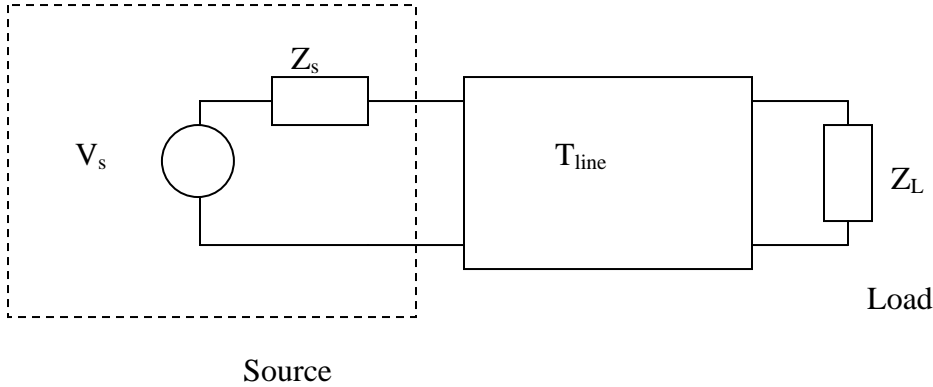




### Some General Comments on Lossless Transmission Lines

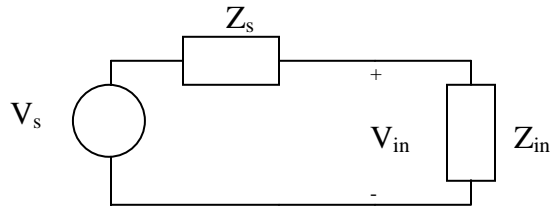


For the transmission line, we need  $f, d, u, Z_o, \beta, \omega, \lambda, l$  and  $c$  per unit length, and possibly the delay time  $t_D$  (time it takes for a wave or pulse to travel from the input to the output end of the line).

For Steady-State Conditions (single frequency)

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} = Z_o \frac{Z_L + jZ_o \tan \alpha t_D}{Z_o + jZ_L \tan \alpha t_D}$$

Power in from the input voltage  $V_{in} = V_s \frac{Z_{in}}{Z_s + Z_{in}}$



Power out equals power in for a lossless line

$V_{out}$  can be found from the output power and the load impedance.

Special cases for loads include matched load, open circuit load, short circuit load.

Special cases for line length include short line (much shorter than a wavelength), lengths that are odd multiples of a quarter wavelength, lengths that are multiples of a half wavelength.

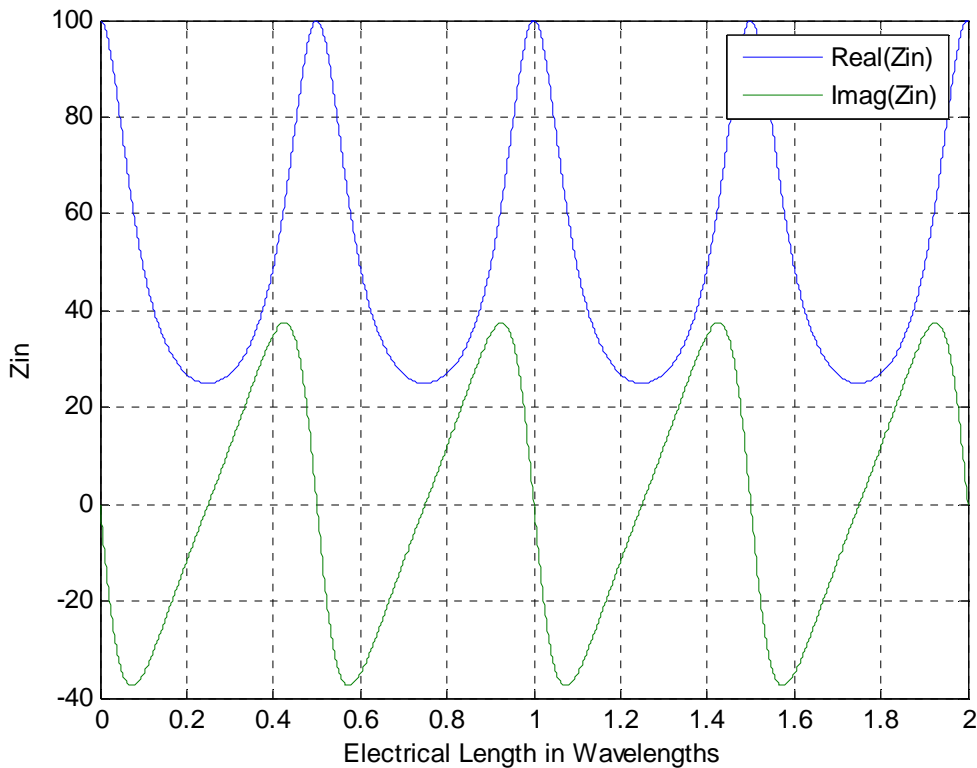
The current for the positive traveling wave is found by dividing the voltage by  $Z_o$ . The current for the negative traveling wave is found by dividing the voltage by  $-Z_o$ .

The general forms of the voltage and current waves are:

$$v(z) = V^+ e^{-j\beta z} + \Gamma_L V^+ e^{+j\beta z} \qquad i(z) = \frac{V^+}{Z_o} e^{-j\beta z} - \frac{\Gamma_L V^+}{Z_o} e^{+j\beta z}$$



The input impedance has significant real and imaginary components, except at particular electrical lengths where the impedance is real if the load is real. For the case where the line impedance is  $50\ \text{Ohms}$  and the load impedance is  $100\ \text{Ohms}$ , the input impedance takes the following form (as determined and plotted in Matlab):



Note that the input impedance is real and  $25\ \text{Ohms}$  at  $\lambda/4, 3\lambda/4, 5\lambda/4, 7\lambda/4$  and real and equal to  $100\ \text{Ohms}$  at  $0, 0.5\lambda, \lambda, 1.5\lambda, 2\lambda$ . Check the formula to see that this is indeed correct.

The Matlab file that generated the plot above is quite simple:

```
% Input Impedance for Lossless Transmission Line
```

```
% Line Parameters
```

```
Zo=50;Zl=100;u=2e8;d=20;
```

```
% Wave Properties
```

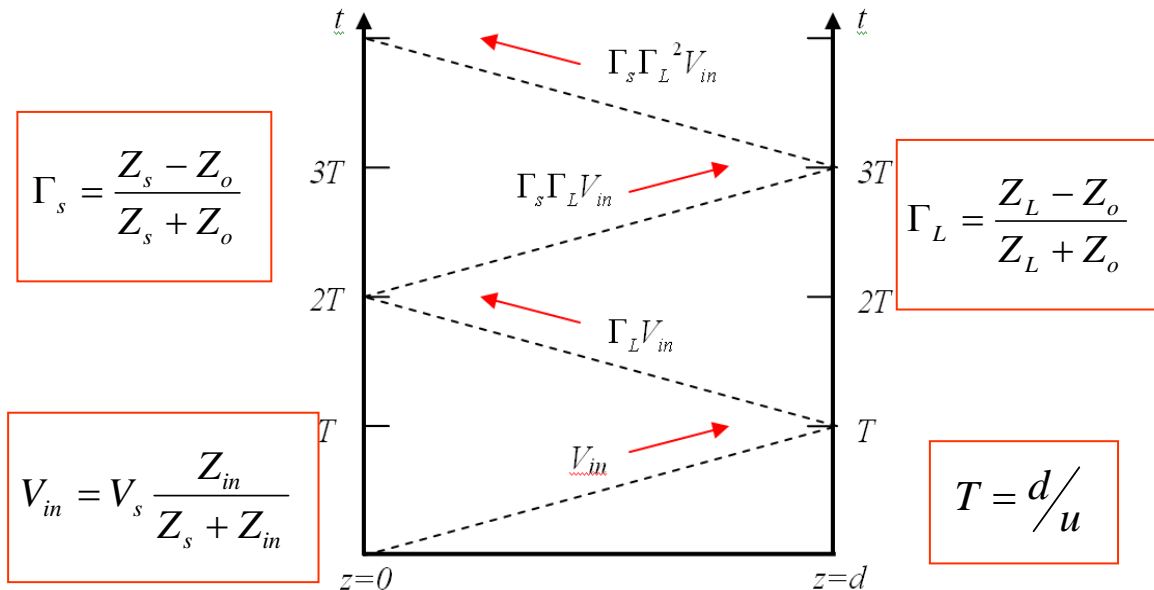
```
f=[0:.01:20].*1e6; w=2.*pi.*f; B=w./u;  
Zin=Zo.*(Zl+j.*Zo.*tan(B.*d))./(Zo+j.*Zl.*tan(B.*d));
```

```
% Plotting Results
```

```
plot(f,real(Zin),f,imag(Zin));grid;xlabel('Frequency'),ylabel('Zin');legend('Real(Zin)','Imag(Zin)');  
figure;plot(B.*d./(2.*pi),real(Zin),B.*d./(2.*pi),imag(Zin));grid;xlabel('Electrical Length in Wavelengths'),ylabel('Zin');legend('Real(Zin)','Imag(Zin)');
```



For pulses on lossless transmission lines, one uses the bounce or lattice diagram to fully characterize the response at either end or at some point in the middle of the line.



Special cases for loads again include matched, open circuit and short circuit. For this case, however, the same apply to the source end, since the source impedance is seen as a load for pulses going the negative z direction.

For line lengths, one can have a very short line (the output and input appear simultaneously), lines that have delay times much larger than the pulse length and lines that have delay times that are much less than the pulse length.

For positive traveling pulses, the current pulse is found by dividing by  $Z_o$ . For negative traveling pulses, the current is found by dividing by  $-Z_o$ .

The power in a pulse is found from the product of the voltage times the current.

The general forms of the voltage and current pulses are:

$$V(z,t) = V^+ \left( t - \frac{z}{u} \right) + V^- \left( t + \frac{z}{u} \right) \qquad I(z,t) = \frac{V^+}{Z_o} \left( t - \frac{z}{u} \right) - \frac{V^-}{Z_o} \left( t + \frac{z}{u} \right)$$