves through it. This signal is quite a bit larger than that from the bridge. However it is not proportional to the position of the beam, rather, it is sensitive to the beam velocity. Since the velocity and the position oscillate at the same frequency, we can use either signal to find the frequency.

Measure the frequency again, since you probably do not have the same beam. Then, measure the frequency three more times using additional masses of your choice. Place the additional masses as close to the end of the beam as possible using a c-clamp – ideally they should be located at the very end of the beam. Orient the c-clamp and mass so that their center of mass is near the end of the beam. For example, attach the c-clamp from the side rather than from the end of the beam. *Make sure that at least two of your masses are greater than 100 grams.* There is a scale available in the studio to measure any mass you choose to add to the beam. Write down the masses you used and the corresponding frequencies in the table below. Fill out the chart on the following page.



Load Used	Mass (kgm)	Frequency (Hz)
None	m_{beam} (unknown for now)	$f_o =$
	$m_1 =$	$f_1 =$
	$m_2 =$	$f_2 =$
	$m_3 =$	$f_3 =$

Also, **produce a plot of the decaying sinusoid you observe using the Agilent-Intuilink software (for the unloaded beam).** For each mass, check the frequency a couple of times, since you should notice that there will be a range of values for the frequency, primarily because of noise and the somewhat non-ideal nature of the sinusoidal voltage. For now, the mass of the beam m_{beam} is not determined.

Now, we will use the information you have just obtained to calculate the spring constant *and* the equivalent mass of the beam. With the four measurements you have made of frequency, you should have more than enough information to determine these two constants of the simple pendulum equation. From a variety of references, it is shown that $k = \frac{Ewt^3}{4l^3}$, where E is Young's Modulus, w is the width, t is the thickness and l is the length of the beam.

To determine the spring constant and the equivalent mass of the beam, rewrite the formula for the frequency as

$$\frac{k}{m_{Total}} = (2\pi f)^2$$

where

$$m_{Total} = 0.23 \cdot m_{beam} + m_{extra}$$

For the unloaded beam, m_{beam} is the mass of the beam while m_{extra} is the mass of the magnet and holder for the magnetic pickup coil. Recall that the pendulum or harmonic oscillator equation holds for point masses located at the end of a massless beam. Since the beam has mass, but its center of mass is not located at the end of the beam, this term is multiplied by 0.23 to give the equivalent mass placed at the end of the beam that produces the same response. While we could figure out both of these masses from the material properties, we will assume for now that we must determine these terms (at least their sum) experimentally. For convenience, designate m as the sum of these two mass terms.

We can write out a k expression for each of the four frequencies you have measured:

$$k = m \cdot (2\pi f_o)^2$$

$$k = (m + m_1) \cdot (2\pi f_1)^2$$

$$k = (m + m_2) \cdot (2pf_2)^2$$

$$k = (m + m_3) \cdot (2pf_3)^2$$

Use these numbers to determine the values of k , and m . We cannot easily separate the beam mass from the mass of the magnet and holder, so just determine the effective total of the two, assuming that they act as if the mass is located at the end of the beam. Note that you are making four measurements to determine two constants. This means that you have some redundancy built in and also that you will not obtain perfect agreement for all four equations. None of your measurements will be perfect, so it is best to have some more measurements than constants to determine to average out measurement error. You should find the values of k and m that come the closest to satisfying all four equations. You can use a trial and error method for this by first approximating the values (make an educated guess) and then keep changing them a little at a time until you think you have the best possible agreement. You can also use some statistical analysis. For example, you could determine the standard deviation of the four expressions for k for a range of realistic values for the beam mass. Also, since the relative mass of the beam is much less for the larger masses, your data for the larger masses should be more reliable. You can calculate k and m using the two larger masses to find an initial guess and then experiment with the values until you get the best match. Once you have obtained what you think are good values for these two constants, plot frequency vs load mass

$$f = \frac{1}{2p} \sqrt{\frac{k}{m + m_{load}}}$$

for loads from 0 to 500 grams and then also plot the four points you have measured to show that your constants are reasonable. Include this plot with your report.

Finally, measure the dimensions of the beam and using the table of Young's modulus for different materials, decide what the beam is made of. You will probably need the width, thickness, length and Young's modulus of the beam to figure out what material is used. It may not be possible to exactly determine what the material is, so you may end up with more than one possibility.

Background on Harmonic Oscillators Made from Inductors and Capacitors

There are many important lessons we can learn from the harmonic oscillator, but perhaps one of the most useful is the value of conservation laws. It is fair to say that *the most powerful problem solving technique* is to first decide which conservation laws hold. Once the conservation laws are identified, they can be used to determine a great deal of information about any system.

In passive electrical systems, there are three kinds of circuit elements: resistors, capacitors and inductors. Resistors turn electrical energy into heat. When a current I flows through a resistor, there will be a voltage drop V across the resistor. The power dissipated by the resistor is equal to the product of I times V . Since resistors produce heat, it should be no surprise that they play the same role as friction in a mechanical system. The ideal pendulum will oscillate forever ... a real pendulum will oscillate until all its stored energy is converted to heat through friction. Thus, if we wish to create a circuit analogous to the ideal harmonic oscillator, it can have no resistors in it. Rather, we will combine only inductors and capacitors.

A typical inductor consists of a coil of wire. If we pass a current through the coil, a magnetic field will be created. Many of us have made simple electromagnets at some time in our lives by wrapping wire around some magnetic material like a nail. When a battery is connected to the wire, it is possible to attract small pieces of iron to the nail. The field created by the coil, the magnetic field, can do work and thus contains energy. The energy stored in an inductor is given by the expression

$$W_M = (1/2) L I^2$$

where we have used the subscript M to indicate that the energy is stored in the magnetic field and L is the inductance in units of Henries. Joseph Henry was honored by using his name for this unit because of his early work in developing practical electromagnets. He began his work here in New York's capital district when he was teaching at the Albany Academy. There is a statue of this great scientist outside the original location of the Academy across the street from the Albany City Hall. Henry was a contemporary of Amos Eaton, the intellectual force behind the founding of RPI. He eventually left Albany for Princeton and the Smithsonian.

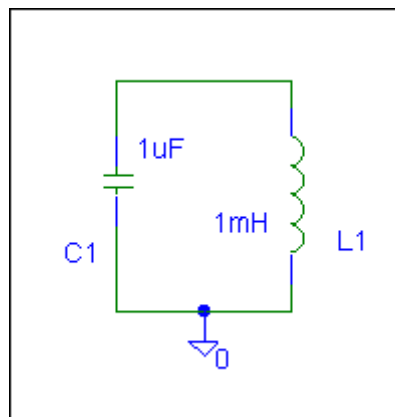
A typical capacitor consists of two metal plates of large area separated by some insulating material such as teflon or some other plastic. When a voltage source is connected to the plates, charge flows from the source to the plates with positive charge deposited on the plate with the highest voltage and negative charge deposited on the plate with the lowest voltage. Since these charges are opposite in sign and since unlike charges attract one another, there is a force between the two plates. Again, the existence of this force tells us that we have to do work to charge up the plates and that there is energy stored. In this case, the energy is stored in the electric field created by the charges. The energy stored by a capacitor is given by the expression

$$W_E = (1/2) C V^2$$

where we have used the subscript E to indicate that the energy is stored in the electric field and C is the capacitance in units of Farads. Michael Faraday also worked on electromagnets and, being British, gained much more fame for his work, since America was a scientific backwater at the time. Henry showed him how to make better magnets, but Faraday's work was much more far reaching. Henry also showed Morse how to build a telegraph!

Aside: It is somewhat interesting to note that neither Henries nor Farads turns out to be much of a common practical unit. One Henry is a huge inductor, rarely seen in practice. One Farad is also rare, now occasionally seen in highly filtered power supplies for computers. We will need to use the prefixes milli-, micro-, nano-, pico-, etc. a lot when dealing with these components. We also do not see one Ohm all that much, but require the other kind of prefixes (kilo-, mega-, etc.).

Consider the simplest possible configuration of a single capacitor and a single inductor connected as shown below. Note that, since there are only two components, one can describe this connection as either in parallel or in series. Also assume that the capacitor has been charged up to some voltage V at time t = 0, at which time it is connected to the inductor. The charge will begin to flow creating a current through the inductor. After a short time, all the charge that was originally on the capacitor plates will be gone and the current through the inductor will reach its maximum value. Thus, we began with all the energy stored in the capacitor and none in the inductor and end with the opposite condition. The current flowing through the inductor will then charge the capacitor back up and the process will begin again. The energy is traded back and forth between the two storage elements.



The total energy of the system must remain constant because there is no dissipative element (no resistor). The total energy is given by

$$W = (1/2) C V^2 + (1/2) L I^2 = \text{constant}$$

Since this is a constant, we can take the time derivative of this expression and set it equal to zero.

$$dW/dt = (1/2) C 2 V dV/dt + (1/2) L 2 I dI/dt = 0$$

or

$$(L dI/dt) I + (C dV/dt) V = 0$$

which we can separate into

$$L dI_L/dt = V_L \text{ and } C dV_C/dt = I_C$$

if we note that $V = -V_C = V_L$ and $I = I_C = I_L$. V is the voltage at the top of the circuit and I is the current flowing around the circuit. The convention for circuit elements is that the current flows into the end with the highest voltage and thus we have to reverse the capacitor voltage to get the correct expression for V and I . These equations relating the current and voltage of capacitors and inductors are analogous to $V = IR$ for a resistor and can be derived or determined empirically. (Please see Chapter 1.4 and 1.5 of Lunn and the electronic textbooks listed on the course webpage.) We can combine the two current-voltage relationships for the capacitor and inductor of our simple circuit to obtain

$$d/dt (C dV_C/dt) = d/dt I_C = dI_L/dt = V_L/L = -V_C/L$$

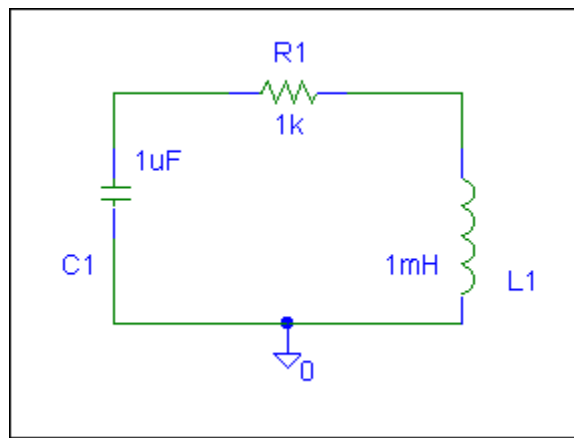
or

$$d^2V_C/dt^2 + (LC)^{-1} V_C = 0$$

which is the harmonic oscillator equation. The frequency can be trivially determined since the form is the same as we saw with the pendulum. Thus,

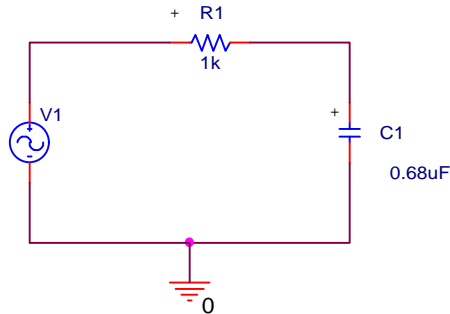
$$\omega = (LC)^{-1/2}$$

If we add a resistor to make a series RLC circuit, we will have a damped oscillator, the circuit for which is shown below.



Part B RC and RLC Circuits – Examples of AC Circuits

We have used the LC oscillator circuit to develop the equations we need to analyze AC circuits. Passive AC circuits can consist of any combination of R, L, or C. We will begin with a combination we have seen before, an RC circuit.



Set up this circuit using PSpice, with a 100mV source voltage and a frequency of 1kHz. Run a transient simulation, using the parameters we have applied previously for a 1kHz source. The capacitor and the resistor form a voltage divider combination in the same manner as one configured with two resistors. From the Probe plot, we want to show that, just like the resistor divider, the source voltage (V_1) is equal to the sum of the voltages across the resistor and the capacitor ($V_R + V_C$). To determine V_1 , place a voltage level marker at the top of the voltage source. To determine V_R and V_C , use voltage differential markers. Place the positive voltage marker at the end of each component marked with a +. This follows the convention that voltage is positive at the end where the current enters. When you run your simulation, you will obtain three voltages. You will have to add a trace that is the sum of the two differential voltages. When you have done this correctly, you should see that the sum will equal the source voltage. **Print out a plot of your results, and include it in your report.**

Repeat for frequencies of 10 Hz and 10 kHz. Change the transient analysis appropriately to obtain a useful plot. As a rule of thumb, it is best to display about three cycles of a sinusoidal signal. Be sure that you change your maximum time step or you might have to wait a long time for your 10Hz results. It is not necessary to make a copy of these plots. **Verify that the voltages add as expected. Describe the qualitative differences you observe in these three cases.**

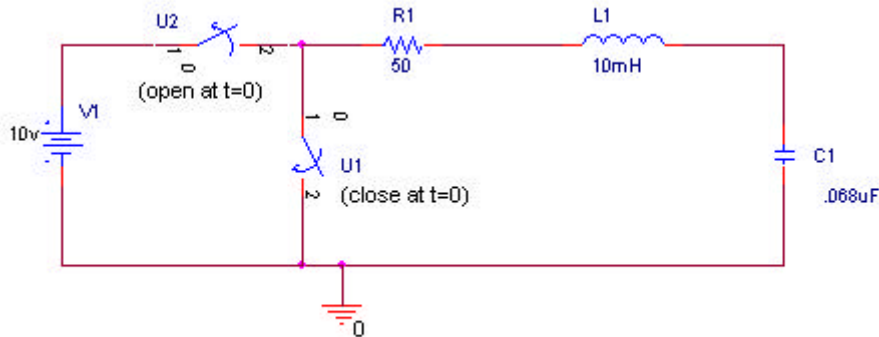
The configuration we have been addressing, where the input voltage is V_1 and the output voltage is taken across the capacitor is a kind of filter. **Run an AC sweep analysis of this circuit in decades (log scale) from 1Hz to 100kHz** to determine what kind of filter it is (low pass or high pass). Do not use the differential voltage markers. Plot the voltages at the top of the voltage source and at the top (marked with a +) of the capacitor. You will then be plotting these voltages with respect to ground. Modify your plot so that you are displaying the magnitude of the capacitor voltage divided by the magnitude of the source voltage. **Produce a plot of your results, and include it in your report.** This plot is the frequency response for this filter.

Now, reverse the order of the resistor and capacitor in the circuit and take the output voltage across the resistor. Perform the same AC sweep and determine what kind of a filter you get when you take the output voltage across the resistor. **Produce a plot of the frequency response for the second type of filter.** That is, plot the magnitude of the output voltage divided by the magnitude of the input voltage. **Include this plot in your report.**

Set up the low pass RC filter on a protoboard. The low pass RC filter is the simple RC circuit with output voltage measured across the capacitor. Connect the function generator as the input AC (sinusoidal) source. Monitor the input and output voltages with the 'scope. Measure the relative amplitude $|V_{out}|/|V_{in}|$ at the three frequencies 100 Hz, 1 kHz, and 10 kHz. You cannot do this directly, since this is not an option with the 'scope. However, you can measure the input and output voltages and then divide the latter by the former. If you have time,

also do the measurement at 10 Hz. This is such a low frequency, that the display on the 'scope is somewhat harder to work with. **Add these experimentally measured points to the AC Sweep PSpice plots. Repeat for the high pass RC filter.** The low pass RC filter is the simple RC circuit with output voltage measured across the resistor.

We will now consider an RLC circuit, with all three kinds of passive components. Here we have used two switches to change the input voltage from zero to 10 volts at time $t = 0$. The switches are in the EVAL library (Sw_tClose will close at a specified time period after $t=0$ and Sw_tOpen will open at a specified time period after $t=0$). **Use PSpice to simulate the transient response of this circuit for a total time of 1msec. Describe the voltages plotted by Probe. What features of the voltages reminds you of the instrumented beam?**



Report and Conclusions:

The following should be included in your report. Everything should be labeled and easy to find. Partial credit will be deducted for poor labeling or unclear presentation.

Part A:

Include following plot:

1. Plot of the decaying sinusoid obtained from 'scope for the unloaded beam. (1 pt)
2. Plot of frequency vs. load mass with four points marked (3 pts)

Answer following questions:

1. Explain how you did your analysis to determine a reasonable value of k and m_{beam} . (2 pts)
2. What are the values for k and m_{beam} that you obtained? Do your results seem plausible? Why? (2 pts)
3. What is the beam made of? Include your beam measurements and calculations. (2 pts)
4. Write a possible form for the equation of motion of the beam that includes friction. The solution to this equation should be a damped sinusoid. That is, our solution for x should be multiplied by $e^{-\alpha t}$, where α is the damping constant. Determine the value of the damping constant from the 'scope trace you obtained for the unloaded beam. (2 pts)

Part B:

Include following plots:

1. PSpice plot (transient analysis) showing that KVL is correct for the RC circuit ($V_1 = V_C + V_R$). (1 pt)
2. PSpice plot from the AC sweep of the RC circuit with experimental points added. (2 pts)
3. PSpice plot from the AC sweep of the RC circuit (with reversed order) with experimental points added. (2pts)

Answer following questions:

1. Write out the mathematical expressions for the three **sinusoidal** voltages for the first RC circuit case you considered, i.e. write them in the form $V(t)=A \text{ Sin}(\omega t+\phi)$. (1 pt)
2. Define the characteristic of a low-pass filter. Define the characteristic of a high pass filter. (1 pt)
3. An RC filter can be either high pass or low pass, explain what determines which type of filter it will function as. (1 pt)
4. Determine the range of frequencies for which the input voltage V_1 is within 5% of the output voltage (either V_R or V_C) for the two configurations. This tells you the range of frequencies that pass through the filter more-or-less unchanged. Signals at other frequencies are attenuated by the filter. (1 pt)
5. Define attenuated. (1 pt)
6. Determine the resonant frequency of the RLC circuit you analyzed with Pspice. (1 pt)

Summarize key points (1 pt)

Discuss mistakes and problems (1/2 pt)

List member responsibilities (1/2 pt)

Total: 25 points for write up

Attendance: 3 classes (5 points) 2 classes (3 points) 1 class (0 points) out of 5 possible points
No attendance at all = No grade for experiment.