Fields and Waves

Lesson 1.4

VECTOR CALCULUS - Surface Integrals and Divergence

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SURFACE INTEGRALS

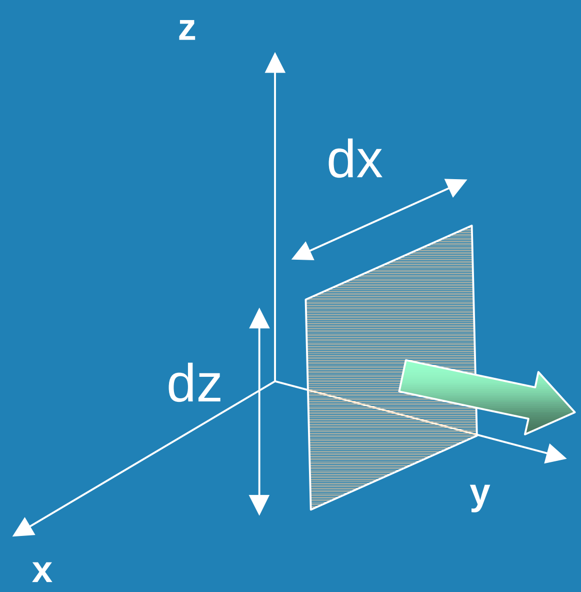
Surface Integrals look like: $\int \vec{A} \bullet d\vec{s}$

Differs from Lesson 1.2 area integrals because of “dot” product

$d\vec{s}$,is a vector and points normal to the surface

- the magnitude is the differential area formed from differential lengths of the sides

SURFACE INTEGRALS



Note that <u>all 3 coordinates</u> are involved

$$d\vec{s} = dx \cdot dz \cdot \hat{a}_y$$

$\int \vec{A} \bullet d\vec{s}$, measures flux of $\vec{A}$, through a surface

SURFACE INTEGRALS

Example - FLUID FLOW

For, $\vec{v} \,//\, d\vec{s}$, there is flow through

But, $\vec{v} \perp d\vec{s}$, there is no flow

Hence, $\vec{v} \bullet d\vec{s}$, measures FLUX

Example: Let y=2, x=0 to 3 and z = -1 to 1

$\vec{A} = xy \cdot \hat{a}_x + z^2 \cdot \hat{a}_y$, then, $\int \vec{A} \bullet d\vec{s} = \int_{-1}^{+1}\int_0^3 z^2 \cdot dx \cdot dz = 3 \cdot \left. \frac{z^3}{3} \right|_{-1}^{+1} = 2$

Do Problem 1

DIVERGENCE

"Global" quantities

$\int \vec{A} \bullet d\vec{s}$ Measures Flux through any surface

$\oint \vec{A} \bullet d\vec{s}$ Measures Flux through closed surfaces

is related to

$\nabla \bullet \vec{A}$, is a "local" measure of flux property

DIVERGENCE

Notation: $div\,\vec{A} = \nabla \bullet \vec{A}$ *NOT a DOT product but has similar features*

Result is a SCALAR, composed of derivatives like:

$$\frac{\partial A_r}{\partial r}, \frac{\partial A_\phi}{\partial \phi}$$

Divergence Theorem:

$$\oint \vec{A} \bullet d\vec{s} = \int \left(\nabla \bullet \vec{A}\right) \cdot dv$$

Volume integral on right is volume enclosed by surface on the left