Fields and Waves

Lesson 2.3

ELECTROSTATICS - POTENTIALS

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MAXWELL'S SECOND EQUATION

Lesson 2.2 looked at Maxwell's 1st equation:

$$\nabla \bullet \vec{D} = \rho$$

$$\oint \vec{D} \bullet d\vec{s} = \int \rho \cdot dv$$

Today, we will use Maxwell's 2nd equation:

$$\nabla \times \vec{E} = 0$$

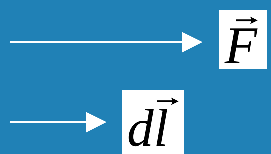
$$or$$

$$\oint \vec{E} \bullet d\vec{l} = 0$$

Importance of this equation is that it allows the use of *Voltage* or *Electric Potential*

POTENTIAL ENERGY

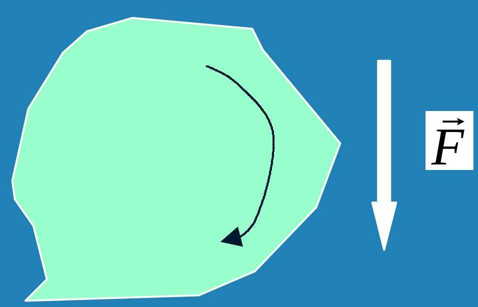
Work done by a force is given by: $\int \vec{F} \bullet d\vec{l}$



If vectors are parallel, particle <u>gains</u> energy - Kinetic Energy

If, $\oint \vec{F} \bullet d\vec{l} = 0$ ⇒ Conservative Force

Example : GRAVITY



- going DOWN increases KE, decreases PE
- going UP increases PE, decreases KE

POTENTIAL ENERGY

If dealing with a conservative force, can use concept of POTENTIAL ENERGY

For gravity, the potential energy has the form **mgz**

Define the following integral:

$$-\int_{P_1}^{P_2} \vec{F} \bullet d\vec{l} = \text{Potential Energy Change}$$

POTENTIAL ENERGY

Since $\oint \vec{E} \bullet d\vec{l} = 0$ and $\vec{F} = q \cdot \vec{E}$

We can define:

Potential Energy = $-\int_{P_1}^{P_2} q \cdot \vec{E} \bullet d\vec{l}$

Also define: Voltage = Potential Energy/Charge

$$V(P_2) - V(P_1) = -\int_{P_1}^{P_2} \vec{E} \bullet d\vec{l}$$

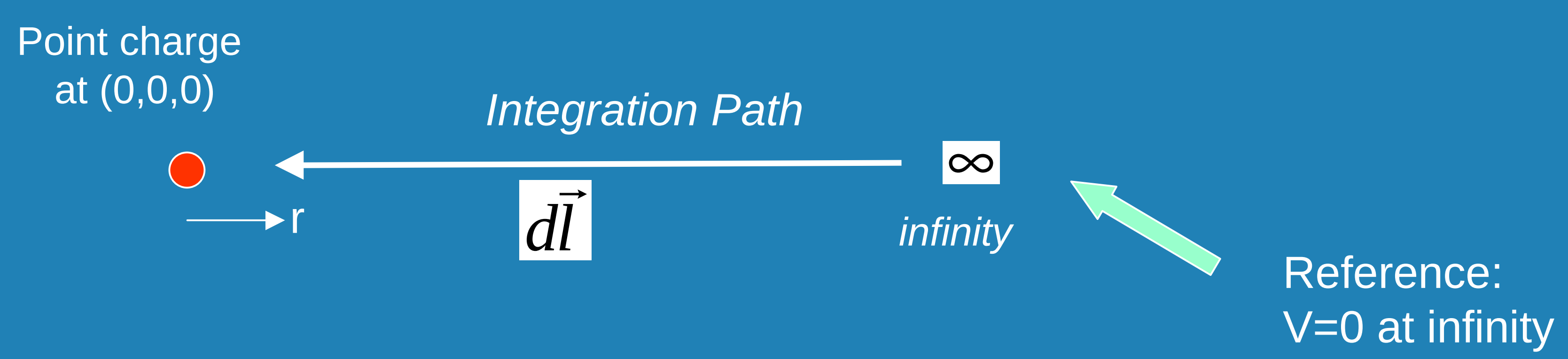
Voltage always needs reference or use voltage difference

POTENTIAL ENERGY

Example: Use case of point charge at origin and obtain potential everywhere from E-field

Spherical Geometry

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \cdot \hat{a}_r$$



POTENTIAL ENERGY

The integral for computing the potential of the point charge is:

$$V(r) - V(r = \infty) = -\int_{r=\infty}^{r} \vec{E} \bullet d\vec{l}$$

0

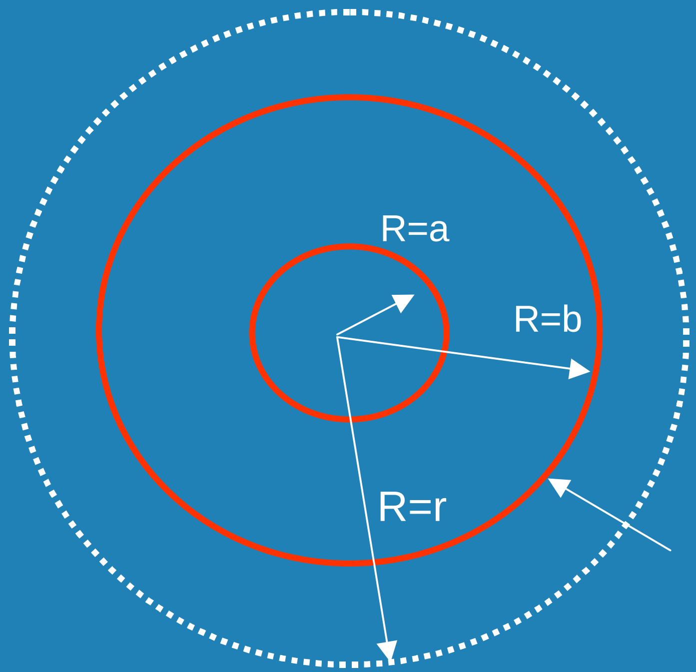
$$\therefore V(r) = -\int_{r=\infty}^{r} E \cdot dr$$

$$= -\int_{r=\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2} \cdot dr \quad \Rightarrow \quad V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

POTENTIAL ENERGY - problems

Do Problem 1a

Hint for 1a:



Use r=b as the reference - Start here and move away or inside r<b region

POTENTIAL ENERGY - problems

For conservative fields:

$$\oint \vec{E} \bullet d\vec{l} = 0$$,which implies that:

$$\oint \nabla \times \vec{E} \bullet d\vec{s} = 0$$, for any surface

$$\therefore \nabla \times \vec{E} = 0$$

From vector calculus:

$$\nabla \times \nabla f = 0$$,for any field *f* ⇨ Can write: $$\vec{E} = \nabla f$$

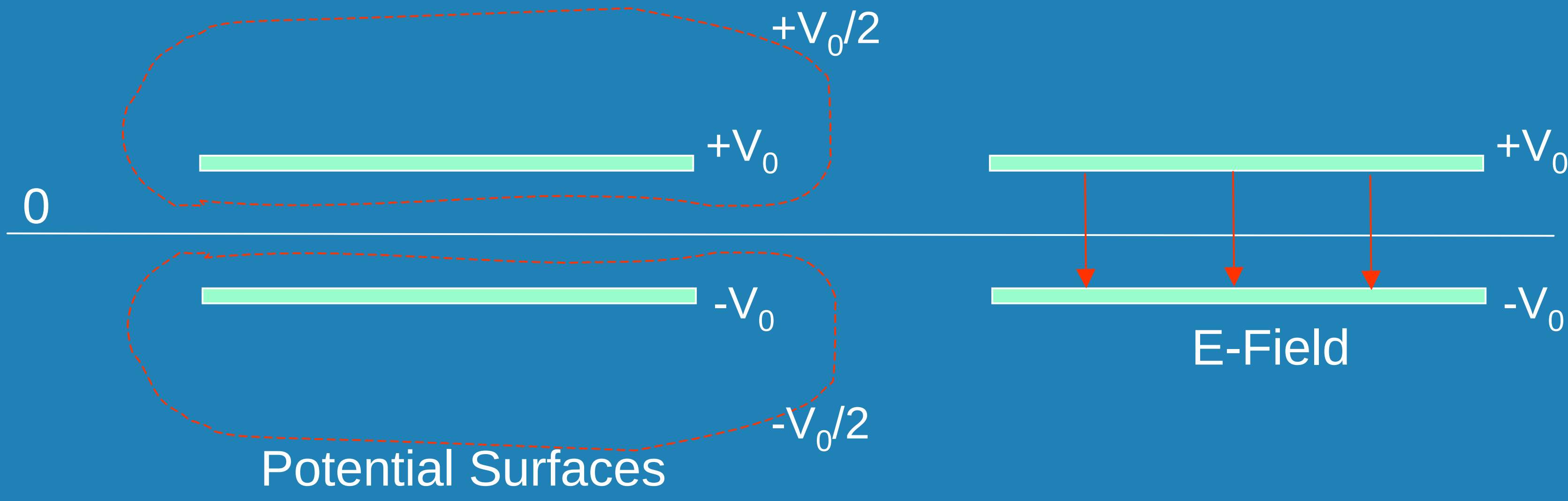
Define: $$\vec{E} = -\nabla V$$

POTENTIAL SURFACES

Potential is a SCALAR quantity

Graphs are done as Surface Plots or Contour Plots

Example - Parallel Plate Capacitor



E-field from Potential Surfaces

From:

$$\vec{E} = -\nabla V$$

Gradient points in the direction of largest change

Therefore, E-field lines are perpendicular (normal) to constant V surfaces

(add E-lines to potential plot)

Do problem 2

Numerical Simulation of Potential

In previous lesson 2.2, problem 3 and today in problem 1,

Given ρ or Q ⟹ (derive) E-field ⟹ (derive) V

Look for techniques so that , given ρ or Q ⟹ (derive) V

Numerical Simulation of Potential

For the case of a point charge:

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}'\right|} = V(\vec{r})$$

Distance from charge

$\vec{r}$, is field point where we are measuring/calculating V

$\vec{r}'$, is location of charge

Numerical Simulation of Potential

For smooth charge distribution:

$$V(\vec{r}) = \iiint \frac{\rho(\vec{r}\,')\cdot dv'}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|}$$

Volume charge distribution

$$V(\vec{r}) = \int \frac{\rho(\vec{r})\cdot d\vec{l}}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|}$$

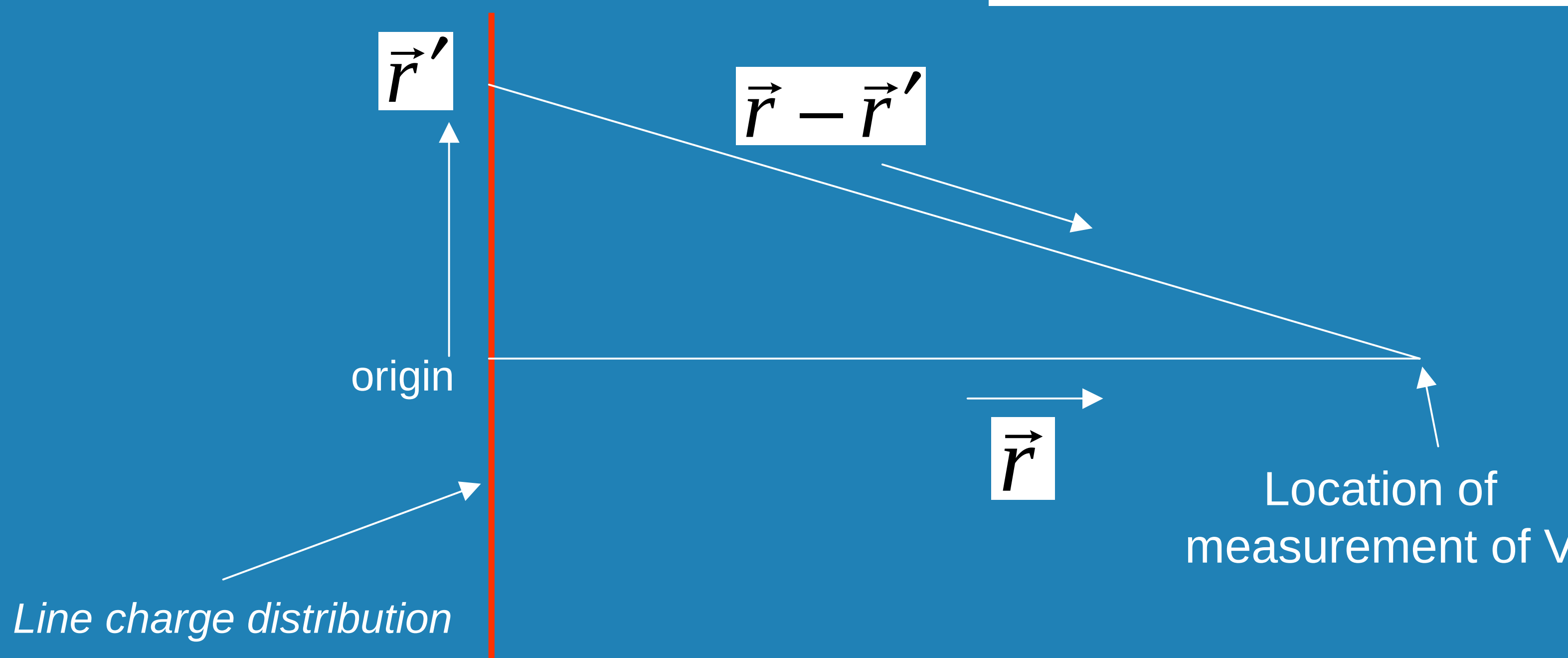
Line charge distribution

Numerical Simulation of Potential
Problem 3

Setup for Problem 3a and 3b

$$V(\vec{r}) = \int \frac{\rho(\vec{r}) \cdot d\vec{l}}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r}\,'\right|}$$

Line charge:

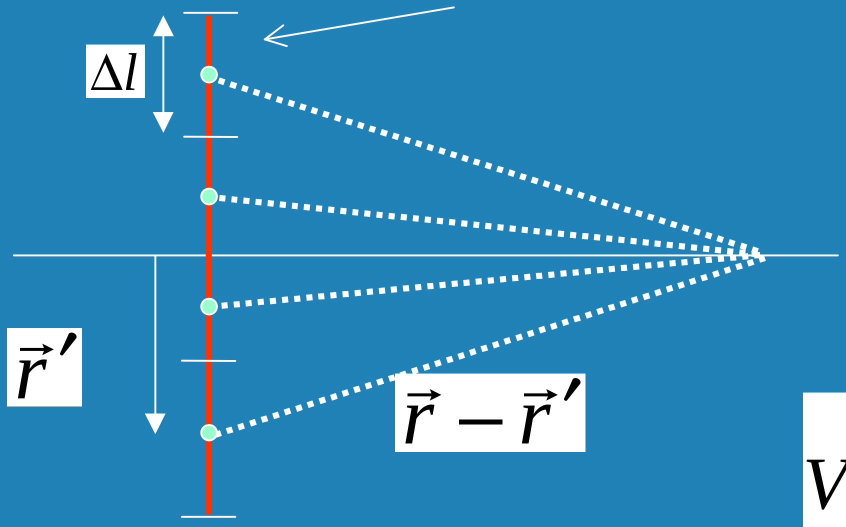


Integrate along charge means *dl* is *dz*

Numerical Simulation of Potential
Problem 3 contd...

Numerical Approximation

Break line charge into 4 segments



Charge for each segment

$$q = \rho_l \cdot \Delta l$$

Segment length

$$V = \sum_{4charges} \frac{q_i}{4\pi\varepsilon_0 \left| \vec{r} - \vec{r}_i \right|}$$

Distance to charge

Numerical Simulation of Potential
Problem 3 contd...

For Part e….

Get V(r = 0.1) and V(r = 0.11)

Use: $$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\hat{a}_r \approx -\frac{\Delta V}{\Delta r}\hat{a}_r$$

So..use 2 points to get ΔV and Δr

- <u>*V* is a SCALAR field</u> and easier to work with
- In many cases, easiest way to get E-field is to first find *V* and then use, $\vec{E} = -\nabla V$