Fields and Waves

Lesson 2.5

ELECTROSTATICS - CAPACITANCE

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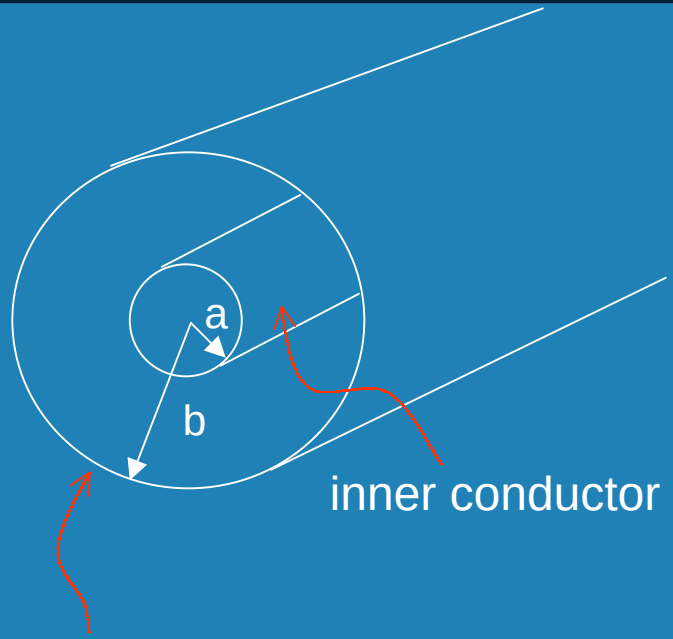
CAPACITANCE of Coaxial Cable



In previous class, for coaxial cable:

$$V_{ab} = \frac{\rho_{sa} \cdot a}{\varepsilon} \cdot \ln\frac{b}{a}$$

Note:

$$V_{ab} \propto \rho_{sa}$$

$$V \propto Q$$

very general result

Define:

$$C = \frac{Q}{V}$$

Q: charge on 1 conductor

V: ΔV between conductors

Note that:

$$\frac{d}{dt}(Q = CV) \Rightarrow I = C\frac{dV}{dt}$$

Calculation of CAPACITANCE

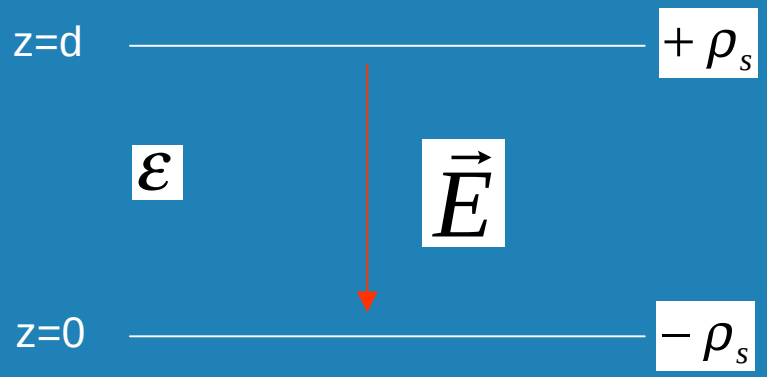
Problems on calculation of C

1. Method - Assume ρ_s (use symmetry)
   - Find Q
   - Find V(ρ_s)

   $$C = \frac{Q}{V}$$

2. Alternate method - Assume V and find Q

CAPACITANCE - parallel plate capacitor



Use Gauss' Law,

$$\vec{E} = -\frac{\rho_s}{\varepsilon} \cdot \hat{a}_z$$

$$V_{top} - V_{bottom} = -\int_0^d \vec{E} \bullet d\vec{l} = \int_0^d \frac{\rho_s}{\varepsilon} \cdot dz$$

$$= \frac{\rho_s \cdot d}{\varepsilon}$$

$$\therefore C = \frac{Q}{V} = \frac{\rho_s \cdot A}{\rho_s \frac{d}{\varepsilon}} = \varepsilon \cdot \frac{A}{d}$$

C of Parallel Plate capacitor

CAPACITANCE - parallel plate capacitor

$$C = \varepsilon \cdot \frac{A}{d}$$

Parallel Plate Capacitance

To get large C →

- increase A
- increase ε
- decrease d

→ This is how electrolytics increase C

Do problem 1a or 2a & 2b

CAPACITANCE - ENERGY METHOD

- energy stored in capacitors is stored in the E-field

Define stored energy: $$W_e = \frac{1}{2} \cdot CV^2$$

Substitute values of C and V for parallel plate capacitor:

$$W_e = \frac{1}{2} \cdot CV^2 = \frac{1}{2} \cdot \left(\varepsilon \frac{A}{d} \right) \cdot \left(\left| \vec{E} \right| \cdot d \right)^2 = \frac{1}{2} \cdot \varepsilon \left| \vec{E} \right|^2 \cdot Ad$$

Energy Density *Volume*

CAPACITANCE - ENERGY METHOD

In general we can write the total stored energy as:

$$W_e = \int \frac{1}{2} \cdot \varepsilon \cdot \left|\vec{E}\right|^2 \cdot dv$$

or

$$W_e = \int \frac{1}{2} \vec{D} \bullet \vec{E} dv$$

CAPACITANCE - ENERGY METHOD

Use the Energy Formulation to compute C for the Parallel Plate Capacitor

We know that, $\vec{E} = -\frac{V_0}{d} \cdot \hat{a}_z$

Compute TOTAL ENERGY:

$$W_e = \frac{1}{2} \cdot \int \varepsilon \cdot \left(-\frac{V_0}{d} \right)^2 \cdot dv = \frac{1}{2} \cdot \varepsilon \cdot \left(\frac{V_0}{d} \right)^2 \cdot Ad$$

$$= \frac{1}{2} \cdot \varepsilon \frac{A}{d} \cdot V_0^2$$

$$C = \varepsilon \frac{A}{d}$$

CAPACITANCE

Do rest of Problems 1 or 2

Any 2 conductors have capacitance

Example:
- lines on circuit board
- Theremin
- wires and cables

- problems 2 and 3 model lines on a circuit board

Do problem 3