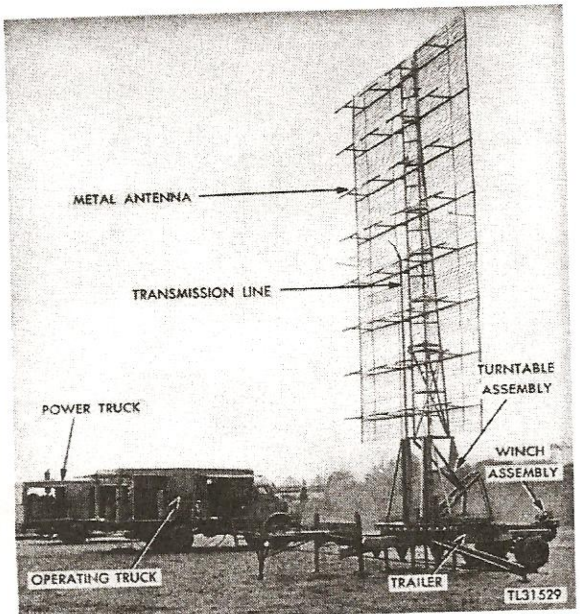
Rensselaer | EE EPE CSE

Department of Electrical, Computer, & Systems Engineering



SCR-270: Similar to the model that detected the attacking Pearl Harbor planes (the actual Opana antenna was nine dipoles high by four wide, instead of the eight-by-four configuration shown here). The scale for reading the direction the antenna is pointing to can be seen at the base. The radar operated at 106 MHz, using a pulse width from 10 to 25 microseconds, and a pulse repetition frequency of 621 Hz. It is now located at the Historical Electronics Museum near Baltimore.

Quiz 3
Fall 2009

Name Solution

Section __________

1. (18 Pts) __________

2. (12 Pts) __________

3. (5 Pts) __________

4. (20 Pts) __________

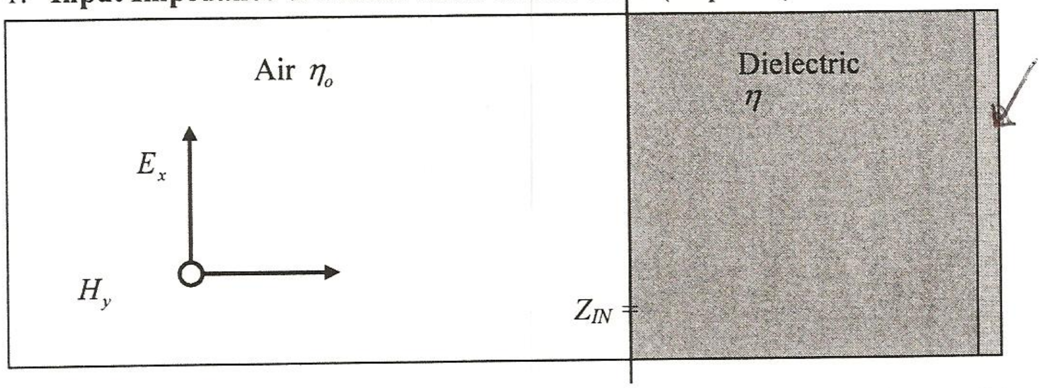
5. (25 Pts) __________

6. (20 Pts) __________

Total __________

Notes:
1. **Please read over all questions before you begin your work. There may be some information in a later question that helps you with an earlier question.**
2. For short answer questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use 4 significant digits.
4. Problem 1 is an almost exact analog to a problem from Quiz 1 and problem 2 is a little different, but largely inspired by another problem from Quiz 1. Thus, it will be worthwhile to think about the transmission line equivalent to these problems when you address them.

1. **Input Impedance of Planar Slabs of Materials** (18 points)



Conductor is a short

Assume a sinusoidal electromagnetic wave is incident normally from air on a slab of dielectric material followed by a perfectly conducting sheet.

a. (6 pts) For what slab thickness will the input impedance observed at the left side of the slab ($z = 0$) be infinite? Circle all correct answers. Odd mult of $\lambda/4$

0 $\lambda/8$ [✓ $\lambda/4$] $3\lambda/8$ $\lambda/2$ $5\lambda/8$ [✓ $3\lambda/4$] $7\lambda/8$ λ $9\lambda/8$ [✓ $5\lambda/4$] $11\lambda/8$ $3\lambda/2$

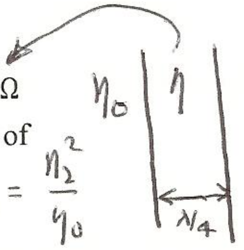
For what slab thickness will the input impedance observed at the left side of the slab ($z = 0$) be zero? Circle all correct answers. Mult of $\lambda/2$ & 0

[✓ 0] $\lambda/8$ $\lambda/4$ $3\lambda/8$ [✓ $\lambda/2$] $5\lambda/8$ $3\lambda/4$ $7\lambda/8$ [✓ λ] $9\lambda/8$ $5\lambda/4$ $11\lambda/8$ [✓ $3\lambda/2$]

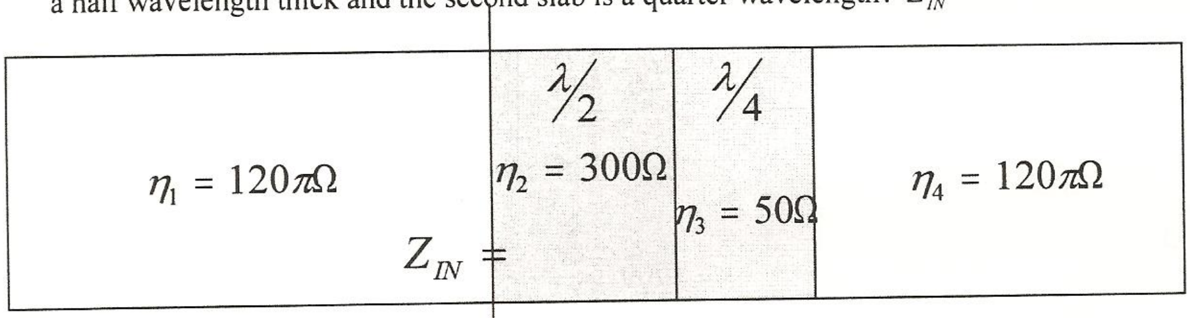
b. (6 pts) Assume the slab has a thickness equal to $\lambda/4$, intrinsic impedance $\eta = 100\Omega$ and the conducting sheet is removed so that the region beyond the right boundary of the slab is free space. What is the input impedance?

$Z_{IN} = \frac{\eta_2^2}{\eta_0} = \frac{100^2}{120\pi} = 26.5\ \Omega$

$Z_{in} = \frac{\eta_2^2}{\eta_0}$

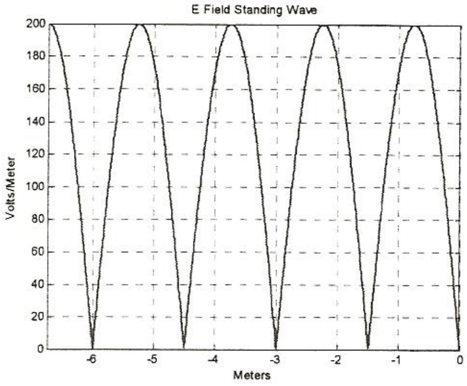


c. (6 pts) What is the input impedance of the configuration below, where the first slab is a half wavelength thick and the second slab is a quarter wavelength? Z_{IN} =

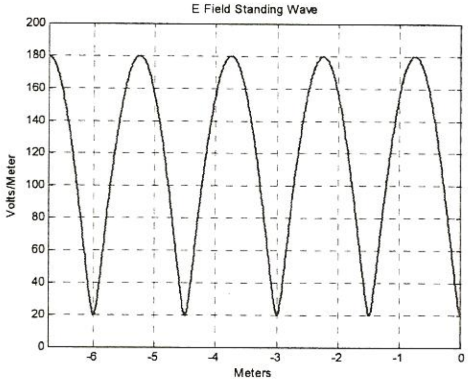


$Z_{in} = \frac{50^2}{120\pi} = 6.63\Omega$ Z_{in} same $\leftarrow$ $Z_{in} = \frac{50^2}{120\pi}$

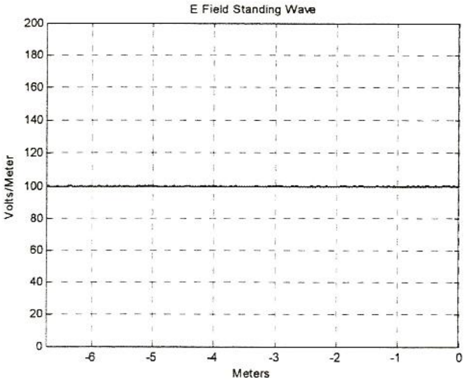
2. Standing Waves (12 points)



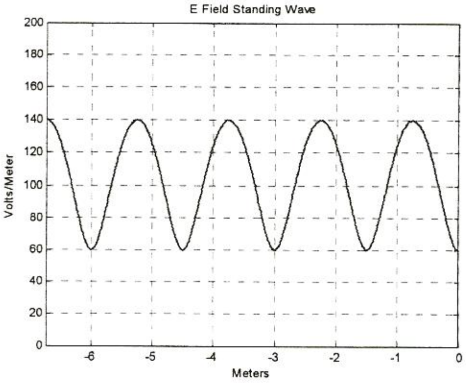
A



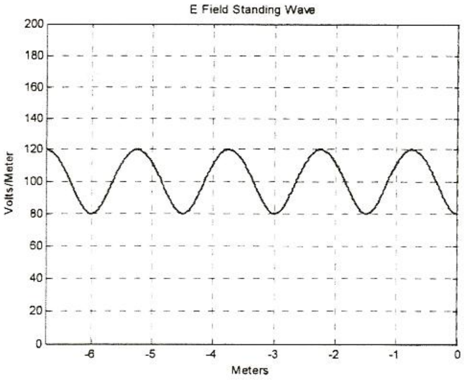
B



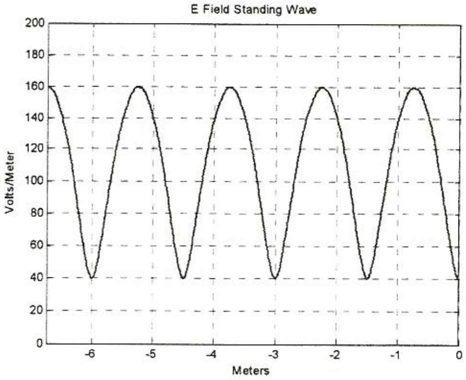
C



D



E



F

Each figure above shows the E-field standing wave pattern observed in free space as a wave reflects off of an unknown material, which can be either a dielectric or a conductor.

Fill out the table at the right, indicating the parameters listed for each case. If the second region is a conductor, just write 'cond' in the cell for ε_{r2} rather than a number. The configuration under study here is shown on the next page.

	A	B	C	D	E	F
ε_{r2}	cond	81	1	5.44	2.25	16
Γ	−1	−.8	0	−.4	−.2	−.6
SWR	∞	9	1	14/6	1.5	4

Calc for Prob 2

$$\Gamma = \frac{1-\sqrt{\varepsilon_{r2}}}{1+\sqrt{\varepsilon_{r2}}}$$

$$\Gamma\left(1+\sqrt{\varepsilon_{r2}}\right) = 1-\sqrt{\varepsilon_{r2}}$$

$$-\Gamma+1 = \sqrt{\varepsilon_{r2}}\left(1+\Gamma\right)$$

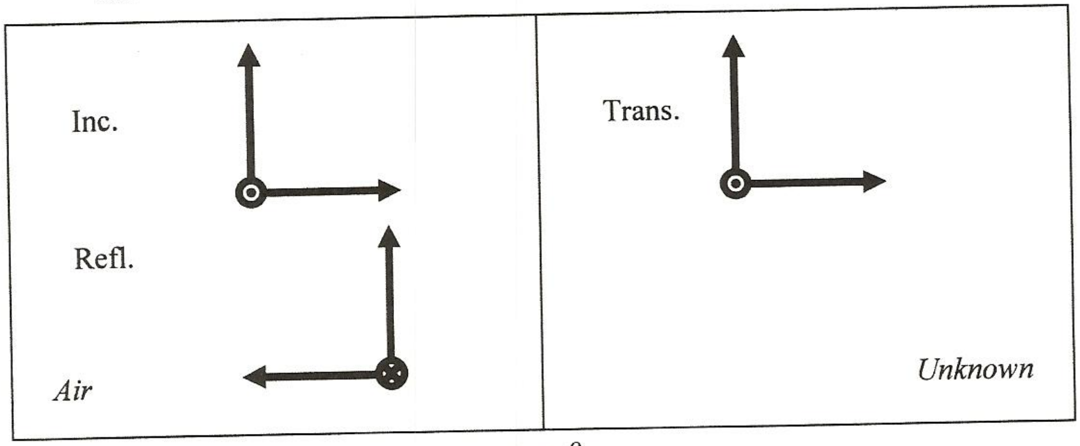
$$\sqrt{\varepsilon_{r2}} = \frac{1-\Gamma}{1+\Gamma}$$

$$\varepsilon_{r2} = \left(\frac{1-\Gamma}{1+\Gamma}\right)^2 = \left(\frac{1.2}{.8}\right)^2 = 2.25$$

$$\varepsilon_{r2} = \left(\frac{1.4}{.6}\right)^2 = 5.44$$

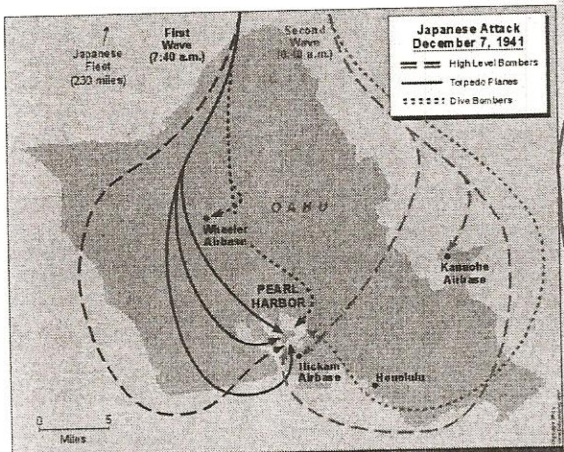
$$\varepsilon_{r2} = \left(\frac{1.6}{.4}\right)^2 = 16$$

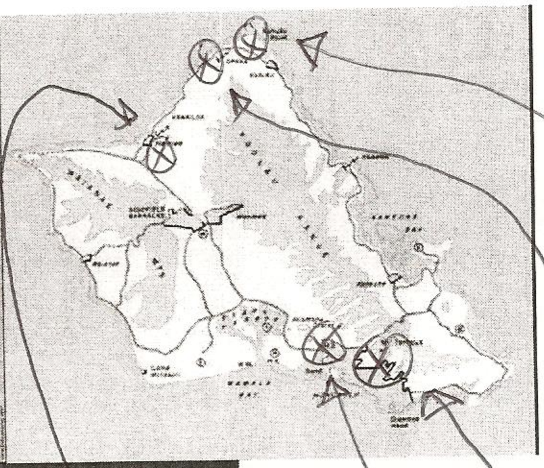
$$\varepsilon_{r2} = \left(\frac{1.8}{.2}\right)^2 = 81$$

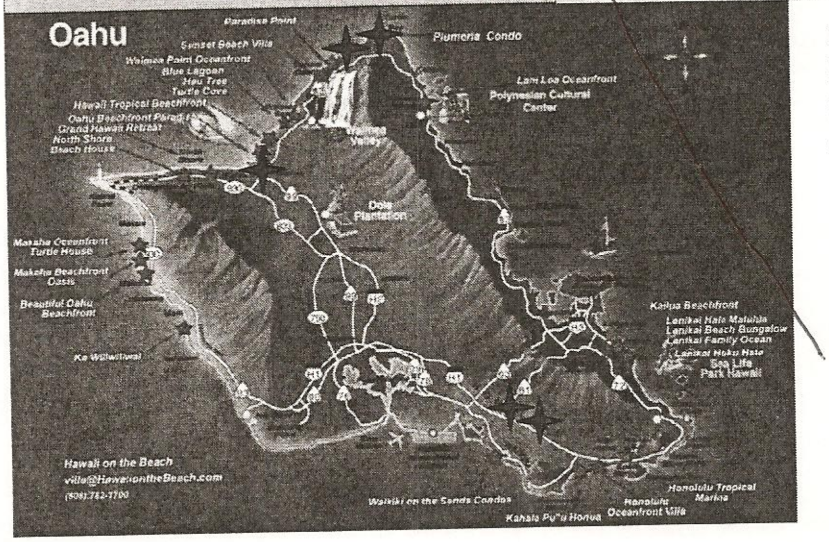


z = 0

3. Radar & WWII in Hawaii (5 points)







Identify the five landmarks in Oahu indicated by large stars at the left.

- EE Dept, U of Hawaii
- Opana Radar
- Turtle Pt Resort
- Matsumoto Shave Ice
- Punchbowl Nat'l Cemetery

4. **Plane Waves in Lossy Media** (20 points)

$\frac{\sigma}{2\pi f \varepsilon} = 1$
$\Rightarrow f = \frac{\sigma}{2\pi\varepsilon}$
$= 8.8\times10^{8}$
$\sim 1\text{GHz}$

Throughout a wide range of frequencies, the permittivity of sea water is approximately $\varepsilon = \varepsilon_r\varepsilon_o = 81\varepsilon_o$. Its conductivity is given in one of the tables at the end of this quiz.

a. For which of the following frequencies will sea water be a good conductor?

Frequency: [✓ *1Hz 100Hz 10kHz 1MHz 100MHz*] *10GHz 1THz*

b. For which of the following frequencies will sea water be a low loss dielectric?

Frequency: 1Hz 100Hz 10kHz 1MHz 100MHz [✓ *10GHz 1THz*]

c. Determine the values of the parameters α, β, and η for a frequency of *100Hz.* Good Conductor

$$\alpha = \beta \simeq \sqrt{\pi f \sigma \mu} = \sqrt{\pi(100)(4)(4\pi\times10^{-7})} \simeq .04\ \text{m}^{-1}$$

$$\eta \simeq (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\sqrt{\frac{\pi\ 100\ 4\pi\times10^{-7}}{4}} = (1+j)(.01)\ \Omega$$

d. Determine the values of the parameters α, β, and η for a frequency of *10GHz.* Low-loss dielectric

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = 83.8\ \text{m}^{-1}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{81}} = 41.9\ \Omega$$

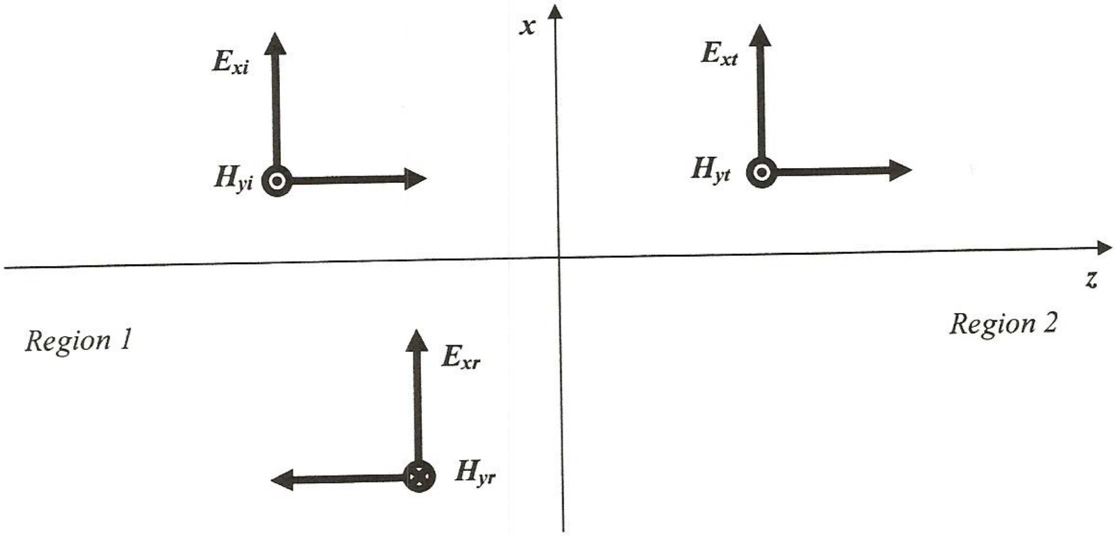
$$\beta \simeq \omega\sqrt{\mu\varepsilon} = \frac{2\pi10^{10}\sqrt{81}}{3\times10^{8}} = 1885\ \text{m}^{-1}$$

e. At which of the two frequencies ([✓ *10Hz*] and *10GHz*), will the field penetrate further into the medium?

$\delta = \frac{1}{\alpha}$ so small $\alpha \to$ greater penetration

5. Plane Waves at Normal Incidence (25 points)

A uniform plane wave is incident normally on the boundary between two lossless media. The incident medium (region 1) is Bakelite and the transmitted medium (region 2) is air.



The magnitude of the incident electric field is *1000V/m* and its frequency $f = 3GHz$.

a. Determine the wave parameters ω, β, η, u, and λ for the two regions.(5 pts)

$\omega = 2\pi f = 2\pi\, 3\times10^9 = 1.88\times10^{10}$

$\beta_1 = \frac{\omega}{u_1} = 140\ m^{-1}$

$\eta_1 = \eta_0/\sqrt{5} = 169\,\Omega$

$u_1 = \frac{c}{\sqrt{5}} = 1.34\times10^8\ m/s$

$\lambda_1 = 2\pi/\beta_1 = 0.045m$

$\beta_2 = \frac{\omega}{c} = 62.7\ m^{-1}$

$\eta_2 = \eta_0 = 120\pi$

$u_2 = 3\times10^8$

$\lambda_2 = \frac{c}{f} = 0.1\ m$

b. Determine the reflection and transmission coefficients Γ and τ. (4 pts)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = .38$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 1.38$$

c. Write the incident electric and magnetic fields in phasor form. (4 pts)

$E_o = 1000\ \text{V/m}$

$\beta_1 = 140\ \text{m}^{-1}$

$\eta_1 = 169\ \Omega$

$$\vec{E}_i = \hat{x}\, E_o\, e^{-j\beta_1 z}$$

$$\vec{H}_i = \hat{y}\, \underbrace{\frac{E_o}{\eta_1}}_{= 5.92}\, e^{-j\beta_1 z}$$

d. Write the reflected electric and magnetic fields in phasor form. (4 pts)

$\Gamma = .38$

$$\vec{E}_r = \hat{x}\, \underbrace{E_o \Gamma}_{380}\, e^{+j\beta_1 z}$$

$$\vec{H}_r = -\hat{y}\, \frac{E_o}{\underbrace{\eta_1}_{2.25}}\, \Gamma\, e^{+j\beta_1 z}$$

e. Write the transmitted electric and magnetic fields in phasor form. (4 pts)

$\beta_2 = 62.7$

$\eta_2 = \eta_0 = 120\pi$

$$\vec{E}_t = \hat{x}\, \underbrace{E_o \tau}_{1380}\, e^{-j\beta_2 z}$$

$$\vec{H}_t = \hat{y}\, \frac{E_o \tau}{\underbrace{\eta_2}_{3.66}}\, e^{-j\beta_2 z}$$

f. Determine the incident, reflected and transmitted average power densities. (4 pts)

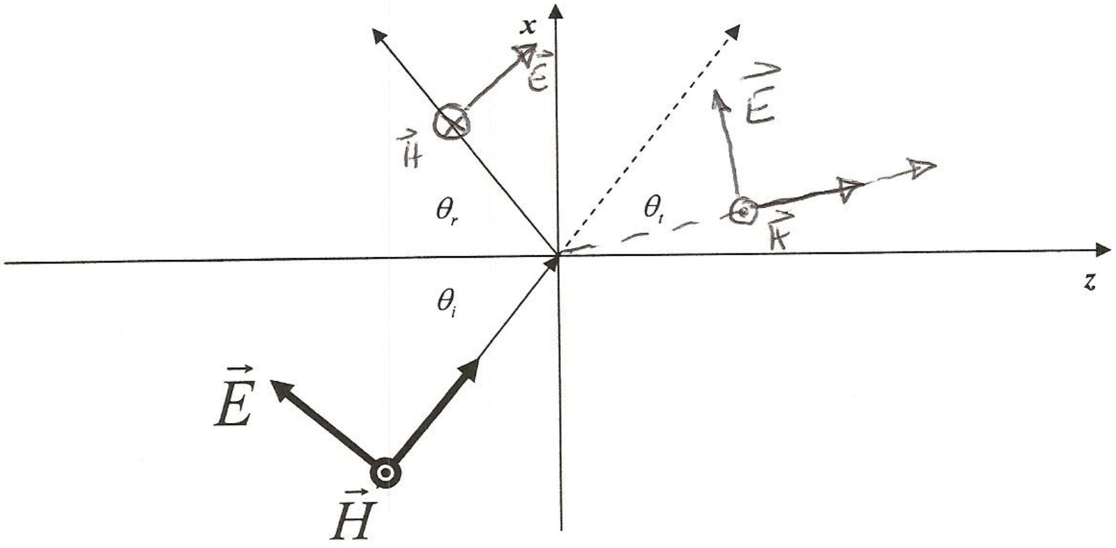
$$S_{av\,i} = \frac{1}{2}\frac{E_o^2}{\eta_1} = \frac{1}{2} E_o H_i = \frac{1}{2}(1000)(5.92) = 2960\ \frac{\text{W}}{\text{m}^2}$$

$$S_{av\,r} = \frac{1}{2}(380)(2.25) = 427.5\ \frac{\text{W}}{\text{m}^2}$$

$$S_{av\,t} = \frac{1}{2}(1380)(3.66) = 2526\ \frac{\text{W}}{\text{m}^2}$$

6. **Plane Waves at Oblique Incidence** (20 points)

A uniform plane wave is incident obliquely in air on Bakelite, as shown below.



a. What is the polarization of this wave? (2 pts)

Parallel because $\vec{E}$ is in plane of incidence

b. In this diagram, the transmitted angle is shown as equal to the incident angle. However, we know that this cannot be the case for a wave incident on Bakelite. Will the transmitted angle be greater $\theta_t > \theta_i$ or less than $\boxed{\theta_t < \theta_i}$ the incident angle? (4 pts)

Angle always smaller in region with larger index

c. The three vector representation is shown for the incident wave. Add the corresponding representation for the reflected and transmitted waves to the diagram. Be sure that you use the information in this diagram when you answer the following questions. (2 pts)

Any choice is good if used consistently.

d. Assume an incident angle $\theta_i = 60^o$. What are the angles of reflection θ_r and transmission θ_t? (4 pts) *Answer in degrees.*

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1}{\sqrt{\varepsilon_r}}\sin\theta_1 = \frac{1}{\sqrt{5}}\sin 60^\circ$$

$$\theta_2 = 22.8^\circ$$

e. Determine the reflection coefficient and transmission coefficient. (4 pts)

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = -.096$$

$$\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = .49$$

checking

$$\tau_2 = (1+\Gamma_{\parallel})\frac{\cos\theta_i}{\cos\theta_t} = .49 \checkmark$$

f. Determine the vector form of the average Poynting vector in the Bakelite region for an incident wave magnitude of E_o = *100V/m.* (4 pts)

$$\vec{E}_t = \hat{a}_{E_t}\tau_{\parallel}E_o e^{-j\bar{k}_2\cdot\bar{r}}$$

$$\vec{H}_t = \hat{a}_{H_t}\frac{\tau_{\parallel}E_o}{\eta_2}e^{-j\bar{k}_2\cdot\bar{r}}$$

$E_o = 100$

$E_t = 49$

$\eta_2 = 1686$

$H_t = .29$

$$\hat{a}_{H_t} = \hat{y} \qquad \hat{a}_{E_t} = \hat{x}\cos\theta_t - \hat{z}\sin\theta_t$$

$$\vec{k}_2 = k_2(\hat{x}\sin\theta_t + \hat{z}\cos\theta_t)$$

$$S_{av\,t} = \frac{1}{2}E_t H_t^* = 7.13\text{ W}$$

$$\vec{S}_{av} = 2.76\hat{x} + 6.57\hat{z}$$

$$S_{av\,i} = \frac{1}{2}\frac{E_o^2}{\eta_o} = 13.3\text{ W}$$

$$S_{av\,r} = .122\text{ W}$$

Checking

$S_{av\,i}A_i = 13.3A\cos\theta_i = 6.63$

$S_{av\,t}A_t = 7.3A\cos\theta_t = 6.57$

so trans power is close to incident power. ✓