

Homework #2
Circuit Transfer Functions

*Due: Friday, 23 February (Can be turned in the studio
or any time before 5 pm at Prof. Connor's Office, JEC 6002)*

In Experiment 5, we addressed circuits that were combinations of resistors, inductors and capacitors. The analysis discussed there included

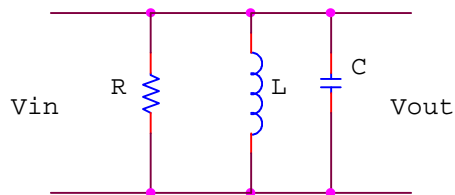
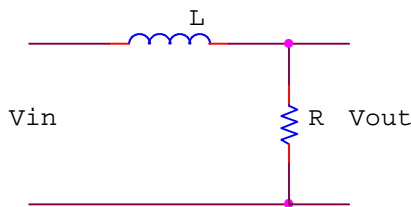
1. Determining the complex transfer function
2. Simplifying the complex transfer function for low and high frequencies and also for characteristic frequencies such as the corner frequency or resonant frequency. Determining the magnitude and phase of these simplified expressions.
3. Simulating the transfer function (magnitude and phase) over some range of frequencies using the AC Sweep Analysis available with PSpice.
4. Comparing the results of these two methods of analysis.
5. Experimentally determining the transfer function and comparing it with the results of a PSpice simulation. Before doing the final PSpice simulation, it is necessary to check the values of the components used to be sure that the model is accurate.

It is possible to do much more extensive analysis of such circuits. However, the steps listed above should be more than sufficient for our purposes. We will primarily be using these circuits as filters or assessing their performance as filters when they are included in a larger circuit for some reason.

To see if you understand this method and to prepare for the next quiz, you are to do some of these steps for most of the simple circuits we are likely to encounter in this course. These include (from Gingrich):

- | | |
|--|---|
| 1 – The low-pass RC filter of figure 3.2 | 2 – The high-pass RC filter of figure 3.3 |
| 3 – The four RLC circuits of figure 3.9 | 4 – The RLC circuit of figure 3.10 |
| 5 – The RLC circuit of figure 3.12 | 6 – The RLC circuit of problem 1 (p 61) |
| 7 – The LR circuit of problem 3 (p 62) | |

In addition, there is the other LR circuit and the parallel RLC circuit:



a. Write an expression for the transfer functions of all twelve of these circuits. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions. To be as general as possible, the input and output voltages are labeled V_{in} and V_{out} , respectively.

$$V_{out} = \frac{1/j\omega C}{1/j\omega C + R} V_{in}$$

$$V_{out} = \frac{R}{R + 1/j\omega C} V_{in}$$

$$V_{out} = \frac{R}{R + j\omega L} V_{in}$$

$$V_{out} = \frac{R}{R + j\omega L + 1/j\omega C} V_{in}$$

$$V_{out} = \frac{j\omega L}{R + j\omega L + 1/j\omega C} V_{in}$$

$$V_{out} = \frac{R + j\omega L}{R + j\omega L + 1/j\omega C} V_{in}$$

b. Simplify your answer to part a for very small frequencies. Identify the magnitude and phase of your expression. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions.

$$V_{out} \approx \frac{1/j\omega C}{1/j\omega C} V_{in} = V_{in} \quad V_{out} \approx \frac{R}{1/j\omega C} V_{in} = j\omega RC V_{in} \quad V_{out} \approx \frac{R}{R} V_{in} = V_{in}$$

$$V_{out} \approx \frac{R}{1/j\omega C} V_{in} = j\omega RC V_{in} \quad V_{out} \approx \frac{j\omega L}{1/j\omega C} V_{in} = -\omega^2 LC V_{in} \quad V_{out} \approx \frac{R}{1/j\omega C} V_{in} = j\omega RC V_{in}$$

Magnitude and Phase:

1,0	ωRC , 90°	1,0
ωRC , 90°	$\omega^2 LC$, 180°	ωRC , 90°

c. Simplify your answer to part a for very large frequencies. Identify the magnitude and phase of your expression. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions.

$$V_{out} \approx \frac{1/j\omega C}{R} V_{in} = -j \frac{1}{\omega RC} V_{in} \quad V_{out} \approx \frac{R}{R} V_{in} = V_{in} \quad V_{out} \approx \frac{R}{j\omega L} V_{in} = -j \frac{R}{\omega L} V_{in}$$

$$V_{out} \approx \frac{R}{j\omega L} V_{in} = -j \frac{R}{\omega L} V_{in} \quad V_{out} \approx \frac{j\omega L}{j\omega L} V_{in} = V_{in} \quad V_{out} \approx \frac{j\omega L}{j\omega L} V_{in} = V_{in}$$

Magnitude and Phase:

$\frac{1}{\omega RC}$, -90°	1, 0°	$\frac{R}{\omega L}$, -90°
$\frac{R}{\omega L}$, -90°	1, 0°	1, 0°

d. Write the corner frequency or the resonant frequency for each circuit. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions.

e. For the circuits of figure 3.12 and problem 3 (p 62) rewrite your answers to parts a, b, c, and d using the following values for the components: $R = 100$, $L = 100\mu\text{H}$ and $C = 10\mu\text{F}$.

f. While you are in class, perform a PSpice simulation for the two circuits of part e. Do an AC Sweep from low frequencies to high frequencies and produce a plot of the transfer function (ratio of output to input voltage) magnitude and phase. Follow the procedure given in Experiment 5. You will have to decide what is a low frequency and what is a high frequency. Low frequencies should be a lot smaller than the corner frequency and high frequencies should be a lot larger. Produce a plot of your result. On your plot, show that your expressions from part e are consistent with your PSpice plot.

Real Form of Transfer Functions: Recall that phasor expressions are converted back into real, time-varying expressions by first multiplying by $e^{j\omega t}$ and then taking the real part of the resulting expression. Assuming that $V_{in} = V_o \cos \omega t$, the phasor form of the input voltage is $V_{in} = V_o$ and each of the six low frequency cases shown above becomes

$$V_{out} \approx V_o \cos \omega t \qquad V_{out} \approx \operatorname{Re}(-j\omega R C V_o e^{j\omega t}) = \omega R C V_o \sin \omega t$$

$$V_{out} \approx V_o \cos \omega t \qquad V_{out} \approx \operatorname{Re}(-j\omega R C V_o e^{j\omega t}) = \omega R C V_o \sin \omega t$$

$$V_{out} \approx -\omega^2 L C V_o \cos \omega t \qquad V_{out} \approx \operatorname{Re}(-j\omega R C V_o e^{j\omega t}) = \omega R C V_o \sin \omega t$$

while the six high frequency cases become

$$V_{out} \approx \operatorname{Re}\left(-j \frac{1}{\omega R C} V_o e^{j\omega t}\right) = \frac{1}{\omega R C} V_o \sin \omega t \qquad V_{out} \approx V_o \cos \omega t$$

$$V_{out} \approx \operatorname{Re}\left(-j \frac{R}{\omega L} V_o e^{j\omega t}\right) = \frac{R}{\omega L} V_o \sin \omega t \qquad V_{out} \approx \operatorname{Re}\left(-j \frac{R}{\omega L} V_o e^{j\omega t}\right) = \frac{R}{\omega L} V_o \sin \omega t$$

$$V_{out} \approx V_o \cos \omega t \qquad V_{out} \approx V_o \cos \omega t$$

Note: Recall that this analysis is based on Euler's Identity $e^{jq} = \cos q + j \sin q$. When we apply this expression specifically for phasors we have $e^{j\omega t} = \cos \omega t + j \sin \omega t$. Thus, $\operatorname{Re}(e^{j\omega t}) = \cos \omega t$ and $\operatorname{Re}(-j e^{j\omega t}) = \operatorname{Re}(-j \cos \omega t - j(j \sin \omega t)) = \sin \omega t$. When the relationship between the input and output is real and positive, there is no phase shift between the two voltages. When the relationship is negative imaginary, the output is 90° behind the input, since $\sin \omega t$ lags $\cos \omega t$ by 90° . One of the many advantages of using phasor notation is that one can determine the phase shift without converting the expressions back into real time-varying form. If we plot the transfer function in the complex plane, we can obtain the phase by noting the angle between it and the real axis. The four points 1, j, -1, -j are plotted below on the complex plane:

