

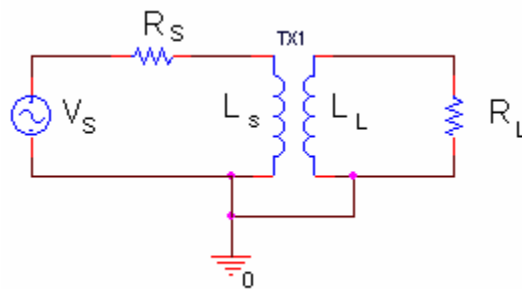
Experiment 9

Circuit Components and Magnetic Fields – Inductors, Transformers, and ...

Purpose: Partly as preparation for the next project and partly to help develop a more complete picture of voltage sources, we will return to considering inductors. The extension we are primarily concerned with is the mutual inductor or transformer. The transformer has three uses: stepping up or down voltages, stepping up or down currents, and transforming impedances. Like other devices we have considered, the transformer does not work in an ideal manner for all circumstances. We will first consider the basic ideas behind the transformer.

Equipment Required:

- HP 34401A Digital Multimeter
- HP 33120A 15 MHz Function / Arbitrary Waveform Generator
- HP 54603B 2 Channel 60 MHz Oscilloscope
- Enameled Magnet Wire
- Iron Ring or Core, Nail, Paper Tube

**Background**

In the circuit above, the voltage source (V_S) and the 50 ohm resistor (R_S) represent a sinusoidal voltage source like the function generator we use in the studio. R_L is the load resistor (also 50 ohms) and TX1 is the transformer. We make transformers by winding coils of wire around some kind of a core material. Sometimes the core material is just air, as when we wind the wire around a paper tube, for example. Most of the time, the core material is iron or some other magnetic material. Inductors have larger values when the core material is a magnetic material like iron. The value of the inductance will also depend on the geometry of the core material. Just for simplicity, we will address only one geometry, the cylindrical core. This produces the kind of inductors we have used in previous experiments. If you do not recall what they look like, get one out and look at it. If the core cylinder has a radius equal to r_c and we wind a coil N times around the cylinder to cover a length d , the inductor will have an inductance equal to:

$$L = \frac{(\mu_0 N^2 \pi r_c^2)}{d} \text{Henries}$$

where $\mu_0 = 4\pi \times 10^{-7}$ Henries/meter. If the core is not air, but rather some magnetic material, replace μ_0 with μ , which is usually many times larger than μ_0 . By many times we can mean as much as 10^5 times larger. You should know that this formula only works well when the length d is much larger than the radius r_c .

What if you have a coil in which d is not very large compared to r_c ? This coil would look more like a finger ring and have the shape of a coin. In this case, the above equation would over-estimate the value of the inductance and, thus, it is only useful to find a ballpark number. However, you can use the equation on the following page to get a better estimate of the inductance of a ring-shaped coil.

$$L \cong \mu N^2 r_c \left\{ \ln\left(\frac{8r_c}{r_w}\right) - 2 \right\}$$

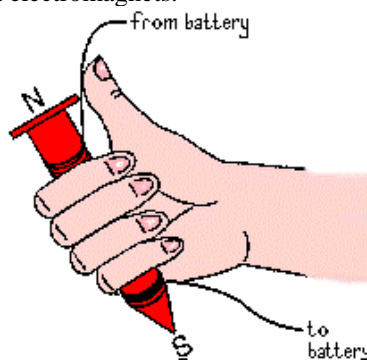
where r_c is the major radius of the coil and r_w is the radius of the wire. Each physical coil geometry has a unique equation to calculate its inductance. Calculations based on this and other inductance expressions can be found at <http://emcsun.ece.umn.edu/new-induct/> from the University of Missouri-Rolla Electromagnetic Compatibility Laboratory. (Electromagnetic Compatibility, or EMC, refers to the ability of a device or system to function without error in its intended electromagnetic environment. Electromagnetic Interference, or EMI, refers to electromagnetic emissions from a device or system that interferes with the normal operation of another device or system. Both are very big issues indeed in electromechanical systems.)

When one makes an inductor, it is also obvious that the wires used can have a large variety of cross sectional areas. There are some inductors made with very thick wires, while others are made with very thin wires. Thin wires permit one to wind many more turns of wire around a core and thus increase the inductance. Thick wires have lower resistance for any given length. All wires have a resistance given by the expression

$$R = \frac{l}{\sigma A}$$

where l is the length of the wire, A is the cross sectional area of the wire (thickness), and σ is the conductivity of the wire material. For copper, the conductivity is about 6×10^7 Siemens/meter. The unit of Siemens is $1/\Omega$. Sea water has a conductivity of 5 Siemens/meter. There are many handbooks like the [CRC Handbook of Chemistry and Physics](#) that have the resistance of different diameter wires per mile (or another unit of distance). It is also quite easy to calculate the resistance of a piece of wire using the formula above. The links on our webpage on topics like wires and cables nearly all have information of this type.

Now that we have seen how the inductors we have used are made, we should also notice that they look a lot like grade-school-science-project electromagnets.



Inductors work by creating a magnetic field. If one inductor is placed near another inductor, then the magnetic fields of the two inductors will interact with one another. If you have ever built an electromagnet, you will know that the magnets will attract or repel one another if the current in the coil is large enough. Even when we cannot sense that they are interacting, a current in one coil will induce a current in a nearby coil. To maximize this interaction, we usually wind the coils onto the same core and make them look as similar as possible. When we build a device to maximize this interaction, we call it a transformer. Transformers only work for time varying currents and voltages. Note that when the coils are as similar as possible, all the geometric terms in the formulas for the inductance will be the same for the primary and the secondary coils. Only the number of turns will be different. We can demonstrate this using our equation for the inductance of a long, thin coil.

$$\frac{L_1}{L_2} = \frac{\frac{(\mu_0 N_1^2 \pi r_c^2)}{d}}{\frac{(\mu_0 N_2^2 \pi r_c^2)}{d}} = \frac{N_1^2}{N_2^2}$$

Since we try to make μ_0 , r_c and d the same, these terms cancel out and we are left with the ratio of the inductances depending only upon the ratio of the number of turns squared.

To analyze just how a transformer works, we have to add an additional kind of inductance, called mutual inductance. If a coil of wire of inductance L_1 is very near another coil of inductance L_2 , there will be a mutual inductance M between the two coils, where $M^2 = k^2 L_1 L_2$. The constant k is the coupling coefficient. If the coils are perfectly coupled, $k = 1$. Usually k is a little less than 1 in a good transformer. The two loop or mesh equations that apply to the two current loops in the PSpice circuit on page 1 are

$$\begin{aligned} \text{i) } V_S &= I_S(R_S + j\omega L_S) - I_L(j\omega M) \\ \text{ii) } 0 &= -I_S(j\omega M) + I_L(R_L + j\omega L_L) \end{aligned}$$

where L_S is the inductance of the primary (source) coil and L_L is the inductance of the secondary (load) coil. By convention, the coil connected to the source is called the primary coil and the coil connected to the load is called the secondary coil. The L and M terms have opposite signs because the loop currents go in opposite directions. Thus, the voltage generated by one coil will influence the other in the opposite direction. We can solve for the input impedance of the transformer by taking the ratio of the voltage across the primary $V_S - I_S R_S$ to the primary current I_S

$$Z_{in} = V_S / I_S - R_S = j\omega L_S + (\omega^2 L_S L_L) / (R_L + j\omega L_L)$$

For an ideal transformer, the coupling constant $k = 1$ and then $M^2 = L_S L_L$ and

$$Z_{in} = j\omega L_S + (\omega^2 L_S L_L) / (R_L + j\omega L_L)$$

Now, let both L_S and L_L become very large (tend to infinity), but their ratio should remain the same.

$$L_L = a^2 L_S$$

then,

$$Z_{in} = R_L / a^2$$

Thus, the transformer transforms the load resistance R_L by the square of the turns ratio, $a = N_L / N_S$ where N_S is the number of turns in the primary coil and N_L is the number of turns in the secondary coil. This is the first, and most stringent, relationship for an ideal transformer. Remember that Z_{in} is the ratio of the input voltage to the input current. In any circuit that we build or simulate, we can determine Z_{in} by finding this ratio.

The relationship between the primary and secondary currents in the transformer can be found from the second loop equation. Solving again for the case where the inductances become very large, we find that

$$I_L / I_S = 1/a \quad a^2 = L_L / L_S \quad \text{and} \quad N_S I_S = N_L I_L$$

The voltages across the primary and secondary coil connections are given by

$$V_S N_L = V_L N_S$$

Note that both the voltage and current relationships shown actually contain no sign information. Depending on how the transformer is wired, it is possible for minus signs to appear in these expressions. Thus, you should consider that they hold only for magnitudes. As with Z_{in} , we can determine these ratios by

measuring the voltages and currents separately and then taking their ratios. When we design a particular transformer, we usually have a turns ratio in mind. For example, in the DC power supplies that come with consumer electronics (also called wall worts), there is a step-down transformer that takes the 120 volt line voltage and steps it down to a smaller voltage, like 6 or 12 volts. In such devices, N_S/N_L is chosen to be 10 or 20, depending upon the desired output voltage. This voltage is then rectified with a full-wave rectifier and sometimes regulated with a zener diode to produce a DC voltage.

Note that for a transformer to be working properly, the expressions for Z_{in} , V_S/V_L , and I_L/I_S must all be satisfied.

Part A: Simulation of Transformer

Set up the circuit shown on page 1 in PSpice. For the voltage source V_S , use VSIN and assume an amplitude of 1 volt, a frequency of 1k, and a DC offset of 0. Also set the AC voltage in the spread sheet to 1 volt. Set the input resistance to 50 ohms and the load resistance to 5K ohms. For the transformer (XFRM_LINEAR in the ANALOG library), you need to set the coupling coefficient. Choose 1 for perfect coupling. You also need to set the primary and secondary inductances, $L_1 = L_S$ and $L_2 = L_L$. For your first simulation, set L_1 and L_2 both to 1 mH. Perform an AC sweep from 1 Hz to 10 Meg Hz. For what range of frequencies, if any, does this transformer work like an ideal transformer?

In order to determine this, we need to determine the frequencies where the relationships for Z_{in} , V and I are satisfied. Recall this means that V_S/V_L and I_L/I_S must be equal to a constant, a . This constant can be found using L_1 and L_2 . Remember that the ratio of N_S to N_L is determined from the ratio of the square roots of L_1 and L_2 . The third criteria we must prove is that Z_{in} satisfies the relationship $Z_{in} = R_L/a^2 = V_S/I_S - R_S$. For the purposes of this simulation, we will assume R_S is negligible compared to R_L . Given this assumption and solving for a , we find that if $a = \sqrt{R_L / (V_S/I_S)}$ is satisfied, then the third criteria is satisfied. To find where our transformer works, we can plot these three relationships and determine where all three are satisfied. To avoid dividing by zero current, we will plot $1/a$. Add a trace of V_1/V_2 , where V_1 is the voltage across the primary coil and V_2 is the voltage across the secondary coil. Then add a trace of I_2/I_1 , where I_2 is the current through the secondary load resistor and I_1 is the current through the primary load resistor. Because Pspice is picky about polarities, this ratio may be negative. Multiply it by -1 or change the polarity of one of the resistors to make it positive. In order for the third criteria to be satisfied, $1/a$ should also be equal to $\text{SQRT}[(V_1/I_1)/R_2]$. Add a trace to plot this expression as well. **Print out the plot with all three ratios. Mark the frequency range where all three criteria are satisfied on the plot.**

By trying a variety of values for L_1 and L_2 , find a transformer for which the three relationships are satisfied and $a=10$ ($1/a=1/10$). In this case, the voltage across the primary coil will be 1/10 the voltage across the secondary coil and the other two expressions will also be satisfied. *Use the smallest possible inductances you need to make the transformer work properly at 10K Hz and above.* (To change the effective frequency, you must increase or decrease both L_1 and L_2 , while keeping the ratio of the square roots the same (1/10).) **Write down the values for L_1 and L_2 that you have selected and print your PROBE plot with the three ratios that demonstrate that the transformer works as specified.**

Now repeat the task of the last paragraph for frequencies of 100 Hz and up. **Write down the values for L_1 and L_2 that you have selected and print your PROBE plot with the three ratios that demonstrate that the transformer works as specified.** What design advantages exist for circuits that are to work at higher frequencies rather than lower frequencies?

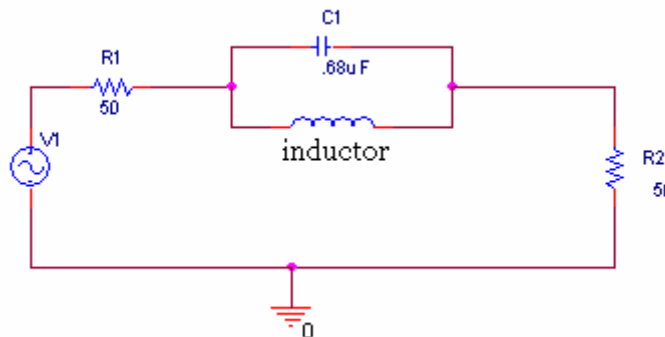
Part B: Making an Inductor

First, we need something to wind our coil on. Take a sheet of paper and roll it into a tight cylinder about an inch in diameter. Tape the roll so it does not unwind. Next, take at least 2 feet of enameled wire. Note the gauge of the wire you are using. Measure its exact length. Then, wrap it tightly *in a single layer* around your roll of paper. Carefully keep track of the number of times you wind the wire around the paper roll. This is the number of turns of your inductor, N . Leave about 1 inch of wire at each end of the coil so you will have something to connect to. Tape the wire in place. Remove some of the enamel from the ends of the wire. The enamel is the insulation for this wire, so you cannot make electrical contact unless it is removed. There should be some sandpaper for this purpose or you can use a knife. You will be provided with wood blocks to do your sanding on. **Please do not sand the table tops when you remove the enamel!**

Now you have made an inductor. You can estimate its resistance by looking up the dimensions of the wire in a table of wire properties listed by gauge and calculating the resistance using the equation on page 2. Next, measure the resistance using the multimeter. When you measure small resistances, it is important to first measure the resistance of the wires you are using to connect the coil to the meter. Then, you add the coil and see if there is a difference. The resistance of the inductor will be the difference between the resistance of the wires alone and the resistance of the wires with the inductor. Does your measured value agree at least roughly with the calculated value? Estimate the inductance of your coil using both of the formulas for inductance found at the beginning of the write-up. Based on the shape of your coil, which should give a closer estimate of the actual inductance? Measure the inductance of your coil directly with the impedance bridge. Which equation gave you a better estimate?

Part C: Measurement of Inductance

Now we will use another method to estimate the inductance of our coil. In the circuit below, $V1$ and $R1$ represent the function generator, $R2$ is a 50 ohm load connected to the function generator by a parallel combination of an inductor and a capacitor.



The inductor will have a positive imaginary impedance given by $j\omega L$ while the capacitor will have a negative imaginary impedance given by $\frac{1}{j\omega C}$ or equivalently $\frac{-j}{\omega C}$. At the resonant frequency,

$\omega_0 = \frac{1}{\sqrt{L1 \cdot C1}}$, the voltage difference across the combination of inductor and capacitor will achieve its

maximum value. Measure the capacitance of a $0.68 \mu\text{F}$ capacitor using the impedance bridge. Use the measured value for the capacitance and the mathematical value you calculated for the inductance in part A. (Remember that we are treating the inductance as unknown, so we cannot use the inductance you measured with the impedance bridge.) Now we can use PSpice to find a closer estimate for the resonant frequency. Simulate the above circuit in Pspice. Again, use the measured value for the capacitance and the mathematically calculated value for the inductance. Recall that inductors do not have negligible resistance. Therefore, you should include another resistor (with the value you measured for resistance of the inductor) in series with the calculated inductance. Look at the AC sweep and determine where the plot reaches a maximum. Record this frequency. Calculate a mathematical estimate for L using the resonant frequency

you found with PSpice, $f_0 = \omega_0/2\pi$. **Print out the plot of the AC sweep. Mark the highest point. Also write the new estimate for L on your plot.**

Set up the circuit as shown on your proto-board. Use the capacitor you measured and other components with values as close as you can get to the ones shown. Adjust the the frequency until the impedance of the LC combination is the largest. This is the resonant frequency of the LC combination. It may not be very easy to find. Start looking around the estimated frequency you obtained using PSpice. You can observe the voltage out of the function generator on one 'scope channel and across the 50 ohm resistor, R2, with the other channel. There should be a very distinctive effect at around the frequency $f_0 = \omega_0/2\pi$. Enter this frequency in the table below and also list the peak-to-peak values for V_{in} and V_{out} . Find a frequency below and above resonance where the LC combination is more-or-less negligible (looks like a short circuit). Enter these frequencies and voltages in the table. **Take a picture using the Agilent software of the input and output at the resonant frequency.**

	Frequency	V_{in} (peak-to-peak)	V_{out} (peak-to-peak)
Below Resonance			
At Resonance			
Above Resonance			

Use the resonant frequency and $f_0 = \omega_0/2\pi$ to find another estimate for the inductance of the coil.

Try adjusting the value of L in PSpice, until the peak frequency matches the resonant frequency of your actual circuit. **Include this plot. Write the value of L which worked best on it.**

In this experiment, you have found impedance six ways: mathematically using the 2 equations for inductance; using the impedance bridge; using Pspice, with an estimated value for L, to find f_0 and then using the equation for resonant frequency to recalculate L; finding the resonant frequency in a protoboard circuit and using the resonant frequency equation to calculate L; and finally, returning to PSpice and adjusting L to give you the resonant frequency you found on the protoboard. How do the other five values compare to the inductance you found using the impedance bridge?

Even though we have an approximate expression for the inductance, the ideal model of the cylindrical inductor will generally over-estimate the inductance. We can get a larger inductance and also a more predictable inductance by winding the coil on a piece of iron rather than on a paper tube. The permeability of iron is many times larger than that of air.

Part D: Making a Transformer

Since you now have one coil, the addition of a second coil will result in a configuration that can be used as a transformer. Wind a second coil on your paper tube that is as similar as possible to your first coil. Try to get the two coils to be as close as possible to one another. That will make the coupling better. The best method is to *wind the second coil on top of the first coil*. Be sure that you count the number of turns in both your coils. Mark the ends of each coil, so that you know how to hook them up. Again remove enamel from the ends of your second coil so you can make good electrical contact. You have now built a transformer where one of your coils is the primary and one is the secondary. Calculate the value of a using the number of turns in the two coils. How should this value affect the magnitude of your output voltage relative to your input voltage? Using the function generator, find a frequency for which the transformer works more-or-less as expected. This may be very high. Justify your choice of frequency. (**Obtain plots of your results**). Be sure that you display the input to the transformer on one 'scope channel and the output on the other.

Note – the two coils near one another work in a manner similar to the pickup coil we used on the cantilever beam. On the beam, a time-varying magnetic field was created by moving a permanent magnet up and

down. The pickup coil sensed this time-varying field and produced a voltage. In a transformer, the time-varying field is produced by an electromagnet with a time-varying current in it (coil #1) and sensed a similar coil (coil #2). As far as a stationary coil is concerned, it is not possible to tell whether the time-varying magnetic field is produced by a stationary electromagnet or a moving permanent magnet, as long as the field produced oscillates in time. The time-varying magnetic field generates a voltage and current in the second coil. This is the basic principle of electrical generators. Electrical motors, as we have seen with the cantilever beam, are mostly just generators run backwards. In the next project, we will build a motor using a permanent magnet and a coil like the ones you have just wound. You should recall the analysis done here when you do the project so you will be able to figure out the resistance and inductance of your motor coil.

Report and Conclusions:

The following should be included in your report. Everything should be labeled and easy to find. Partial credit will be deducted for poor labeling or unclear presentation.

Part A:

Include Following plots:

1. PSpice plot of the initial 3 traces of the ratios for the transformer when $L1=1m$ and $L2=1m$. (1 pt)
2. PSpice plot of the three ratios that prove the transformer works at 10kHz and up. (1 pt)
3. PSpice plot of the three ratios that prove the transformer works at 100kHz and up. (1 pt)

Answer Following questions:

1. What are the values of $L1$ and $L2$ you chose to obtain the best transformer operation over the two specified frequency ranges? (1 pt)
2. Use your output traces and the three design criteria equations to describe how you know these values satisfy the criteria for an ideal transformer. (2 pts)
3. What design advantages exist for circuits that are to work at higher frequencies rather than lower frequencies? (1 pt)

Part B:

Answer following questions:

1. What value did you calculate for the resistance of the inductor? How did this compare to the measured resistance? (2 pts)
2. What two values did you calculate for the inductance of the inductor? How did these compare to the measured inductance? Which equation worked better? Why? (2 pts)

Part C:

Include Following plots:

1. PSpice sweep of RLC circuit using calculated value of impedance with resonant frequency marked and recalculated value of inductance written on it. (1 pt)
2. PSpice sweep of RLC circuit with value of L on it that was best for finding a resonant frequency equal to the one you found with the protoboard circuit. (1 pt)
3. Agilent plot of the input and output to the RLC circuit at the resonant frequency. (1 pt)

Answer following questions:

1. What are the six calculated and/or measured resistances of your air core inductor? (2 pts)
2. Which of the five other methods gave you an inductance closest to the one measured with the impedance bridge? Why do you think this gave you the best estimate? (2 pts)

Part D:

Include following plot:

1. 'Scope plot with the input and output of your transformer. (1 pt)

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Answer following questions:

1. At what frequency did your transformer work as expected? How do you know this? To answer this question, you will need to analyze the voltages observed and show how closely they satisfy the basic formulas for the transformer. (2 pts)
2. List at least 2 things that you could do to the design of your coil to improve you results? (2 pts)

Summarize key points (1 pt)

Discuss mistakes and problems (1/2 pts)

List member responsibilities (1/2 pts)