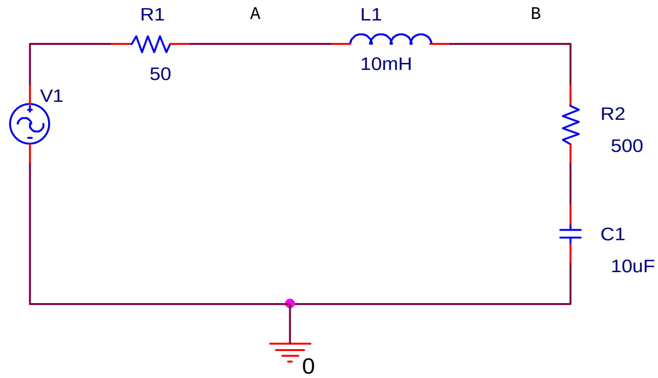
Electronics and Instrumentation

Name ____Solution___ ENGR-4220 Spring 1999 Section ____________

Homework #2
Circuit Transfer Functions

Due: Friday, 8 October (Can be turned in the studio or any time before 5 pm at Prof. Connor's Office, JEC 6002)

There is a discussion of series RLC circuits in section 3.6.1 of Gingrich. Read the two examples in this section thoroughly before you start to analyze this similar circuit. You don't have to worry about how to do a hand plot of the transfer functions, since we will be doing that with PSpice.



In this circuit, the input is applied between point A and ground. This you should recognize as the connection between a function generator and the three series impedances to the right. The output voltage is to be taken at point B across the combination of the resistor R2 and the capacitor C1. Even though values are provided for the components in this circuit, the first few questions you are to consider should be answered generally in terms of V1, R1, R2, L1, and C1. Remember that these questions are based on the example.

a. Write and expression for the transfer function of the circuit shown above.

Answer: $$H(jw) = \frac{R2 + 1/jwC1}{jwL1 + R2 + 1/jwC1}$$

b. Simplify your answer to part a for very small frequencies. Identify the magnitude and phase of your expression.

Answer: $$H(jw) = 1$$

The magnitude is 1 and the phase is zero.

c. Simplify your answer to part a for very large frequencies. Identify the magnitude and phase of your expression.

Answer: $$H(jw) = \frac{R2}{jwL2}$$

The magnitude is R2/(wL2) and the phase is –90^o.

d. Simplify your answer to part a for the resonant frequency $w = w_o = 1/\sqrt{L1C1}$. Identify the magnitude and phase of your expression.

Answer: $H(jw) = 1 + \frac{1}{jwR2C1} = 1 + \frac{\sqrt{L1}}{jR2\sqrt{C1}}$

The magnitude is given by the square root of the real part squared plus the imaginary part squared. The phase is equal to the inverse tangent of the ratio of the imaginary part to the real part.

Thus, the magnitude is $\sqrt{1 + \frac{L1}{R2^2 C1}}$ *and the phase is* $\tan^{-1}\left(-\frac{\sqrt{L1}}{R2\sqrt{C1}}\right)$.

e. Rewrite your answers to parts b, c, and d using the given values for the circuit components.

Answer: The answer to b is trivially the same. The answer to c is $\frac{-j50000}{w}$. *The answer to d is* $1 - j0.063$.

f. While you are in class, perform a PSpice simulation of the circuit. Do an AC Sweep from low frequencies to high frequencies and produce a plot of the transfer function (ratio of output to input voltage) magnitude and phase. Follow the procedure given in Experiment 5. You will have to decide what is a low frequency and what is a high frequency. Low frequencies should be a lot smaller than the resonant frequency and high frequencies should be a lot larger. Produce a plot of your result. On your plot, show that your expressions from part e are consistent with your PSpice plot. *The answer follows. Note that at low frequencies, the output and the input are the same. At high frequencies (greater than about 10kHz), the output is very small and the phase is about –90^o. At resonance (about 500Hz), the input and output are about the same, which agrees with the answer to e since 0.063 is much smaller than 1. Note that the output at resonance a bit bigger than the input. This is typical of RLC circuits, as you will notice from Quiz 2 of Fall 1998. Here the effect is very small because the resistance is relatively large.*