Experiment 5
Bridges, Potentiometers, and Harmonic Oscillation

Purpose: In the following exercises, you will learn what a bridge is and how it can be used to measure small changes in resistance. You will also learn how to balance a bridge using a potentiometer. Then you will use the bridge to measure small resistances from a strain gauge mounted to an oscillating cantilever beam. You will use the oscillation frequency and your knowledge of cantilever beams to determine the material the beam is made of. Finally, we will extend the theory of oscillation to an electrical system, an oscillating circuit.

Equipment Required:

- **DMM** (HP 34401A 6-1/2 Digit Multimeter)
- **Rensselaer IOBoard Rev D** (with Mobile Studio Desktop)
- **Oscilloscope** (Rensselaer IOBoard)
- **Function Generator** (Rensselaer IOBoard)
- **Instrumented Beam**
- **Parts Kits**

Helpful links for this experiment can be found on the links page for this course:
http://hibp.ecse.rpi.edu/~connor/education/EILinks.html#Exp5

Part A – Bridge Circuits

Background

Bridges and Voltage Dividers: In Experiment 1, we looked at one of the simplest useful circuits – the voltage divider. In many simple applications of electronics, we have only a small number of standard voltages in whatever circuits we are building. When we use a 9 volt battery as our source we have only one voltage level available, unless we use a voltage divider to get smaller voltages. We can also use a divider, as pictured in figure A-1, to measure resistance, if we have some device with an unknown resistance. For example, if we connect an unknown resistor in series with a known resistor, then the voltage across the unknown resistor can tell us the value of the resistance. An even better measurement can be done by combining two voltage dividers in a configuration like the one shown in the following figure. Note that if R1 = R2 = R3 = R4, the voltages at the two points marked Vleft and Vright will be equal to half of the source voltage. Thus, their difference should be zero. Whenever the voltage difference across a bridge is zero, we say that the bridge is *balanced.*
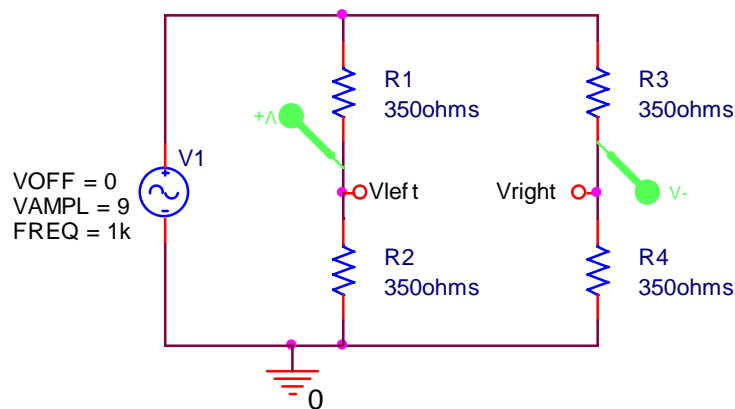

Figure A-1

The following equations apply to the bridge circuit. When dV = 0, the bridge is balanced:

$$V_{left} = \frac{R2}{R1+R2}V1 \qquad V_{right} = \frac{R4}{R3+R4}V1 \qquad dV = V_{left} - V_{right}$$

$$V_{left} = \frac{1K}{1K+1K}V1 \qquad V_{right} = \frac{1K}{1K+1K}V1 \qquad dV = V_{left} - V_{right} = \frac{V1}{2} - \frac{V1}{2} = 0$$

Experiment

Modeling a bridge in PSpice
In this part, we will set up a bridge in PSpice and look at the effect of a small change in one of the resistors on the difference across the bridge.

- A.1 Look at the behavior of a balanced bridge circuit.
    - Set up the circuit on the previous page as shown.
    - Use a 9V amplitude, 1KHz frequency and no DC offset.
    - Place voltage markers at Vleft and Vright, or use the "voltage difference markers." Then run a transient analysis. (You have done enough transients now to be able to find a reasonable "run to time" and "step size".)
    - Add a *Trace* of the difference between the two voltages, (Vleft-Vright) (It will already be there if you used the "voltage difference markers."
    - Is the difference zero?

- A.2 Look at the behavior of an unbalanced bridge circuit.
    - Now, change R4 to be equal to 362 Ohms, a 2% change in R.
    - Do the analysis again and add the trace of the difference between the two voltages.
    - What is the amplitude of the difference voltage as a percentage of the source voltage?
    - Label the plot, print it and include it with your report.

- A.3 Analyze this circuit by hand
    - Use voltage dividers to find the voltages at the two points and their difference. Make sure that your answer agrees with the PSpice simulation.
    - Assume that R1 = R2 = R3 are known resistors equal to R, and that R4 is unknown. Derive a formula for R4 in terms of R , the source voltage V1, and the voltage difference between the two divider voltages (dV=Vleft-Vright). [Hint: Substitute voltage divider expressions in for Vleft and Vright and solve for R4.]

- A.4 Parameter sweep
  - o In PSpice, it is possible to run simulations for several values of component. Above you ran two separate simulations, one with R4=350Ω and one with R4=362Ω. Now you will do one run with 11 values for R4. This is called a parameter sweep.
  - o Change the resistance value of R4 from 362Ω to a variable name set with in the curly brackets, {Rvar} is used in the figure below. You can make the name anything you want, but use something that reminds us what you are doing. The resistance value is now a variable.
  - o Next we have to tell PSpice that we are using a parameter. To do this, we go to the parts list and select PARAM, which we will find in a library called *Special.* Place this item in an uncluttered spot on your schematic.
  - o The <u>PARAMETERS:</u> "part" is a list of variables. You can now create and assign value to your variable *Rvar* using the following procedure:
    - Double click on the word <u>PARAMETERS:</u> to display the spreadsheet
    - Click on New Column.
    - In the Property name textbox, enter your chosen name (*Rvar* without the curly brackets).
    - Enter 1k for the default value. This value is only used to set a dc operating point, so the actual value doesn't matter.
    - While this cell is still selected, click *Display.*
    - In the *Display Format* window select *Name and Value,* then click *OK*.
    - Click *Apply* to update all the changes to the PARAM part.
    - Close the Parts spreadsheet.
    - When you have finished, you will see the parameter and its default value listed under <u>PARAMETERS:.</u>
    - Now your circuit should look like the figure below. When you have finished, you will see the parameter and its default value listed under Parameters. (Refer to figure A-2 below.)
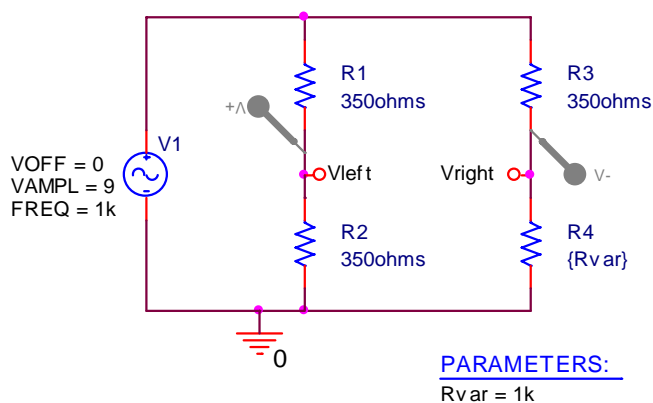

Figure A-2

  - o Next we will set up the analysis (See figure A-3 on the following page). Create a simulation.
    - o Select Time Domain (Transient)
    - o Check Parametric Sweep
    - o Check the Global parameter box
    - o Enter the variable name in the Parameter name box
    - o Pick a linear sweep, start value of 340, steps of 2 and end value of 360.
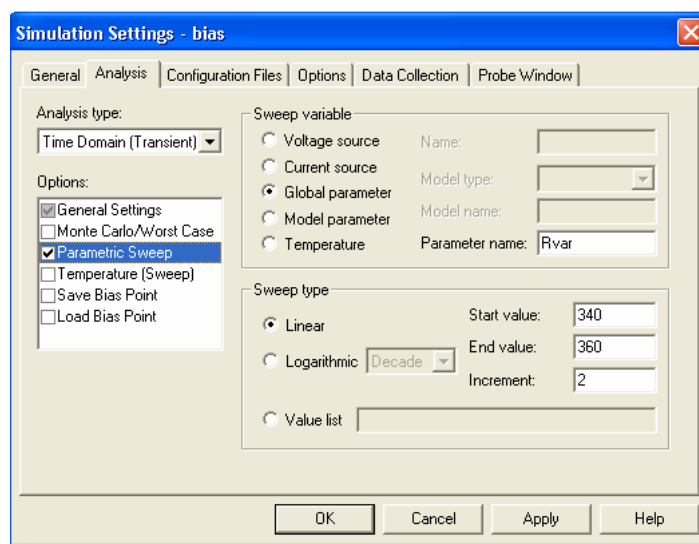
Figure A-3

- o Perform the simulation
    - • You will have all 11 simulations listed in the Probe window
    - • Use the All option
    - • Plot Vleft-Vright
    - • Note: if you right click on a trace, click on information, you can tell which value of R4 is plotted.
    - • <u>Print this page and include it in your report.</u>
- o Delete all traces from the plot window. Plot just Vright, for all values of R4.
    - • Include this plot in your report
    - • Comment on if it is easier to see the effect of a change in R4 using the bridge (Vleft-Vright) compared to just using one side (Vright.)

- • A.5 Sensitivity calculation
    - o Use the results from the plot to determine the sensitivity of the bridge circuit to changes in R4.
    - o What is the change in (Vleft-Vright) divided by the change in R4?

Summary

A bridge allows you to compare two voltages and to detect relatively small changes in a component value. We will use it in part B to observe small voltage differences caused by very small changes in the resistance of a strain gauge.

Part B – Strain Gauges

Background

Strain Gauges: A strain gauge measures the displacement of a surface when it is subjected to stress. The gauge is mounted securely to a surface (usually with super glue). When the surface stretches because of an external force, the strain gauge also stretches. The conductors of the strain gauge are either stretched or compressed, strained. This results in a small change in the strain gauge resistance. In our case, the change in resistance of the strain gauge will give a signal proportional to the displacement at the end of the beam. Figure B-1 shows a typical strain gauge.

National Instruments has a very good discussion of stain gauges and our application at:
http://zone.ni.com/devzone/cda/tut/p/id/3642
Go to the link above and read up to Figure 7, Full-bridge Circuit. We will use the Half-Bridge Circuit.
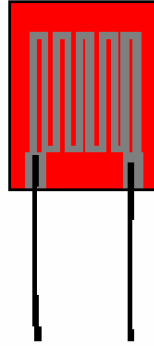

Figure B-1

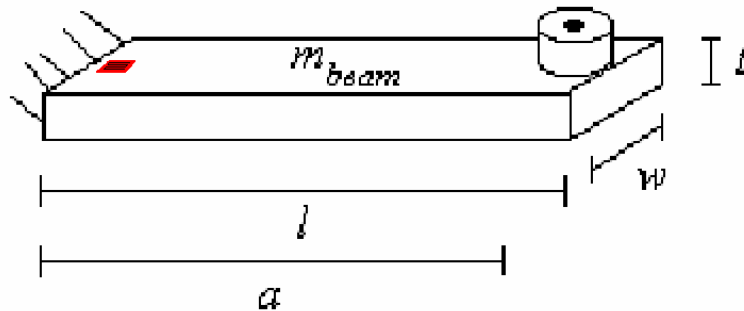*The Cantilever Beam:* Figure B-2 shows the cantilever beam we use in class.

Figure B-2

Please note: The beams to be used in the Fall 2006 have two strain gauges, one on the top of the beam and one on the bottom. This allows for the use of the Half-Bridge Circuit discussed on the NI website. These beams look somewhat different than the figure above but the principals are the same.

The beam has two stain gauges wired together, replacing R3 and R4 in figure A-1 of this document. It also has two matched 350Ω resistors. These are R1 and R2 of figure A-1. The signals from the stain gauges are small, so a difference amp with a gain of 100 is required.

Experiment:

Strain Gauges
We will now combine our study of strain gauges, bridge circuits and op-amps in a practical hardware application. We use the two resistors and the two strain gauges mounted to a beam to create a bridge circuit to measure the deflection of the beam. The signals are small so the output of the bridge will be amplified using a difference amplifier op-amp circuit.

B.1) Before you build the circuit, use the multimeter to determine the resistance of one strain gauge in its rest position.
   o   *Do not make this measurement when the beam is connected to the circuit. Do not over extend the beam upwards or downwards.*

- o   Measure the resistance of the strain gauge with the DMM when the beam is at rest. This is the resistance measured between the gray and red leads without anything else connected to the circuit.
- o   Deflect the beam down until about half way to the support plate and measure the resistance with the DMM.
- o   Deflect the beam up an equal amount and measure the resistance with the DMM.
- o   Note that the resistances you measure are proportional to how much you move the beam.
- o   <u>Write down the maximum and minimum resistance of your strain gauge.</u> You should now see why we indicate this resistance with a variable resistor in the figure below. You will also note that the total change in the resistance is small. You will need a sensitive circuit to be able to detect this change.

B.2) Wire the circuit shown in figure B-3.
- o   The circuit in the dashed box is wired on your protoboard.
  - ▪   ***This should be wired neatly.*** It will be used for this experiment and for the next project.
- o   Use the LF351 Op-amp
- o   The circuit in the solid box is prewired on the instrumented beam.
- o   You need to connect 4 wires between the beam and the protoboard.
- o   Don't connect the batteries until you are ready to make measurements.
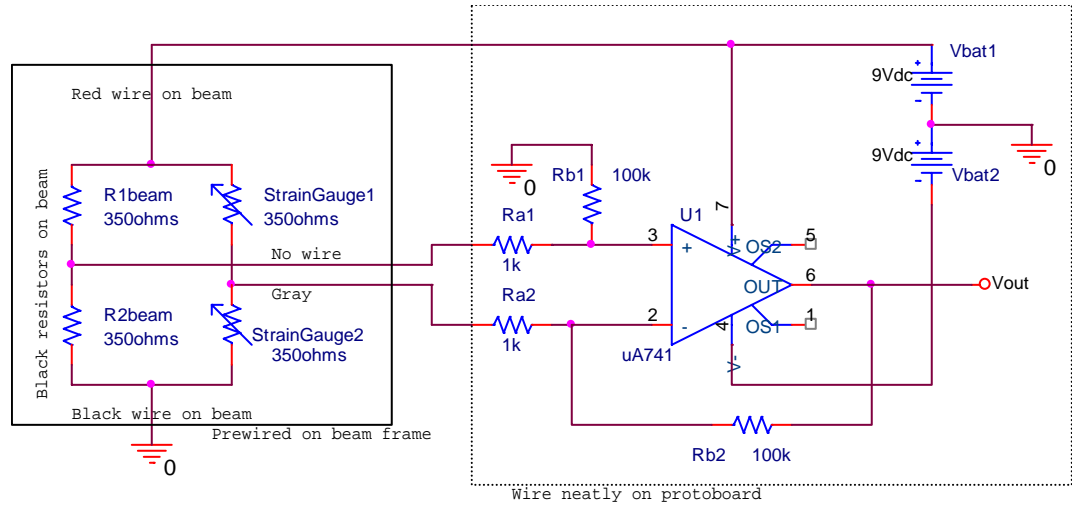- o   ***Disconnect the batteries between measurements.***

Figure B-3
Use the LF351 op-amp
Wire the circuit neatly; it will be used for the next project.

B.3) Repeat the deflection measurements of part B-1.
- o   Read the output voltage without any beam deflection.
- o   Read the output voltage with the beam deflected about halfway to the plate.
- o   Read the output voltage with the beam deflected an equal amount up.
- o   Record the three voltage measurements.

Damped Sinusoids: The damped sinusoid in figure B-4 is governed by the equation: $v(t) = Ce^{-\alpha t}\sin(\omega t)$, where α is called the damping constant that determines the rate of decay. To find the damping constant, choose two points at extreme ends of the sinusoid and use the following equation and solve for α:
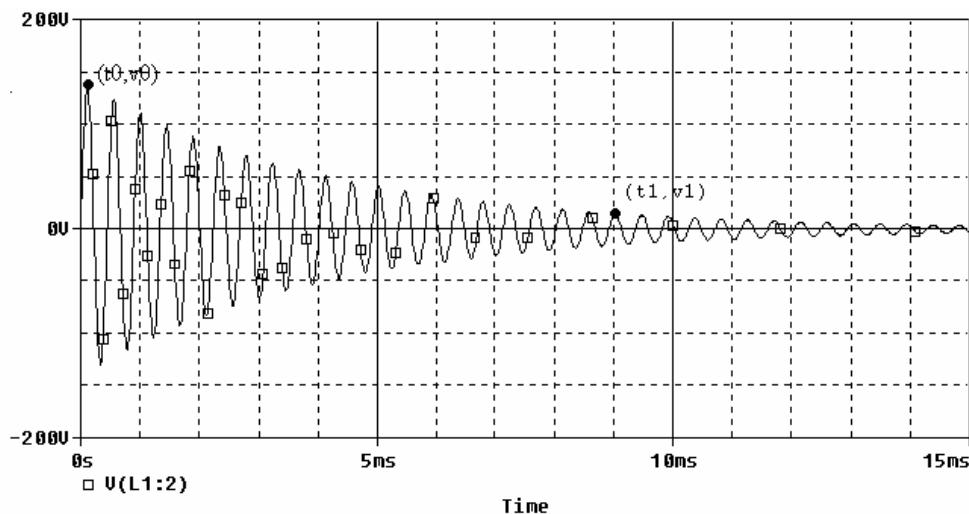
$$v1 = v0e^{-\alpha(t1 - t0)}$$

Figure B-4

B.4 When you have everything hooked up, set the beam into free oscillation.
- o You should observe a decaying sinusoidal voltage.
- o It will take a little experimenting to get a good trace on the screen.
  - Start with DC coupling for the scope channel
  - Start with time base of 10ms/div
  - Pull the beam down about half way and note if the voltage has dropped or increased.
  - If the voltage has increased, swap the two wires connected to Ra1 and Ra2 of figure B-3.
  - Pull the beam up about the same amount 1cm (or 1/2"). Note how much the voltage changed relative to no deflection. Call this $V_{trigger}$
  - Now set the channel coupling to AC
  - Adjust the trigger level so it is above zero by about $V_{trigger}$ or a little less.
  - Now set the trigger mode to "Normal"
  - Pull the beam down and release
    - If the IOBoard scope triggered then you are set
  - Now set the time to 100ms/div
  - Pull the beam down and release

- o Once you have obtained a clear signal of a decaying sinusoid, save the data to a file and plot it with Excel.
- o You can also use the cursors on the IOBoard scope to measure points on the trace.
- o <u>Determine the frequency of oscillation and the decay constant.</u>
- o <u>Include this plot with your report. Indicate on it how you found the beam frequency and the decay constant. Note that a more accurate estimation of the frequency can be found by averaging over many cycles.</u>

Note that there is a very good reference on strain gauges from the Vishay Measurements Group, Inc available in the links page.

Summary

The position of a cantilever beam can be measured by using strain gauges. The signal from the strain gauges is relatively small so they are typically used in a bridge circuit and with amplification. In this experiment, we used these to observe the oscillation of a cantilever beam.

Part C – Instrumented Beam as a Harmonic Oscillator

Background

Simple Pendulums: Before we address harmonic circuits, we will review some of the properties of the simple pendulum. The instrumented beam is a very good example of a simple pendulum, even though it looks more like a small diving board. Let us assume that the end of the beam moves in the x-direction. Obviously, this is a simplification, since it really travels along the arc of a circle. When the beam is stationary, we will assume that it is horizontal and at x=0. Again, this is an approximation because the beam must bend downward slightly due to its own weight. When the beam is bent, it experiences a restoring force like a spring, $F = -kx$, where k is the spring constant. From Newton's Law, we can relate this force to the acceleration, a, velocity v, and displacement, x, of the beam.

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} = -kx$$

Using only the terms related to the displacement, we can derive the harmonic oscillator equation: $\dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$

In standard form, the harmonic oscillator equation is $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ where ω is the frequency of oscillation in

rad/sec. Thus, the beam will oscillate at $\omega = \sqrt{\dfrac{k}{m}}$. The solution to this equation is: $x = x_0 \cos(\omega t + \phi)$, where

x_0 is the initial deflection of the beam and ϕ_0 is the initial phase. If you do not recall that this is the solution, plug the expression for x into the differential equation and you will see that it works. For simplicity, there is no need to include ϕ_0. Therefore, we can model the oscillation of a beam (with no friction) using the now familiar sinusoidal equation: $x = x_0 \cos(\omega t)$. Using this equation, we get an oscillation that will go on forever, rather than decaying slowly away, like the actual beam.

We can use the conservation of energy to verify this result. We know that the pendulum or any harmonic oscillator works by exchanging energy between two different forms. Not all forms of energy can be easily converted to another state and then back again, but we know this is trivial with the kinetic and potential energy of a mass.

The kinetic energy of a mass (the beam) is given by the equation: $KE = \dfrac{1}{2}mv^2$ and the potential energy of a spring

system (like the beam) is given by $PE = \dfrac{1}{2}kx^2$. Upon initial deflection, the energy of the beam is all potential.

Since we have assumed no dissipation (no friction or other damping force), the total energy will be conserved.

Therefore, the total energy, W, will be constant and given by, $W = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$ at any one point in the beam's

oscillation.

We can start at this expression of energy conservation to determine the equations of motion of the beam or any other simple pendulum. Since the total energy is a constant, we can take the time derivative of the entire expression and set it equal to zero:

$$\frac{dW}{dt} = \frac{1}{2}m(2)v\frac{dv}{dt} + \frac{1}{2}k(2)x\frac{dx}{dt} = 0$$

Since $v = \frac{dx}{dt}$, we can substitute for dx/dt and write $mv\frac{dv}{dt} + kxv = 0 \quad \rightarrow \quad m\frac{dv}{dt} = -kx$

which brings us back to Newton's second law: $F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2} = -kx$ Thus, once we have a conservation law, we can use it even to find out how things change with time.

Young's Modulus: We can use frequency to determine the properties of an oscillating mechanical system. In this part, we will use it to determine Young's Modulus for an oscillating beam and use this information to guess what type of material the beam is made of. We can do this using the relationship between ω and the properties of the oscillating system. We already know the relationship between force and the properties of a spring. We can solve for –k. $F = -kx \quad and \quad -k = \frac{F}{x}$. For the oscillating beam, x corresponds to the displacement at the end of the beam. This relationship is defined by the physical properties of the beam, $x = \frac{Fl^3}{3EI}$, where l is the length of the beam, E is Young's Modulus, and I is the moment of inertia of the beam. If we solve this relationship for F/x, we will have a second expression for our constant, $-k = \frac{F}{x} = \frac{3EI}{l^3}$. Now, we can look up the moment of inertia for an object with a rectangular cross-section and substitute it in for I.: $I = \frac{wt^3}{12} \quad and \quad -k = \frac{3Ewt^3}{12l^3} = \frac{Ewt^3}{4l^3}$.

Recalling our relationship for frequency from before and solving for k, we find:

$\omega = 2\pi f = \sqrt{\frac{k}{m}} \quad and \quad \frac{k}{m} = (2\pi f)^2 \quad so \quad k = m(2\pi f)^2$. Since this is an oscillating system, we will ignore the negative sign. This gives us our final result:

$$k = \frac{Ewt^3}{4l^3} = m(2\pi f)^2 \qquad E = \frac{4ml^3(2\pi f)^2}{wt^3}$$

Therefore, if we can come up with a reasonable estimate for the mass of the beam and its resonant frequency, we should be able to find Young's Modulus and use that to look up the material from which the beam is made.

The mass of the cantilever beam: Recall that the pendulum or harmonic oscillator equation holds for point masses located at the end of a massless beam. Since the beam has mass, but its center of mass is not located at the end of the beam, this term is multiplied by 0.23 to give the equivalent mass placed at the end of the beam that produces the same response. The beam also has a sensor attached to the end which adds extra mass. When we talk about "m" in this experiment we are referring to the actual mass of the beam migrated to the end. Thus m is the effective mass of the beam with no load and $m = 0.23 \times m_{beam}$.

Experiment

Frequency of a loaded beam
In the last experiment, you should have measured the oscillation frequency of the unloaded beam using the strain gauge and the bridge circuit.

- Use the circuit from part B to find the frequency of the beam with varying mass
  o Set the beam into oscillation.
  o Produce a plot of the decaying sinusoid you observe and save it using the IOBoard software.
  o Record the frequency in the table below.

Mass of object	Mass of clamp	m_n = Total mass at end of beam	Frequency (Hz)
0 kg	*0 kg*	$m_0 =$	$f_o =$
		$m_1 =$	$f_1 =$
		$m_2 =$	$f_2 =$
		$m_3 =$	$f_3 =$

- Measure the frequency three more times using additional masses of your choice.
  o Choose three objects from about 100 to about 500 grams. You should try to get a good distribution to get discernable data. The actual mass of the beam becomes less important with the heavier masses. So one mass should be heavy (around 500 grams). *Don't load the beam with a mass so heavy that it permanently bends the beam.*
  o Measure the masses of the objects you have chosen with the scale in the studio. Enter them into the table. Be sure to include the mass of the paper clips used to hold the added mass.
  o The total mass at the end of the beam, m_n, is the sum of the object plus the clamp.
  o Place each mass as close to the end of the beam as possible using a paper clip.
  o Find the beam frequency for each mass and record it in the table. Try to be as accurate as possible. Your values should have at least one decimal point. You should check the frequency a couple of times, since you should notice that there will be a range of values for the frequency, primarily because of noise and the somewhat non-ideal nature of the sinusoidal voltage.

Analysis of beam data
In this part of the experiment, we will analyze the frequency data to determine the mass of the beam, Young's Modulus for the beam, and, finally, the material out of which the beam is made.

- Find 4 equations in 2 unknowns
  o We know that our system is governed by the relationship between the oscillation frequency and the properties of a spring.

$$\frac{k}{m + m_n} = (2\pi f_n)^2 \quad n{=}0,1,2,3$$

  o We can write out a k expression for each of the four frequencies you have measured:

$$k = (m + m_0) \cdot (2\pi f_0)^2$$
$$k = (m + m_1) \cdot (2\pi f_1)^2$$
$$k = (m + m_2) \cdot (2\pi f_2)^2$$

$$k = (m + m_3) \cdot (2\pi f_3)^2$$

- Use these numbers to determine the values of *k*, and *m*.
    - o Note that you are making four measurements to determine two constants. This means that you have some redundancy built in and also that you will not obtain perfect agreement for all four equations. None of your measurements will be perfect, so it is best to have some more measurements than constants to determine to average out measurement error. You need to find the values of *k* and *m* that come the closest to satisfying all four equations.
    - o We are going to use Excel to plot the frequency of our system in relation to the mass added to the end. First we must solve for f_n . Note that in the equation below, m_n is the x variable and f_n is the y variable.

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k_{guess}}{m_{guess} + m_n}}$$

    - o We need to determine a good guess for k (the spring constant) and m (the effective mass of the beam) in order to plot this equation. Use the data from only two of your masses and solve two equations in two unknowns to determine a guess for k and m. (If you get a negative mass, try using your smallest and largest mass to do the calculation.) Keep in mind that these are just guesses, so you don't need to get carried away solving all combinations of all the equations. You could use some type of statistical analysis instead to get your guesses for k and m. For example, you could determine the standard deviation of the four expressions for *k* for a range of realistic values for the effective beam mass.
    - o Now you can plot the equation in Excel. Use k_{guess} and m_{guess} you just calculated. Choose values for m_n between 0 and 600 grams. You are plotting a general function and matching your data to it. m_n is the domain of your function, you just need a set of x (m_n) values so you can calculate y (f_n). Have Excel calculate values for f_n for each m_n and plot the results.
    - o Place your four data points on the plot. How well do these points fit the curve you generated? If your guesses are exactly perfect, they will lie right on the curve. Since they are only guesses, there is probably room for improvement.
    - o Now it is time to adjust k_{guess} and m_{guess} to get the curve to match your data as closely as possible. Adjusting one will move the plot up and down. Adjusting the other will cause the curvature to change. Play with the numbers until the curve matches your 4 data points as closely as possible. When you are doing this, keep in mind that the location where the graph crosses the y axis represents the unloaded frequency of the beam. The function goes up very quickly near zero mass. What is a reasonable estimate for the unloaded beam frequency? Include the final plot with the general curve and the four data points marked in your report.
    - o Use the values of k_{guess} and m_{guess} that give you the closest match in your final calculations for the beam mass and Young's Modulus.

- Final results
    - o Calculate the mass of the beam using your best guess for m . $m = 0.23 \times m_{beam}$
    - o CAREFULLY measure the dimensions of your beam. A small inaccuracy in your measurements can lead to a large discrepancy in your results.
    - o Extrapolate the frequency for the beam (with no load at all on the end) from your plot. This is the point at which mass at the end of the beam is 0 kg.
    - o Calculate Young's Modulus using your best estimate for k and m.
    - o Look up Young's Modulus in the table of your choice and find some possible materials for your beam. (There may be more than one possibility.)
    - o If you find more than one possible value for the material, think about other properties of the beam that may narrow the possibilities.

Summary

In this part of the experiment, you used the oscillation frequency and other physical properties of a cantilever beam to find information about the beam that you could not measure. You also learned how to use curve fitting to find a solution when there are more equations than unknowns.

Part D – Oscillating Circuits

Background

Energy storage in inductors and capacitors: In passive electrical systems, there are three kinds of circuit elements: resistors, capacitors and inductors. Resistors turn electrical energy into heat. When a current I flows through a resistor, there will be a voltage drop V across the resistor. The power dissipated by the resistor is equal to the product of I times V. Since resistors produce heat, it should be no surprise that they play the same role as friction in a mechanical system. The ideal pendulum will oscillate forever … a real pendulum will oscillate until all its stored energy is converted to heat through friction. Thus, if we wish to create a circuit analogous to the ideal harmonic oscillator, it can have no resistors in it. Rather, we will combine only inductors and capacitors.

A typical inductor consists of a coil of wire. If we pass a current through the coil, a magnetic field will be created. Many of us have made simple electromagnets at some time in our lives by wrapping wire around some magnetic material like a nail. When a battery is connected to the wire, it is possible to attract small pieces of iron to the nail. The field created by the coil, the magnetic field, can do work and thus contains energy. The energy stored in an inductor is given by the expression

$$W_M = (1/2)\ L\ I^2$$

where we have used the subscript M to indicate that the energy is stored in the magnetic field and L is the inductance in units of Henries. Joseph Henry was honored by using his name for this unit because of his early work in developing practical electromagnets. He began his work here in New York's capital district when he was teaching at the Albany Academy. There is a statue of this great scientist outside the original location of the Academy across the street from the Albany City Hall. Henry was a contemporary of Amos Eaton, the intellectual force behind the founding of RPI. He eventually left Albany for Princeton and the Smithsonian.

A typical capacitor consists of two metal plates of large area separated by some insulating material such as teflon or some other plastic. When a voltage source is connected to the plates, charge flows from the source to the plates with positive charge deposited on the plate with the highest voltage and negative charge deposited on the plate with the lowest voltage. Since these charges are opposite in sign and since unlike charges attract one another, there is a force between the two plates. Again, the existence of this force tells us that we have to do work to charge up the plates and that there is energy stored. In this case, the energy is stored in the electric field created by the charges.   The energy stored by a capacitor is given by the expression

$$W_E = (1/2)\ C\ V^2$$

where we have used the subscript E to indicate that the energy is stored in the electric field and C is the capacitance in units of Farads. Michael Faraday also worked on electromagnets and, being British, gained much more fame for his work, since America was a scientific backwater at the time. Henry showed him how to make better magnets, but Faraday's work was much more far reaching. Henry also showed Morse how to build a telegraph!

Aside: It is somewhat interesting to note that neither Henries nor Farads turns out to be much of a common practical unit. One Henry is a huge inductor, rarely seen in practice. One Farad is also rare, now occasionally seen in highly filtered power supplies for computers. We will need to use the prefixes milli-, micro-, nano-, pico-, etc. a lot when dealing with these components. We also do not see one Ohm all that much, but require the other kind of prefixes (kilo-, mega-, etc.).

Conservation of Energy in a Harmonic Circuit: There are many important lessons we can learn from the harmonic oscillator, but perhaps one of the most useful is the value of conservation laws. It is fair to say that *the most*

powerful problem solving technique is to first decide which conservation laws hold. Once the conservation laws are identified, they can be used to determine a great deal of information about any system.
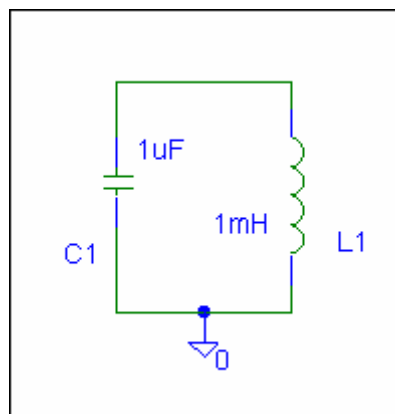

Figure D-1

Consider the simplest possible configuration of a single capacitor and a single inductor connected as shown in figure D-1. Note that, since there are only two components, one can describe this connection as either in parallel or in series. Also assume that the capacitor has been charged up to some voltage V at time t = 0, at which time it is connected to the inductor. The charge will begin to flow creating a current through the inductor. After a short time, all the charge that was originally on the capacitor plates will be gone and the current through the inductor will reach its maximum value. Thus, we began with all the energy stored in the capacitor and none in the inductor and end with the opposite condition. The current flowing through the inductor will then charge the capacitor back up and the process will begin again. The energy is traded back and forth between the two storage elements. There is an excellent applet on the links page which illustrates this principle.

The total energy of the system must remain constant because there is no dissipative element (no resistor). The total energy, W, is a constant equal to the energy stored in the capacitor added to the energy stored in the inductor. Thus,

$W = \dfrac{1}{2}CV_C^2 + \dfrac{1}{2}LI_L^2$. Since this is a constant, we can take the time derivative of this expression and set it equal to zero.

$$\dfrac{dW}{dt} = \dfrac{1}{2}C(2)V_C\dfrac{dV_C}{dt} + \dfrac{1}{2}L(2)I_L\dfrac{dI_L}{dt} = 0 \quad \rightarrow \quad L\dfrac{dI_L}{dt}I_L + C\dfrac{dV_C}{dt}V_C = 0$$

We want to do with this equation what we did with the energy conservation equation of the beam. This time, though, instead of expressing the equation in terms of the displacement, *x*, we want to express it in terms of the voltage, *V*.

We can use the general equations for the behavior of the capacitor and inductor to make these substitutions:

$$V_L = L\dfrac{dI_L}{dt} \qquad I_C = C\dfrac{dV_C}{dt}$$

For the circuit shown above, V is the voltage at the top of the circuit and I is the current flowing around the circuit. Since this is a series circuit with only two elements: $V = V_C = V_L$ and $I = I_C = I_L$. Making these

simplifications, our equations become : $V = L\dfrac{dI}{dt} \qquad I = C\dfrac{dV}{dt}$

We can further conclude from the capacitor equation that, by solving for dv/dt, $\dfrac{dV}{dt} = \dfrac{I}{C}$ and, by takingthe time

derivative of both sides, $\dfrac{dI}{dt} = C\dfrac{d^2V}{dt^2}$. Substituting into the conservation of energy equation, we get

$$L\frac{dI}{dt}I + C\frac{dV}{dt}V = 0 \quad \rightarrow \quad LC\frac{d^2V}{dt^2}I + C\frac{I}{C}V = 0 \quad \rightarrow \quad LC\frac{d^2V}{dt^2} + V = 0$$

This leads us directly to the harmonic equation for oscillating circuits: $\dfrac{d^2V}{dt^2} + \dfrac{1}{LC}V = 0$

If we compare this to the general expression for harmonic oscillation, $\dfrac{d^2V}{dt^2} + \omega_0^2 V = 0$,

we can determine the resonant frequency of the circuit: $\omega_0 = \dfrac{1}{\sqrt{LC}}$.

Also recall that this is the equation for the resonant frequency of any simple RLC circuit.

Damped Oscillation: The circuit we modeled above is unrealistic because it has no resistance at all. This is analogous to a mechanical system with no friction. To make a more realistic system, we must add some resistance, as shown in figure D-2.
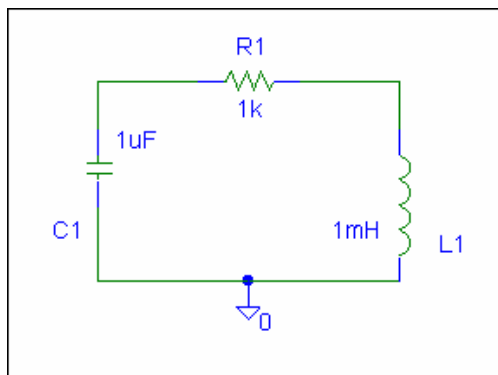

Figure D-2

The circuit will still oscillate, however, the oscillation energy will gradually dissipate because of the resistance. The output signal will be similar to the oscillation behavior of the beam -- a damped sinusoid.. Note that this circuit has no voltage source. It needs to have an initial amount of energy placed into it. This is similar to the initial displacement you place on the beam to make it oscillate.

The damped circuit has the following oscillation equation

$$\frac{d^2V}{dt^2} + 2\alpha\frac{dV}{dt} + \omega_0^2 V = 0$$

where α is the damping constant. It can be shown that in an ideal damped oscillation circuit, α is given by the following equation below:

$$\alpha = \frac{R}{2L}$$

Experiment

Modeling a damped oscillator
We will now consider an RLC circuit, with all three kinds of passive components and observe the damped oscillations.
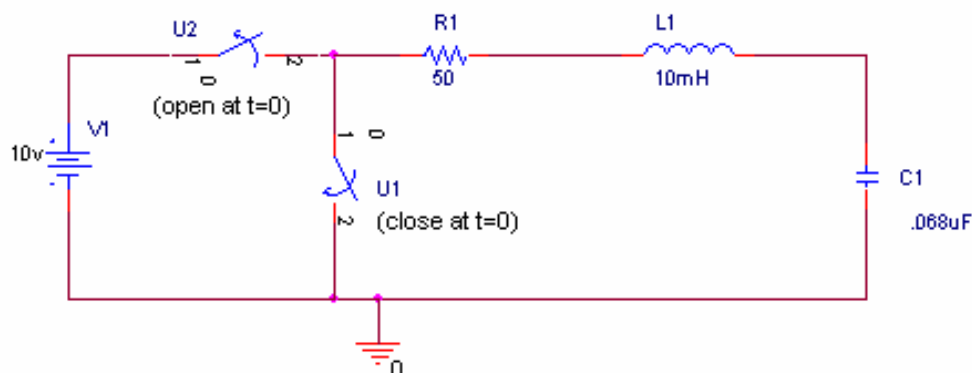

Figure D-3

- Create the oscillating circuit in figure D-3 above in PSpice
    - o In order to place some initial energy into the circuit, we use two switches. Before time t=0, switch U2 is closed and switch U1 is open. This places an initial charge on the capacitor. Now the capacitor has enough energy stored in it to start the oscillation. At time t=0, we disconnect the voltage source from the circuit and let it oscillate on its own. (Just like you release the beam and watch it vibrate until it stops.)
    - o The switches are in the EVAL library (Sw_tClose will close at a specified time period after t=0 and Sw_tOpen will open at a specified time period after t=0).
    - o Place voltage markers on the circuit between R1 and L1 and between L1 and C1.
    - o Use PSpice to simulate the transient response of this circuit for a total time of 1ms.
    - o Print out your results and include them in your report.
    - o What features of the voltages reminds you of the instrumented beam?
    - o Use the transient plot to find the oscillation frequency of your circuit. How does it compare it to the calculated value of $f = 1/[2\pi\sqrt{(LC)}]$?
    - o Use the plot to determine the damping constant of the circuit. In a simple RLC circuit, such as this one, the damping constant can also be found mathematically using the expression $\alpha=R/(2L)$. Calculate the damping constant and compare it to the one you found using the plot.

Summary

In this part of the experiment, you have related your knowledge of oscillating mechanical systems to an oscillating electrical system and created an oscillating circuit.

Report and Conclusions

The following should be included in your report. Everything should be labeled and easy to find. Partial credit will be deducted for poor labeling or unclear presentation.

Part A (12 points)

Include the following
1. A.1 nothing to plot (0 pt)
2. A.2
   a. Plot of Vleft-Vright with R4 modified (1pt)
   b. Fraction of output signal compared to Vac source (1pt)
3. A.3
   a. Analysis of circuit in part A.2 (2pt)
   b. Derivation of formula for Vout as a function of R4 (2pt)
4. A.4
   a. Plot of all 11 Vout traces (2pt)
   b. Plot of just Vright for all 11 cases (1pt)
   c. Comment on the advantage of using a bridge (Vleft-Vright) vs. using just a divider (Vright)(2pt)
5. A.5 What is the sensitivity of this circuit (change in Vout/change in R). (1pt)

Part B (16 points)

Include the following:
1. B.1 Min and Max resistance measured (2 pt)
2. B.2 nothing
3. B.3 Voltage measurements for three beam positions (3pt)
4. IOBoard plot of beam oscillation with calculations of beam frequency and damping constant on it. (6 pt)

Answer the following questions:
1. What is the resonant frequency of the beam? What value did you find for the damping constant? Write an equation for the decaying sinusoid output of the beam in the form $v(t)=Ce^{-\alpha t}\sin(\omega t)$. (5 pt)

Part C (30 points)
Include following plots:
1. Two IOBoard plots of the decaying sinusoid obtained with different masses added. For each write the mass added and the frequency measured (6 pt)
2. A table such as the one on page 10 of this document, listing the mass, total mass and frequency. (6pt)
3. Excel plot of frequency vs. load mass with four points marked (6 pt)

Answer following questions:
1. Explain or show your work on how you did your analysis to determine a reasonable first guess for k and m. (2 pt)
2. What are the values for k and m that you obtained by making the plot in Excel? (2 pt)
3. Calculate the mass of the beam. (2 pt)
4. Calculate Young's Modulus for the beam. Clearly indicate the values you measured for the beam's dimensions. (2 pt)
5. What do you conclude the beam could be made of? Why? (4 pt)

Part D (12 points)
Include following plots:
1. PSpice plot of output from oscillating circuit. (4 pt)
2. Determine the resonant frequency and damping constant of the circuit you analyzed using the PSpice output plot 1. Write an equation for the output in the form $v(t)=Ce^{-\alpha t}\sin(\omega t)$. (4 pt)

1. What value did you calculate for f using the equation for the resonant frequency? How close of an estimate is this to the resonance you found in the plot? (2 pt)
2. What value did you calculate for α using the equation? How close of an estimate is this to the damping constant you found in the plot? (2 pt)

Summary (10 points)

1. Organization, completeness, ordering. Is this easy to grade? (8 pts)
2. List member responsibilities (2 pts)

Total: 80 points for write up + 20 for attendance = 100 points

Attendance: 3 classes (20 points)   2 classes (10 points)   1 class (0 points) out of 20 possible points
                Minus 5 points for each late.
                No attendance at all = No grade for experiment.