# Fields and Waves

## Lesson 2.6

## ELECTROSTATICS - Numerical Simulation

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# Direct Computation of *V*

If we can express entire problem in terms of V then:

- we can solve directly for V
- derive all other quantities e.g. E-field, D-field, C and ρ

This approach can be used if conductor defines Outer Boundary

- can be SYMMETRIC or NON-SYMMETRIC systems

Why is this a useful approach??

- V is a scalar field - easier to manipulate than E-field
- We can control V on conductors
- Can apply numerical methods to solve problem

Use of Laplace and Poisson's Equations

Start with 2 of MAXWELL's equations:

$$\nabla \bullet \vec{D} = \rho \qquad \nabla \times \vec{E} = 0 \qquad \& \qquad \vec{D} = \varepsilon \cdot \vec{E}$$

$$\nabla \bullet \varepsilon \cdot \vec{E} = \rho \qquad \vec{E} = -\nabla V$$

$$\nabla \bullet \varepsilon \cdot \vec{E} = \nabla \bullet \varepsilon \cdot (-\nabla V) = -\varepsilon \cdot \nabla \bullet \nabla V$$

$$= \nabla^2 V$$

In rectangular coordinates:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

# Use of Laplace and Poisson's Equations

Poisson's equation:

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

Laplace's equation: (when ρ = 0)

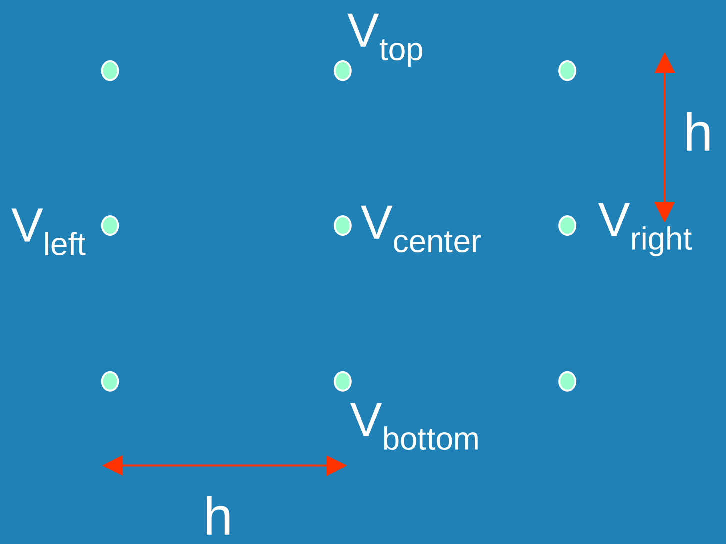
$$\nabla^2 V = 0$$

*Do Problem 1*

# Numerical Solution: Finite Difference Method

Use the FINITE DIFFERENCE Technique for solving problems

Solve for approximate *V* on the Grid - for 2-D Problem

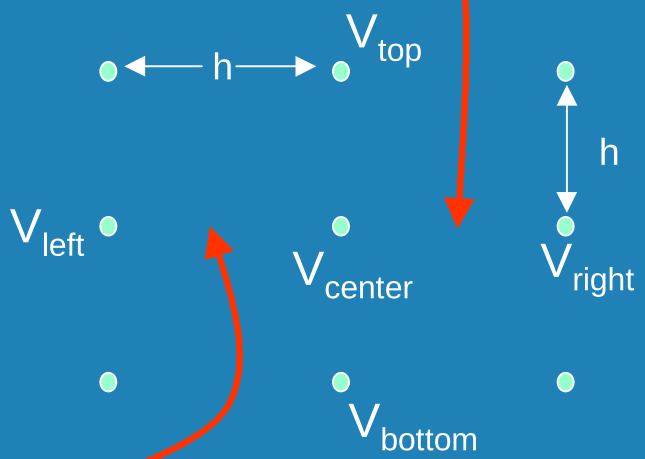



$V_{center}$ at (x,y) = (0,0)

# Numerical Solution: Finite Difference Method

At (x,y) = (h/2,0)

$$E_x = -\frac{\partial V}{\partial x} \approx -\frac{\Delta V}{\Delta x} = -\frac{(V_{right} - V_{center})}{h}$$




At (x,y) = (-h/2,0)

$$E_x = -\frac{\partial V}{\partial x} \approx -\frac{\Delta V}{\Delta x} = -\frac{(V_{center} - V_{left})}{h}$$

# Numerical Solution: Finite Difference Method

$$\nabla^2 V = \nabla \bullet \nabla V = -\nabla \bullet \vec{E} = -\frac{\partial E_x}{\partial x} - \frac{\partial E_y}{\partial y} - \frac{\partial E_z}{\partial z}$$

0

Now,

$$\frac{\partial E_x}{\partial x} \approx \frac{\Delta E_x}{\Delta x} = \frac{E_x\left(\frac{h}{2},0\right) - E_x\left(-\frac{h}{2},0\right)}{h} = \frac{2 \cdot V_{center} - V_{right} - V_{left}}{h^2}$$

Can get similar expression for $\frac{\partial E_y}{\partial y}$

## Numerical Solution: Finite Difference Method

Finally we obtain the following expression:

$$\nabla^2 V = \frac{V_{right} + V_{left} + V_{top} + V_{bottom} - 4 \cdot V_{center}}{h^2} = -\frac{\rho}{\varepsilon}$$

Rearrange the equation to solve for $V_{center}$ :

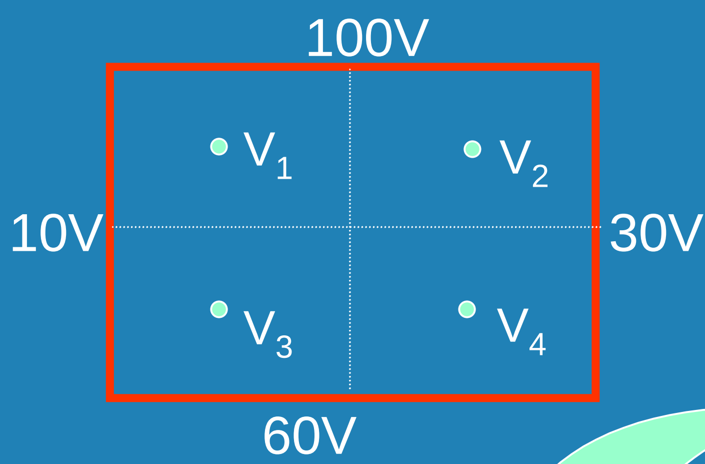
$$V_{center} = \frac{1}{4} \cdot \left( \sum V_{neighbors} + \frac{\rho \cdot h^2}{\varepsilon} \right)$$

⇐ *Poisson Equation Solver*

$$V_{center} = \frac{1}{4} \cdot \left( \sum V_{neighbors} \right)$$

⇐ *Laplace Equation Solver*

# Numerical Solution: Example




Solution Technique - by Iteration

Guess a solution : V=0 everywhere except boundaries

$V_1 = V_2 = V_3 = V_4 = 0$

Put new values back

*Start:*

$$V_1 = \tfrac{1}{4}\cdot(100+10+V_2+V_3) \rightarrow V_1 = \tfrac{1}{4}\cdot(110+0) = 27.5$$

$$V_2 = \tfrac{1}{4}\cdot(100+30+V_1+V_4) \rightarrow V_2 = \tfrac{1}{4}\cdot(130+27.5+0) = 39.375$$

$$V_3 = \tfrac{1}{4}\cdot(10+60+V_1+V_4) \rightarrow V_3 = \tfrac{1}{4}\cdot(70+27.5+0) = 24.375$$

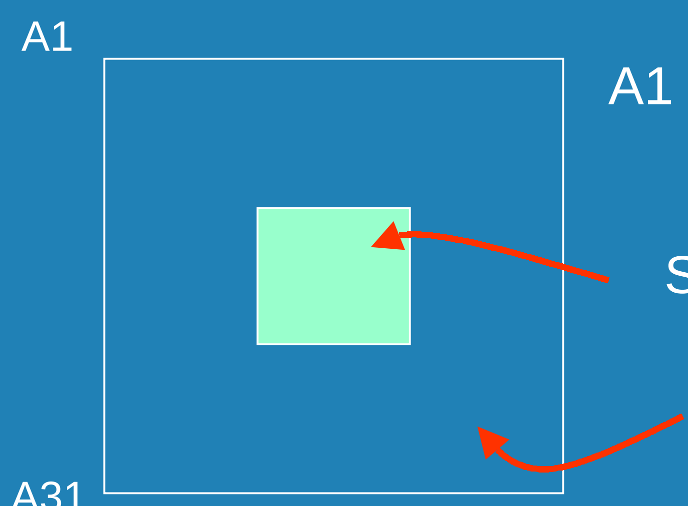
$$V_4 = \tfrac{1}{4}\cdot(30+60+V_2+V_3) \rightarrow V_4 = \tfrac{1}{4}\cdot(90+39.375+24.375) = 38.4375$$

## Numerical Solution - use of EXCEL Spreadsheet

*Do Problem 2*

- To get an accurate solution, need lots of points - one way is to use a SPREADSHEET

In spreadsheet, $B2 = \frac{1}{4} \cdot \left( B1 + B3 + A2 + C2 \right)$




A1 to A31 set boundary voltage = 0Volts

Set these cells to 100

Copy B2 formula to rest of cells

# Numerical Solution: Problems

*Do Problem 3a and 3b*

*Helpful Hints for Problems 3c - 3e*

3c. At point P, what is $\rho_s$ ?

Get $\rho_s$ from Boundary Conditions: $D_n = \varepsilon \cdot E_n = \rho_s$

Approximate $E_n \approx -\frac{\Delta V}{\Delta x}$

3d. Use spreadsheet to add columns: $Q = \sum \rho_s \cdot small\_Area$

$Q = \langle \rho_s \rangle \cdot Full\_Area$

3e. Use C=Q/V