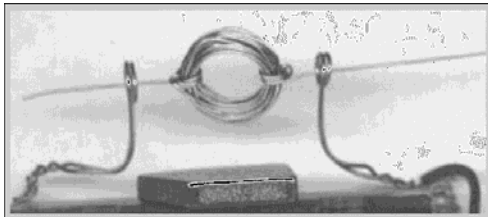
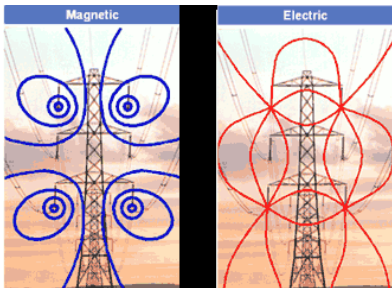
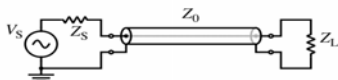
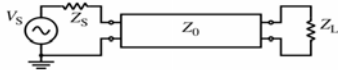
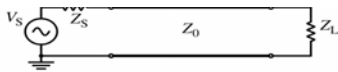
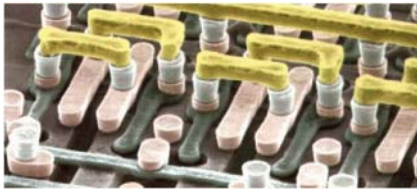
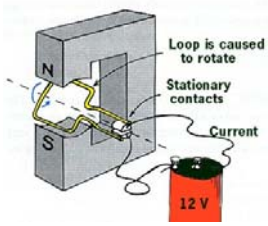




Homework 9

Due 9 December at 6pm



1. Short Questions (8 Points)

- a. (1 Point) _____ {Trans Lines}
- b. (1 Point) _____ {Inductance}
- c. (1 Point) _____ {Waves}
- d. (1 Point) _____ {Magnetic Ckts & Inductance}
- e. (1 Point) _____ {Trans Lines}
- f. (1 Point) _____ {Resistance & Inductance}
- g. (1 Point) _____ {Faraday's Law}
- h. (1 Point) _____ {Boundary Conditions}
- j. (1 Point) _____ {Waves}
- k. (1 Point) _____ {Lossy Materials}
- l. (1 Point) _____ {Misc}
- m. (1 Point) _____ {Vocabulary}

Total _____

2. Electric Fields (3 Points) _____

3. Magnetic Fields (3 Points) _____

4. Transmission Lines (3 Points) _____

5. Capacitance & Laplace's Equation (3 Points) _____

Total (20 Points) _____

Notes:

1. Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.
2. For short answer questions, you should add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use 4 significant digits.

Some Comments and Helpful Info:

In this course, we use two types of notation for unit vectors. Keep in mind that

$$\hat{a}_x = \hat{x} \quad \hat{a}_y = \hat{y} \quad \hat{a}_z = \hat{z}$$

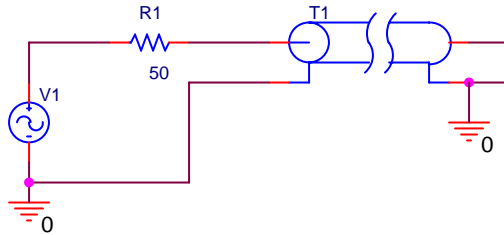
$$\hat{a}_r = \hat{r} \quad \hat{a}_\phi = \hat{\phi} \quad \hat{a}_\theta = \hat{\theta}$$

Also, sometimes R is used for spherical radius instead of r , so R is another term that gets used for more than one purpose. Pay attention to the context of the questions to minimize problems.



1. Short Questions (8 Points)

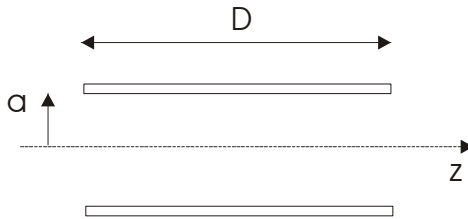
a. (1 Point) Transmission Lines



A 200 meter long air insulated transmission line has a short circuit at its load end. At low frequencies (near $\omega = 0$), does the line look like an inductor, a capacitor or a resistor? (Circle the correct answer.) Which of the following frequencies are low? (Again, circle the correct answers.)

- $f = 100 \text{ Hz}$ $f = 1 \text{ kHz}$. $f = 10 \text{ kHz}$ $f = 100 \text{ kHz}$ $f = 1 \text{ MHz}$ $f = 10 \text{ MHz}$

b. (1 Point) Inductance



A solenoidal inductor can be constructed using any of the following combinations of materials. Which will result in the largest inductance? Assume that the length of the solenoid is equal to D and the radius is equal to a .

- i) The solenoid has 250 turns of wire, length $D = 10\text{cm}$, radius $a = 5.64\text{mm}$ and is wound on mild steel ($\mu_r = 2000$).
- ii) The solenoid has 150 turns of wire, length $D = 20\text{cm}$, radius $a = 5.64\text{mm}$ and is wound on iron ($\mu_r = 5000$).
- iii) The solenoid has 50 turns of wire, length $D = 30\text{cm}$, radius $a = 5.64\text{mm}$ and is wound on mild steel ($\mu_r = 2000$).

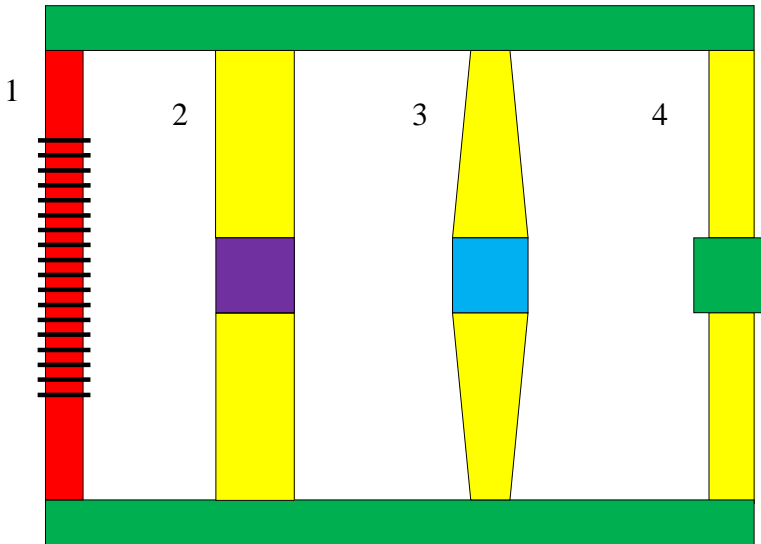


c. (1 Point) Electromagnetic Waves (Circle the correct answers.)

i. The closest distance between a minimum and a maximum in a voltage standing wave pattern is $(\lambda/4, \lambda/2, \lambda, 2\lambda)$.

ii. A uniform plane wave is propagating in water ($\epsilon_r = 1.5$ at optical frequencies) and is incident obliquely at 30° on the water/air surface at $z=0$. The angle of incidence is (equal to, greater than, less than) the angle of reflection. Which of the following is correct? (There is no transmitted wave into air, the wave transmitted into air propagates with an angle greater than 30° , the wave transmitted into air propagates at an angle less than 30°).

d. (1 Point) Magnetic Circuits and Inductance



The magnetic circuit at the left consists of four legs. The leftmost leg is wrapped with N turns of wire carrying a current I . Each of the legs looks the same from each side. The square sections in the middle of the second, third, and fourth legs are identical cubes. The permeability of the cubes is μ_{cube} and the permeability of all legs, including the horizontal legs at the top and

bottom is μ_{legs} . Also, $\mu_{cube} \gg \mu_{legs}$. The dimensions of the four legs are not given, but you can assume their relative sizes correspond to what is shown in the diagram.

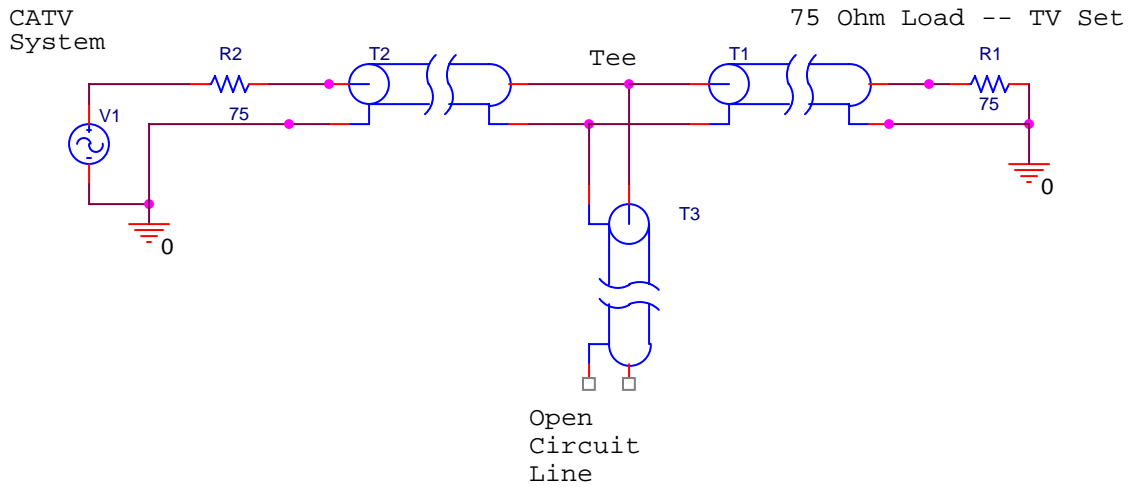
i. Put the reluctances of the four vertical legs in order from largest to smallest.

ii. Put the flux carried by the four vertical legs in order from largest to smallest

iii. You wish to maximize the inductance of this configuration by removing all, some or none of the cubes. What will produce the largest inductance?



e. (1 Point) Transmission Lines

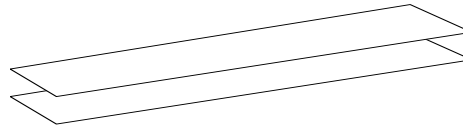


An open-circuited, 0.20 meter long transmission line with $Z_o = 75\Omega$ and $v_{ph} = 2.5 \times 10^8$ m/s connected to a CATV line with a Tee, as shown. Which of the following channels will be blocked by this line? (The frequencies are given in MHz.)

Channel	2	11	19	37	70	97
Frequency	55.25	199.25	151.25	301.25	499.25	103.25

f. (1 point) Resistance and Inductance

A parallel plate transmission line is constructed as shown in the figure. Assume that the length of the line is given by l .



i) Determine the resistance of this line. Circle the correct answer.

$$R = \frac{2l}{\sigma wt}$$

$$R = \frac{2l}{\sigma hw}$$

$$R = \frac{l}{\sigma wt}$$

$$R = \frac{l}{\sigma hw}$$

ii) Determine the inductance of this line. Circle the correct answer.

$$L = \frac{\mu_o t 2l}{w}$$

$$L = \frac{\mu_o h 2l}{w}$$

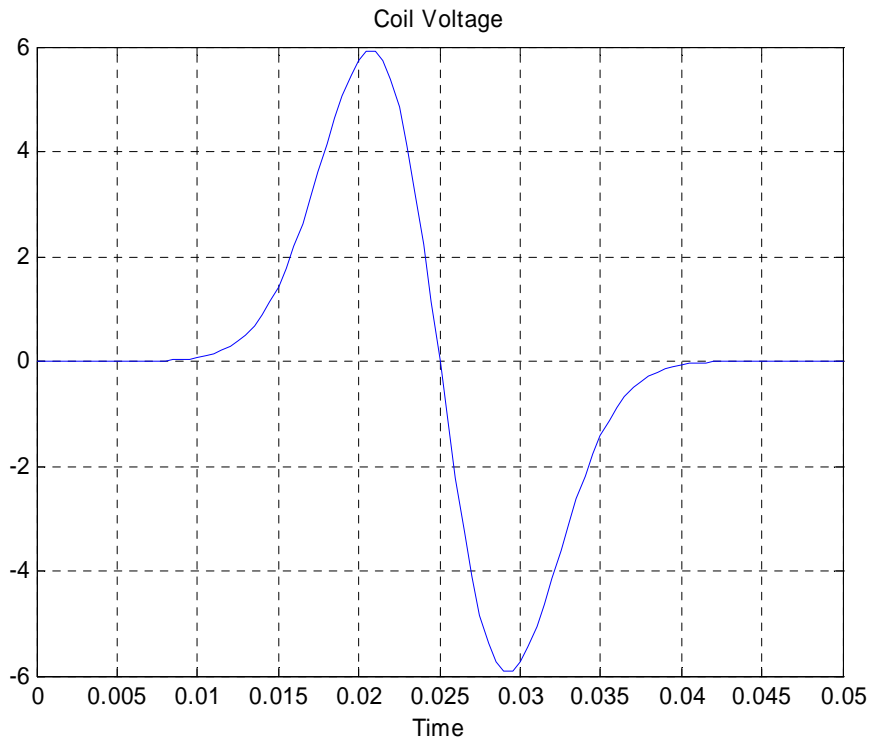
$$L = \frac{\mu_o t l}{w}$$

$$L = \frac{\mu_o h l}{w}$$



g. (1 Point) Faraday’s Law

A small permanent magnet is dropped through a coil like the one we used in the Beakman’s motor project (with 300 turns of wire and a radius of 1 cm). The voltage measured across the terminals of the coil looks like the following figure as a function of time. A good approximation to this voltage is given by the expression $V(t) = A_0te^{-\left(\frac{t}{\tau}\right)^2}$ volts, where $A_0=2300$ and $\tau = 6ms$ (this is from real data taken in 4107)



i. Using the simple relationship between the induced voltage and the flux linked by the coil, $V(t) = -\frac{d\Lambda}{dt}$, determine the magnetic flux linked by the coil $\Lambda(t)$ as a function of time during the entire period of the measurement. Circle the correct answer.

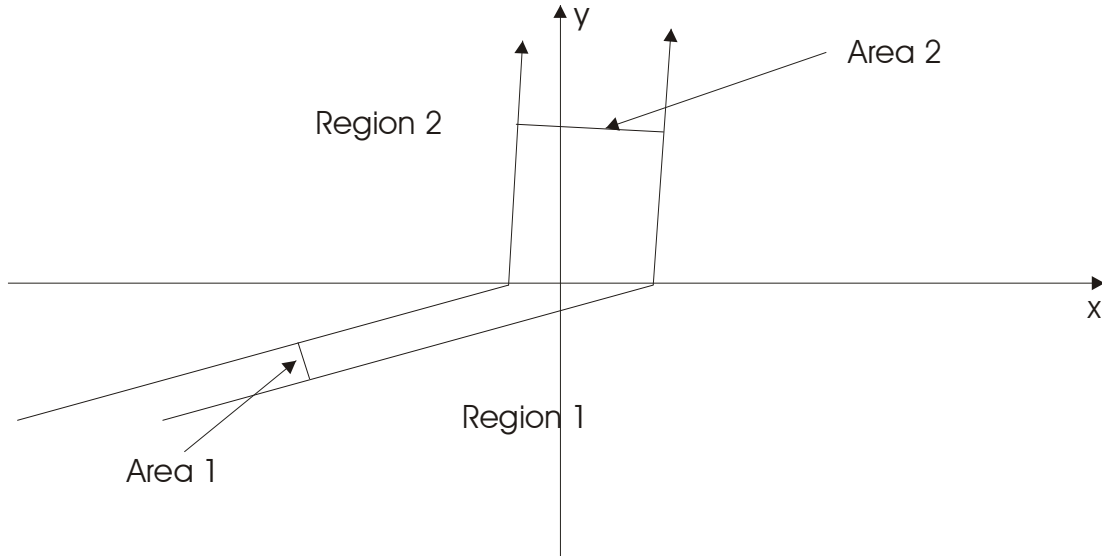
$\Lambda(t) = 4.14 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$, $\Lambda(t) = 0.0414 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$ or $\Lambda(t) = 6.9 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$

ii. Determine the average flux density $B_{axial}(t)$ of the magnet as a function of time. (The coil sensing the magnetic field is coaxial with the magnet axis, so you will be determining the average flux density in the axial direction.) Again, circle the correct answer.

$B_{axial} = 0.44 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$, $B_{axial} = 0.044 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$ or $B_{axial} = 73.2 \exp\left(-\left(\frac{t}{\tau}\right)^2\right)$



h. (1 point) Boundary Conditions



Shown above are two parallel magnetic field (\vec{B}) lines changing direction at the boundary between a magnetic material (with $\mu = \mu_r \mu_o$) and air ($\mu = \mu_o$).

i. Which of the following is correct? Circle the correct answers. (Region 1 is the magnetic material, Region 1 is air, Region 2 is the magnetic material, Region 2 is air)

ii. The angle between the y-axis and the field lines in Region 1 is 70° , while the angle between the field lines and the y-axis in Region 2 is 10° . What is the relative permeability of the magnetic material? Circle the correct answer.

$\mu_r = 156$

$\mu_r = 15.6$

$\mu_r = 312$

$\mu_r = 31.2$

iii. Which of the following is correct? Circle the correct answer. (The flux through area 1 is less than the flux through area 2, the flux through area 1 is equal to the flux through area 2, the flux through area 1 is greater than the flux through area 2)



j. (1 Point) Electromagnetic Waves



A uniform plane wave is incident normally on a dielectric interface between two media. Region 1 is air and region 2 is a dielectric region with $\epsilon = 4\epsilon_0$. The incident electric field is polarized in the x-direction and the magnetic field is polarized in the y-direction. The magnitude of the incident electric field is 100V/m and the frequency of the wave is 10MHz.

i. Write the incident magnetic field vector in phasor form. Be sure that your answer is in the form of a vector.

ii. What is the reflection coefficient Γ ?



k. (1 Point) Lossy Media

In a paper on the electrical loss properties of meat and other foodstuffs, the following table was published.

Dielectric properties (2450 MHz) of 5% aqueous solutions/suspensions of selected ingredients

Ingredient	ϵ''	ϵ'	d_p (cm)	$\tan \delta$	P_r
Deionised water	6.9	76.8	5.0	0.090	0.63
Antioxidant BHA	7.1	68.4	4.5	0.104	0.62
Soya concentrate	7.3	43.3	3.5	0.169	0.54
Natex rusk	7.5	47.9	3.6	0.157	0.56
Potato starch	8.3	77.8	4.2	0.106	0.63
Natex grits	9.0	57.7	3.3	0.156	0.59
Wheat flour	9.1	74.4	3.7	0.122	0.63
Boiled potato starch	9.2	76.1	3.7	0.121	0.63
Sucrose	9.4	76.1	3.6	0.124	0.63
Caseinate	9.9	74.3	3.4	0.133	0.63
Whey protein	10.2	74.2	3.3	0.137	0.63
Soya protein isolate	10.2	75.9	3.3	0.134	0.63
Rusk	10.3	77.2	3.3	0.133	0.63
Wheat gluten	10.3	74.5	3.3	0.139	0.63
Soya protein	10.9	76.3	3.1	0.143	0.63
Gluconolactone	11.5	76.1	3.0	0.151	0.63
Carrageenan	11.6	76.8	2.9	0.152	0.63
Sodium alginate	13.6	76.1	2.5	0.178	0.63
Sodium ascorbate	18.7	74.1	1.8	0.253	0.63
Red 2G	21.2	72.7	1.6	0.291	0.62
MSG	23.2	74.5	1.5	0.311	0.63
Sodium benzoate	24.9	73.1	1.4	0.340	0.63
Potassium sorbate	30.1	74.1	1.1	0.407	0.63
Phosphate (P22)	34.4	73.8	1.0	0.467	0.63
White pudding spice	42.8	71.2	0.80	0.602	0.62
Sodium sulphite anhydrous	44.7	70.7	0.77	0.632	0.62
Nitrate	44.9	70.9	0.76	0.633	0.62
Nitrite	53.1	69.6	0.65	0.763	0.62
Salt	65.5	67.3	0.53	0.973	0.61

ϵ'' : dielectric loss factor; ϵ' : dielectric constant; d_p : penetration depth; P_r : Power reflected.

Reference: J.G. Lyng et al. / Meat Science 69 (2005) 589–602



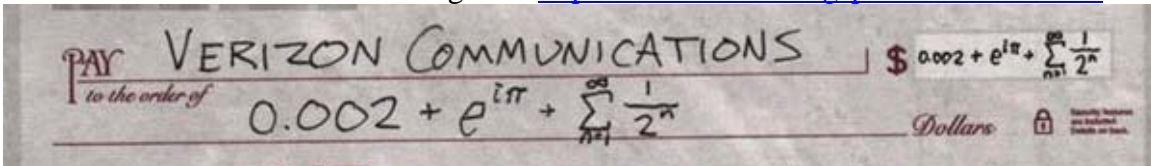
The dielectric constant and the loss factor are the usual material properties we have seen in this course. The penetration depth is equivalent to the skin depth and the reflected power indicates the fraction of the power reflected for a uniform plane wave incident normally on the surface of the liquid solution. The formulas used by the authors are

Reflected Power: $P_r = \left(\frac{\sqrt{\epsilon'} - 1}{\sqrt{\epsilon'} + 1} \right)^2$ Transmitted Power: $P_t = 1 - P_r$

Loss Tangent: $\tan \delta = \frac{\epsilon''}{\epsilon'}$ Penetration Depth: $d_p = \frac{\lambda\sqrt{2}}{2\pi} \left\{ \epsilon' \left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'} \right)^2} - 1 \right] \right\}^{-\frac{1}{2}}$

Note that the properties of two of the foodstuffs are underlined: Wheat Gluten and Nitrite. For one of these, the formula used by the authors for reflected power is in error. Which one is incorrect and what is the correct answer? Their other formulas are correct.

1. (1 Point) In what is now getting to be a somewhat well known story on the internet, an engineer (or physicist, depending on the story) sent the following check to Verizon when they found that no one they talked to at Verizon could do the math to explain the roaming rate in Canada. There is even a blog on it <http://verizonmath.blogspot.com/index.html>



The question here is what is the value of this check?



m. Vocabulary (1 Point) Draw lines connecting the word or phrase on the left with its definition on the right. The definitions are essentially word for word from Wikipedia.

- Dissipation factor
- Faraday's Law
- Continuity equation
- Magnetic flux
- Reluctance
- Charge density
- Inductance
- Divergence
- Propagation constant
- Phase velocity

- The power loss rate for a wave propagating in some material.
- The magnetic analog of electrical resistance.
- The logarithmic rate of change, with respect to distance in a given direction, of the complex amplitude of any field component.
- The speed at which the phase of any one frequency component of the wave travels.
- A measure of quantity of magnetism, taking into account the strength and the extent of a magnetic field.
- The amount of electric charge in a line, surface, or volume.
- An empirical law expressing (local) charge conservation
- An operator that measures the magnitude of a vector field's source or sink at a given point
- The ratio of the magnetic flux to the current.
- The induced electromotive force or EMF in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit.

Note: this question was added to the final to remind you that you should be able to access a lot of good information through online sources like Wikipedia now that you have some basic understanding of electromagnetic theory. This is an exceptionally good way to expand one's knowledge base, as long as one remains reasonably skeptical about the accuracy of the information. As a general rule, most information found online is wrong, but the truth is relatively easy to identify if a few sources are considered.



2. Electric Fields (3 Points)

A uniform volume charge distribution with $\rho_v = \rho_o$ exists in the spherical region $r < R$. There are no conductors or other materials involved in this problem.

a) Using the integral form of Gauss' Law, determine the electric field of this charge distribution in the region $r > R$ and in the region $r < R$.



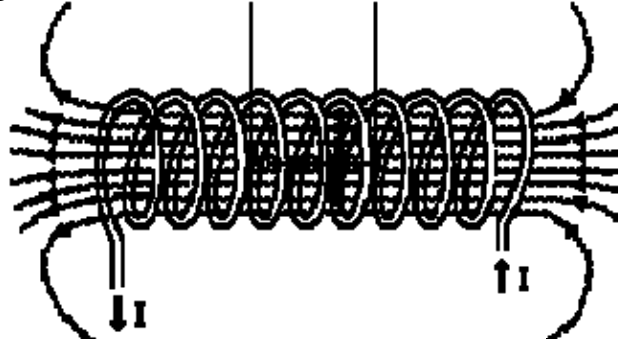
b) Determine the electric scalar potential at $r = a$, $V(a)$. Assume that the voltage goes to zero as $r \rightarrow \infty$.

c) Determine the electric field energy stored in the region $r > R$ or in the region $r < R$ (your choice).



3. Magnetic Fields (3 points)

A simple solenoid is constructed by winding $N=100$ turns of wire around a hollow cylindrical tube. For simplicity, assume that the tube is a toilet paper tube with a radius of $a = 2\text{cm}$ and a length of $d=12\text{cm}$.



a. Making the assumption of no fringing effects (that is, treating the solenoid as a piece of an infinitely long cylindrical solenoid), determine the magnetic flux density \mathbf{B} everywhere in space. For now, leave your answer in terms of a , d , and N . Do not plug in the numbers.

b. What is the magnetic energy stored in the field of this inductor? Again, leave your answer in terms of the parameters a , d , and N .



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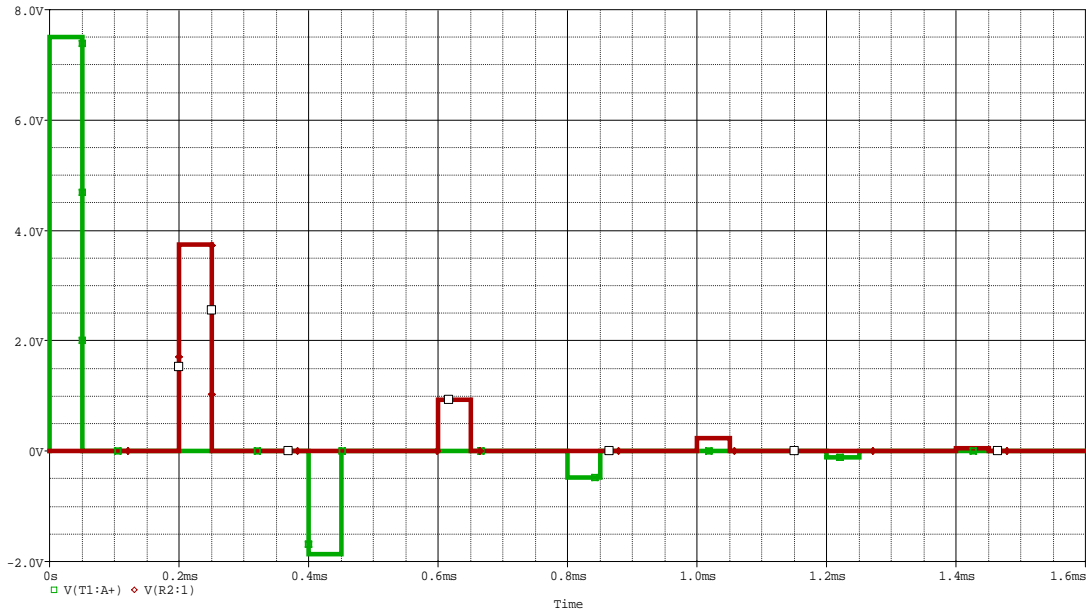
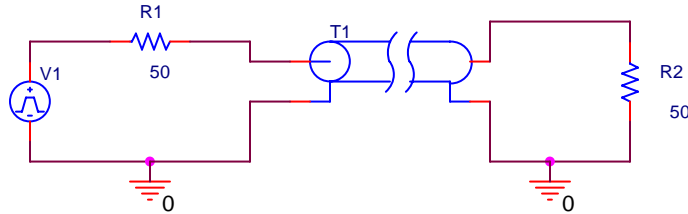
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c. Determine the inductance of this solenoid. First, express your answer in terms of the parameters and then plug in the given numbers.



4. Transmission Lines (3 Points)

A lossless transmission line is driven by a pulsed voltage source and connected to a resistive load, as shown. The source produces a 50 microsecond pulse. The voltages at the input and output end of the transmission line are shown on the same plot. Neither the characteristic impedance Z_o nor the insulating material used in the cable are known. However, we do know that the length of the line is 50 kilometers. (Ideal conditions are assumed.)



a. First, identify the input and output voltages in the diagram above. Then, determine the time it takes for the pulse to propagate from the input to the output of the transmission line. From this information, determine the relative permittivity ϵ_r of the insulating material used in the cable.



b. Determine the characteristic impedance Z_o of the transmission line. To determine this parameter, you must first generate the bounce or reflection diagram for this transmission line and then use the voltage data given above. Draw the bounce diagram on the back of the preceding page or on the next page.

c. Now that you have determined the characteristic impedance of this line, determine its input impedance if it was driven by a sinusoidal source with a frequency of 25kHz. Begin by first determining the propagation constant β . Then find Z_{in} .



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4. Capacitance and Laplace's Equation (3 Points)

a. The two-dimensional parallel plate structure shown below consists of two electrodes with conducting fins aligned with the z-direction (into the paper). The region in the lower half of the diagram is filled with an insulator. Two cases are shown, one in which $\epsilon_r = 20$ and one in which $\epsilon_r = 1$. First, identify, which case is which.

Excel spreadsheet showing a grid of numerical data values for various cases. The grid consists of columns labeled A17 to AE and rows numbered 1 to 32. The values range from -10 to 10, with many cells containing -10.

Note that the two electrodes are at voltages of +10V and -10V.

Second Excel spreadsheet showing a grid of numerical data values, similar to the first table. The grid consists of columns labeled A17 to AE and rows numbered 35 to 66. The values range from -10 to 10, with many cells containing -10.



b. Next, use the information given in the plots to estimate the charge per unit length for this structure (consider both cases). Remember that this is a two-dimensional configuration, so that the dimension of the surfaces into the page should be considered to be 1meter. You should also assume a cell spacing of 1mm.

c. Now, determine the capacitance per unit length for this structure (both cases).



Material Properties (from Ulaby Appendices)

Dielectric Materials

Material	ϵ_r	Material	ϵ_r
Vacuum	1	Dry Soil	2.5-3.5
Air (sea level)	1.006	Plexiglas	3.4
Styrofoam	1.03	Glass	4.5-10
Teflon	2.1	Fused Quartz	3.8-5
Petroleum Oil	2.1	Bakelite	5
Wood (Dry)	1.5-4	Porcelain	5.7
Paraffin	2.2	Formica	6
Polyethylene	2.25	Mica	5.4-6
Polystyrene	2.6	Ammonia	22
Paper	2-4	Sea Water	72-80
Rubber	2.2=4.1	Distilled Water	81
Most Metals	1	All are low frequency values at room temperature (20°C)	

Conductors

Material	$\sigma S/m$	Material	$\sigma S/m$
<i>Conductors</i>		<i>Semiconductors</i>	
Silver	6.2×10^7	Pure Germanium	2.2
Copper	5.8×10^7	Pure Silicon	4.4×10^{-4}
Gold	4.2×10^7	<i>Insulators</i>	
Aluminum	3.5×10^7	Wet Soil	10^{-2}
Tungsten	1.8×10^7	Fresh Water	10^{-3}
Zinc	1.7×10^7	Distilled Water	10^{-4}
Brass	1.5×10^7	Dry Soil	10^{-4}
Iron	1×10^7	Glass	10^{-12}
Bronze	1×10^7	Hard Rubber	10^{-15}
Tin	0.9×10^7	Paraffin	10^{-15}
Lead	0.5×10^7	Mica	10^{-15}
Mercury	0.1×10^7	Fused Quartz	10^{-17}
Carbon	3×10^4	Glass	10^{-17}
Sea Water	4		
Animal Body	0.3	All are low frequency values at room temperature (20°C)	



Magnetic Materials

Material	μ_r	Material	μ_r
<i>Diamagnetic</i>		Titanium	1.0002 \approx 1
Bismuth	0.99983 \approx 1	Platinum	1.0003 \approx 1
Gold	0.99996 \approx 1	<i>Ferromagnetic</i>	<i>Nonlinear</i>
Mercury	0.99997 \approx 1	Cobalt	250
Silver	0.99998 \approx 1	Nickel	600
Copper	0.99999 \approx 1	Mild Steel	2000
Water	0.99999 \approx 1	Iron (Pure)	4000-5000
<i>Paramagnetic</i>		Silicon Iron	7000
Air	1.000004 \approx 1	Mu Metal	\sim 100,000
Aluminum	1.00002 \approx 1	Purified Iron	\sim 200,000
Tungsten	1.00008 \approx 1	Only typical values, actual values depend on material variety	