Electronic Instrumentation

Name ________________ **ENGR-4300 Fall 2001** Section _____________

Due October 9th
Homework #2
Circuit Transfer Functions

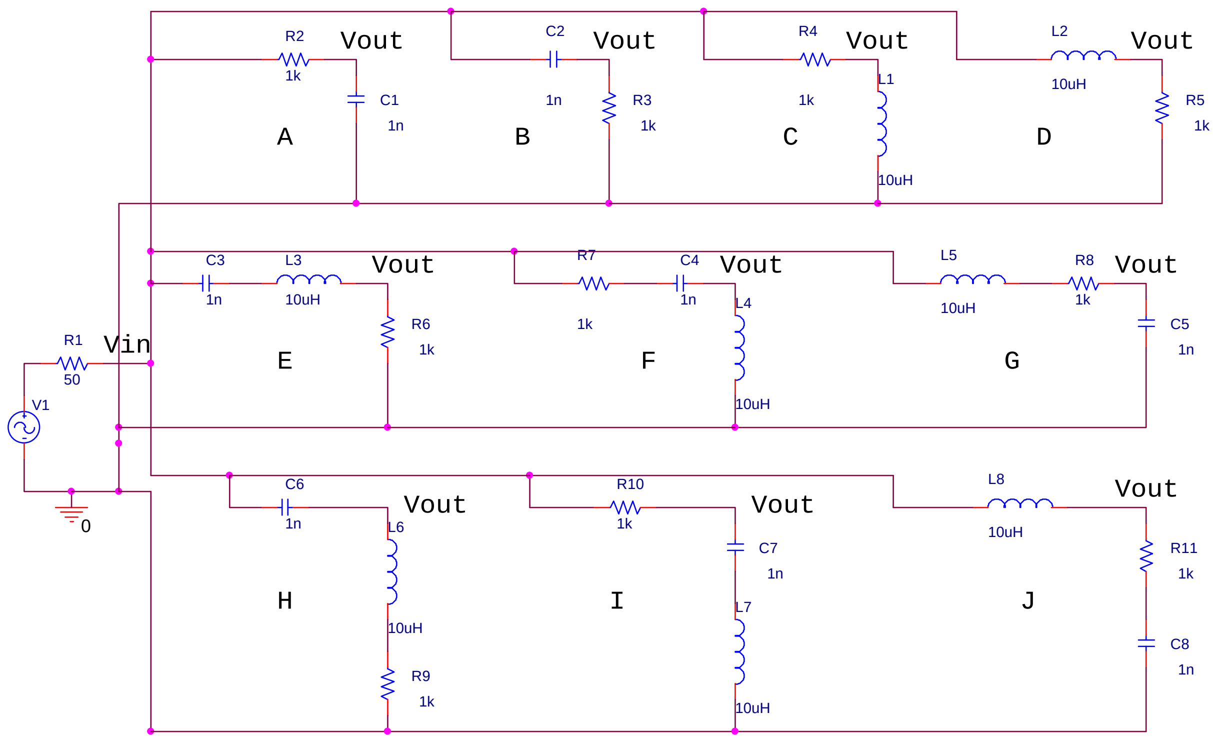
In Experiment 5, we addressed circuits that were combinations of resistors, inductors and capacitors. The analysis discussed there included

1. Determining the complex transfer function
2. Simplifying the complex transfer function for low and high frequencies and also for characteristic frequencies such as the corner frequency or resonant frequency. Determining the magnitude and phase of these simplified expressions.
3. Simulating the transfer function (magnitude and phase) over some range of frequencies using the AC Sweep Analysis available with PSpice.
4. Comparing the results of these two methods of analysis.
5. Experimentally determining the transfer function and comparing it with the results of a PSpice simulation. Before doing the final PSpice simulation, it is necessary to check the values of the components used to be sure that the model is accurate.

It is possible to do much more extensive analysis of such circuits. However, the steps listed above should be more than sufficient for our purposes. We will primarily be using these circuits as filters or assessing their performance as filters when they are included in a larger circuit for some reason.

To see if you understand this method and to prepare for the next quiz, you are to do some of these steps for some of the simple circuits we are likely to encounter in this course. Examples of such circuits are shown in the handouts given in class.

Some of these circuits are shown in the collection below:



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a. **Complex Transfer Functions:** Six example expressions are included below. Identify which goes with which filter circuit in the collection above. To be as general as possible, the input and output voltages are labeled V_{in} and V_{out}, respectively, and the components are not numbered.

$$V_{out} = \frac{1/jwC}{1/jwC + R}V_{in} \qquad V_{out} = \frac{R}{R + 1/jwC}V_{in} \qquad V_{out} = \frac{R}{R + jwL}V_{in}$$

$$V_{out} = \frac{R}{R + jwL + 1/jwC}V_{in} \qquad V_{out} = \frac{jwL}{R + jwL + 1/jwC}V_{in} \qquad V_{out} = \frac{R + jwL}{R + jwL + 1/jwC}V_{in}$$

b. **Transfer Functions at Low Frequencies:** The answers to part a can be simplified for very small frequencies. The magnitude and phase of each expression are also given.

$$V_{out} \approx \frac{1/jwC}{1/jwC}V_{in} = V_{in} \qquad V_{out} \approx \frac{R}{1/jwC}V_{in} = jwRCV_{in} \qquad V_{out} \approx \frac{R}{R}V_{in} = V_{in}$$

$$V_{out} \approx \frac{R}{1/jwC}V_{in} = jwRCV_{in} \qquad V_{out} \approx \frac{jwL}{1/jwC}V_{in} = -w^2LCV_{in} \qquad V_{out} \approx \frac{R}{1/jwC}V_{in} = jwRCV_{in}$$

Magnitude and Phase:

$1,\ 0^o$	$wRC,\ 90^o$	$1,\ 0^o$
$wRC,\ 90^o$	$w^2LC,\ 180^o$	$wRC,\ 90^o$

c. **Transfer Functions at High Frequencies:** The answers to part a can be simplified for very large frequencies. The magnitude and phase of each expression are also given.

$$V_{out} \approx \frac{1/jwC}{R}V_{in} = -j\frac{1}{wRC}V_{in} \qquad V_{out} \approx \frac{R}{R}V_{in} = V_{in} \qquad V_{out} \approx \frac{R}{jwL}V_{in} = -j\frac{R}{wL}V_{in}$$

$$V_{out} \approx \frac{R}{jwL}V_{in} = -j\frac{R}{wL}V_{in} \qquad V_{out} \approx \frac{jwL}{jwL}V_{in} = V_{in} \qquad V_{out} \approx \frac{jwL}{jwL}V_{in} = V_{in}$$

Magnitude and Phase:

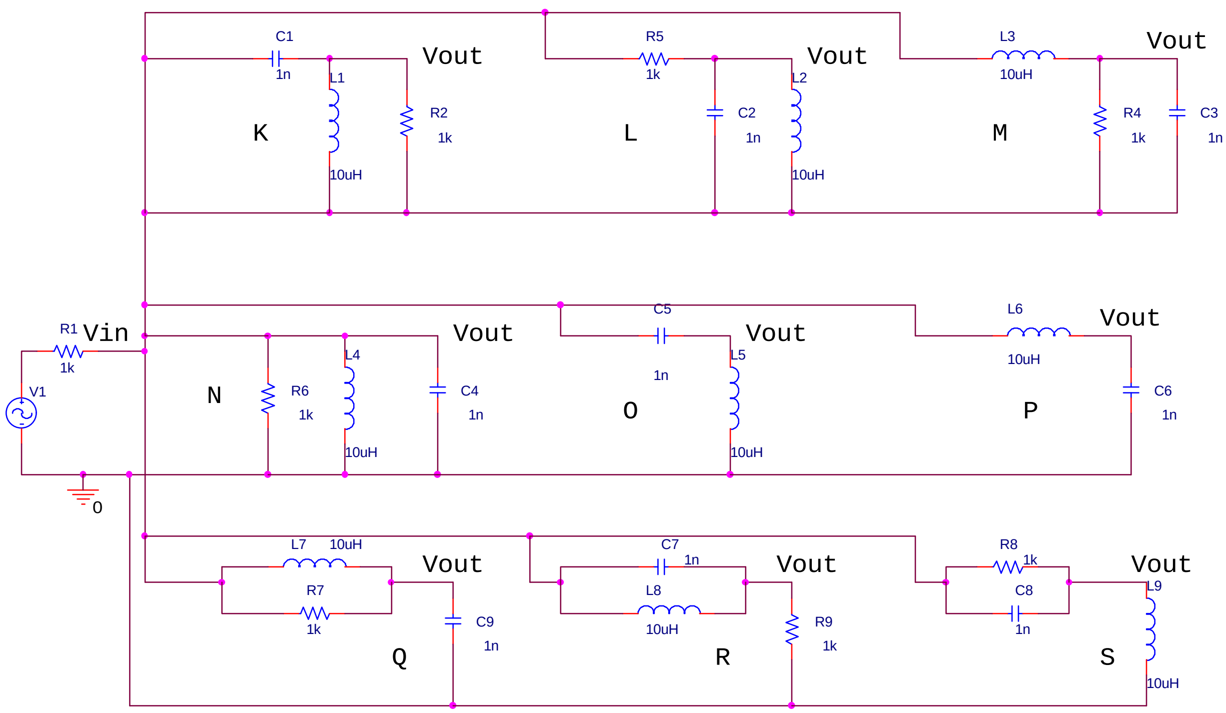
$\frac{1}{wRC},\ -90^o$	$1,\ 0^o$	$\frac{R}{wL},\ -90^o$
$\frac{R}{wL},\ -90^o$	$1,\ 0^o$	$1,\ 0^o$

d. **Corner and/or Resonant Frequencies:** The corner frequency or the resonant frequency for each circuit are included below.

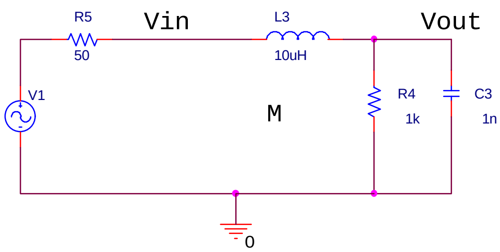
$$w_c = \frac{1}{RC} \qquad w_c = \frac{1}{RC} \qquad w_c = \frac{R}{L}$$

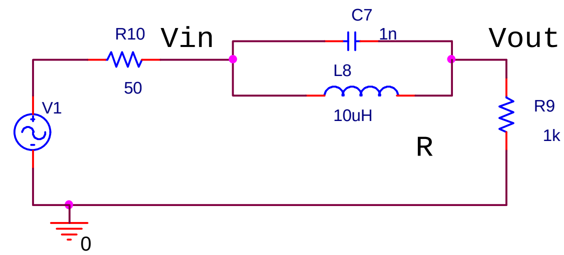
$$w_o = \frac{1}{\sqrt{LC}} \qquad w_o = \frac{1}{\sqrt{LC}} \qquad w_o = \frac{1}{\sqrt{LC}}$$

Additional Examples of Filter Circuits: There are several more simple filters that can be analyzed in this manner. Pick any two and find the complex transfer functions, magnitude, and phase.



f. Repeat the analysis shown above for each of the following circuits (repeat steps a through d). Also, perform a PSpice simulation of each circuit, producing a plot of V_{out} over V_{in}, showing both the magnitude and phase. On your plot, show that your expressions for magnitude and phase of the transfer functions are consistent with your PSpice plot.



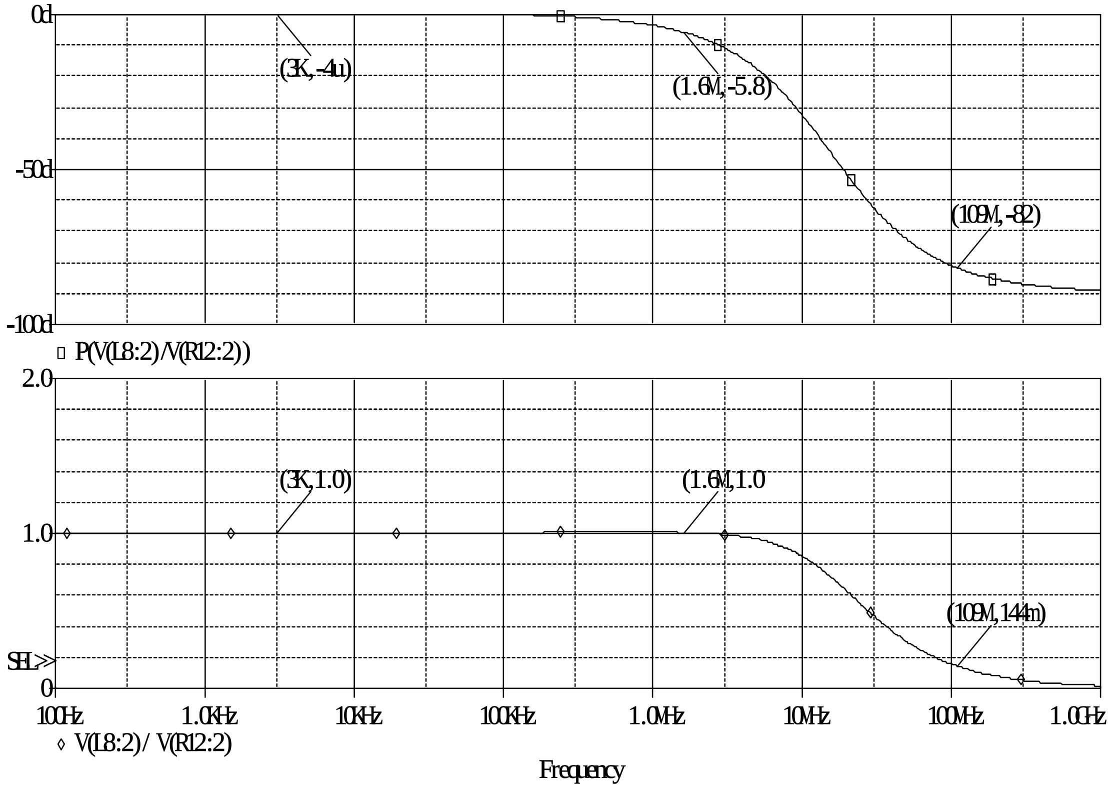


As an example, consider filter circuit J. This is a low pass filter circuit where the magnitude of V_{out} over V_{in} should be about unity at low frequencies and about zero at high frequencies. The phase should be zero at low frequencies and –90^o at high frequencies. A PSpice plot showing phase (top plot) and magnitude (bottom plot) are given below. The high and low frequency behavior looks fine. Note that it was not possible to place the label on the plot at a really high frequency, since it takes up so much room. However, you can see that the phase is going to be –90 degrees at high frequencies, as expected. The resonant frequency for this circuit is about 1.6MHz. At this frequency the transfer function should be

$$V_{out} = V_{in}\frac{R + {}^{1}\!/\!_{jwC}}{R} = V_{in}\left(1 - j\frac{1}{w_o RC}\right) = V_{in}\left(1 - j\frac{\sqrt{LC}}{RC}\right) = V_{in}\left(1 - j\frac{\sqrt{{}^{L}\!/\!_{C}}}{R}\right)$$

For the component values in filter circuit J, this expression is equal to

$V_{out} = V_{in}(1 - j0.1) = V_{in}(1)\exp(-5.7^o)$, which agrees very well indeed with the numbers shown on the plots. The agreement would be perfect if I had placed the marker exactly at the resonant frequency.



The labeled points are somewhat difficult to read, so they are repeated below:

At a frequency of 3kHz, the amplitude is 1.0 and the phase is very small (4 microdegrees)

At a frequency of 1.6MHz, the amplitude is slightly larger than 1 and the phase is –5.8 degrees

At a frequency of 109MHz, the amplitude is small (144mV) and the phase is –82 degrees.
These values are approaching zero and –90 degrees, respectively.