Electronic Instrumentation

Name ________________ ENGR-4300 Spring 2000 Section _____________

Homework #2
Circuit Transfer Functions

Due: Friday, 23 February (Can be turned in the studio or any time before 5 pm at Prof. Connor's Office, JEC 6002)

In Experiment 5, we addressed circuits that were combinations of resistors, inductors and capacitors. The analysis discussed there included

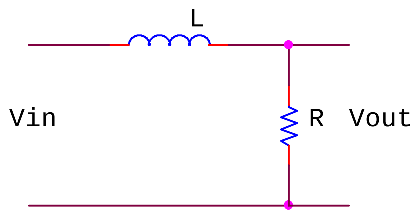
1. Determining the complex transfer function
2. Simplifying the complex transfer function for low and high frequencies and also for characteristic frequencies such as the corner frequency or resonant frequency. Determining the magnitude and phase of these simplified expressions.
3. Simulating the transfer function (magnitude and phase) over some range of frequencies using the AC Sweep Analysis available with PSpice.
4. Comparing the results of these two methods of analysis.
5. Experimentally determining the transfer function and comparing it with the results of a PSpice simulation. Before doing the final PSpice simulation, it is necessary to check the values of the components used to be sure that the model is accurate.

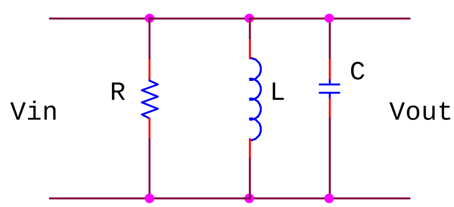
It is possible to do much more extensive analysis of such circuits. However, the steps listed above should be more than sufficient for our purposes. We will primarily be using these circuits as filters or assessing their performance as filters when they are included in a larger circuit for some reason.

To see if you understand this method and to prepare for the next quiz, you are to do some of these steps for most of the simple circuits we are likely to encounter in this course. These include (from Gingrich):

1 – The low-pass RC filter of figure 3.2
2 – The high-pass RC filter of figure 3.3
3 – The four RLC circuits of figure 3.9
4 – The RLC circuit of figure 3.10
5 – The RLC circuit of figure 3.12
6 – The RLC circuit of problem 1 (p 61)
7 – The LR circuit of problem 3 (p 62)

In addition, there is the other LR circuit and the parallel RLC circuit:





a. Write an expression for the transfer functions of all twelve of these circuits. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions. To be as general as possible, the input and output voltages are labeled V_{in} and V_{out}, respectively.

$$V_{out} = \frac{1/jwC}{1/jwC + R} V_{in} \qquad V_{out} = \frac{R}{R + 1/jwC} V_{in} \qquad V_{out} = \frac{R}{R + jwL} V_{in}$$

$$V_{out} = \frac{R}{R + jwL + 1/jwC} V_{in} \qquad V_{out} = \frac{jwL}{R + jwL + 1/jwC} V_{in} \qquad V_{out} = \frac{R + jwL}{R + jwL + 1/jwC} V_{in}$$

b. Simplify your answer to part a for very small frequencies. Identify the magnitude and phase of your expression. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions.

$$V_{out} \approx \frac{1/jwC}{1/jwC} V_{in} = V_{in} \qquad V_{out} \approx \frac{R}{1/jwC} V_{in} = jwRCV_{in} \qquad V_{out} \approx \frac{R}{R} V_{in} = V_{in}$$

$$V_{out} \approx \frac{R}{1/jwC} V_{in} = jwRCV_{in} \quad V_{out} \approx \frac{jwL}{1/jwC} V_{in} = -w^2 LCV_{in} \quad V_{out} \approx \frac{R}{1/jwC} V_{in} = jwRCV_{in}$$

Magnitude and Phase:

1,0	wRC, 90°	1,0
wRC, 90°	$w^2 LC$, 180°	wRC, 90°

c. Simplify your answer to part a for very large frequencies. Identify the magnitude and phase of your expression. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions.

$$V_{out} \approx \frac{1/jwC}{R} V_{in} = -j\frac{1}{wRC} V_{in} \qquad V_{out} \approx \frac{R}{R} V_{in} = V_{in} \qquad V_{out} \approx \frac{R}{jwL} V_{in} = -j\frac{R}{wL} V_{in}$$

$$V_{out} \approx \frac{R}{jwL} V_{in} = -j\frac{R}{wL} V_{in} \qquad V_{out} \approx \frac{jwL}{jwL} V_{in} = V_{in} \qquad V_{out} \approx \frac{jwL}{jwL} V_{in} = V_{in}$$

Magnitude and Phase:

$\frac{1}{wRC}$, -90°	1, 0°	$\frac{R}{wL}$, -90°
$\frac{R}{wL}$, -90°	1, 0°	1, 0°

d. Write the corner frequency or the resonant frequency for each circuit. Half of the expressions are included below. Identify which goes with which circuit and then find the remaining expressions.

e. For the circuits of figure 3.12 and problem 3 (p 62) rewrite your answers to parts a, b, c, and d using the following values for the components: R = 100, L = 100uH and C = 10uF.

f. While you are in class, perform a PSpice simulation for the two circuits of part e. Do an AC Sweep from low frequencies to high frequencies and produce a plot of the transfer function (ratio of output to input voltage) magnitude and phase. Follow the procedure given in Experiment 5. You will have to decide what is a low frequency and what is a high frequency. Low frequencies should be a lot smaller than the corner frequency and high frequencies should be a lot larger. Produce a plot of your result. On your plot, show that your expressions from part e are consistent with your PSpice plot.

Real Form of Transfer Functions: Recall that phasor expressions are converted back into real, time-varying expressions by first multiplying by e^{jwt} and then taking the real part of the resulting expression. Assuming that $V_{in} = V_o \cos wt$, the phasor form of the input voltage is $V_{in} = V_o$ and each of the six low frequency cases shown above becomes

$$V_{out} \approx V_o \cos wt \qquad V_{out} \approx \mathrm{Re}\left(-jwRCV_o e^{jwt}\right) = wRCV_o \sin wt$$

$$V_{out} \approx V_o \cos wt \qquad V_{out} \approx \mathrm{Re}\left(-jwRCV_o e^{jwt}\right) = wRCV_o \sin wt$$

$$V_{out} \approx -w^2 LCV_o \cos wt \qquad V_{out} \approx \mathrm{Re}\left(-jwRCV_o e^{jwt}\right) = wRCV_o \sin wt$$

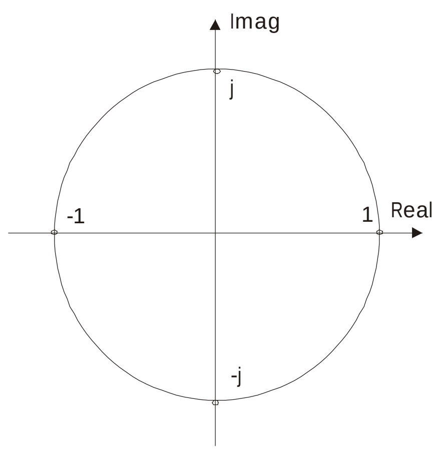
while the six high frequency cases become

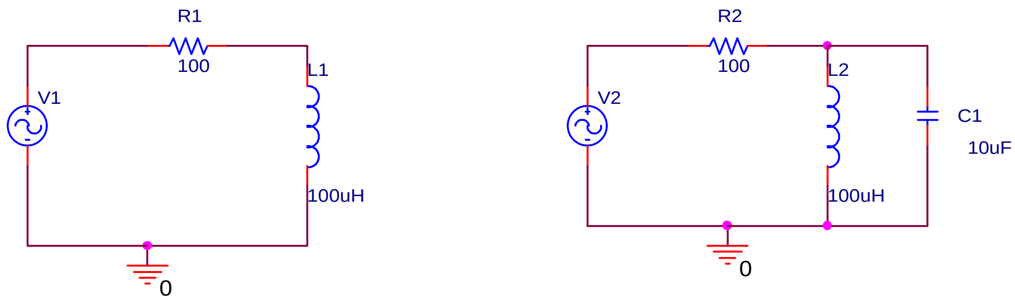
$$V_{out} \approx \mathrm{Re}\left(-j\frac{1}{wRC}V_o e^{jwt}\right) = \frac{1}{wRC}V_o \sin wt \qquad V_{out} \approx V_o \cos wt$$

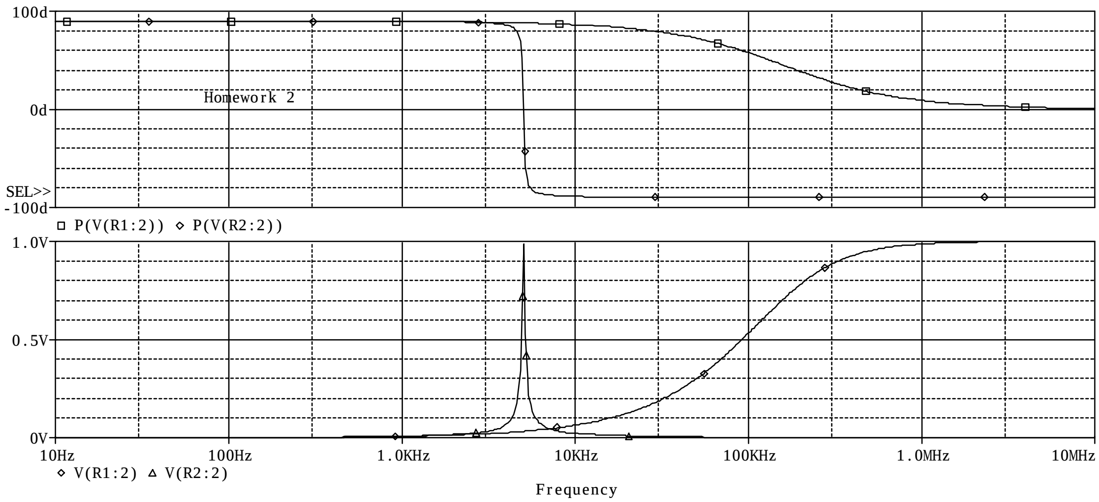
$$V_{out} \approx \mathrm{Re}\left(-j\frac{R}{wL}V_o e^{jwt}\right) = \frac{R}{wL}V_o \sin wt \qquad V_{out} \approx \mathrm{Re}\left(-j\frac{R}{wL}V_o e^{jwt}\right) = \frac{R}{wL}V_o \sin wt$$

$$V_{out} \approx V_o \cos wt \qquad V_{out} \approx V_o \cos wt$$

Note: Recall that this analysis is based on Euler's Identity $e^{jq} = \cos q + j\sin q$. When we apply this expression specifically for phasors we have $e^{jwt} = \cos wt + j\sin wt$. Thus, $\mathrm{Re}\left(e^{jwt}\right) = \cos wt$ and $\mathrm{Re}\left(-je^{jwt}\right) = \mathrm{Re}\left(-j\cos wt - j\left(j\sin wt\right)\right) = \sin wt$. When the relationship between the input and output is real and positive, there is no phase shift between the two voltages. When the relationship is negative imaginary, the output is 90^0 behind the input, since $\sin wt$ lags $\cos wt$ by 90^o. One of the many advantage of using phasor notation is that one can determine the phase shift without converting the expressions back into real time-varying form. If we plot the transfer function in the complex plane, we can obtain the phase by noting the angle between it and the real axis. The four points 1, j, -1, -j are plotted below on the complex plane:




R1
100
L1
V1
100uH
0
R2
100
L2
V2
C1
10uF
100uH
0


100d
0d
SEL>>
-100d
Homework 2
P(V(R1:2)) P(V(R2:2))
1.0V
0.5V
0V
10Hz
100Hz
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
V(R1:2) V(R2:2)
Frequency