Fields and Waves

Lesson 3.1

MAGNETOSTATICS

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Current

We will be looking at

- sources of magnetic fields ie. currents
- description of resistance

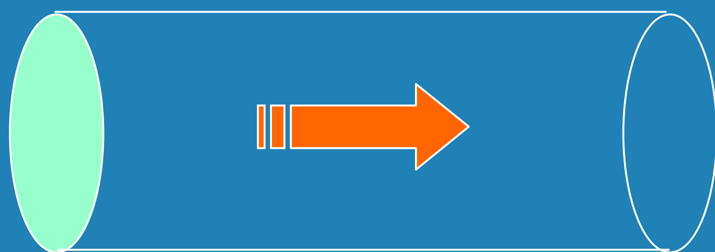
We will consider two parameters:

I - Current (in Amps)

J - Current Density (in Amps/unit area)

Current & Current Density

Wire with current, I



Definition: $$I = \frac{\Delta Q}{\Delta t}$$

Charge passing through cross-section in time Δt

Define: $$\vec{j} = \frac{I}{Area} \cdot \hat{a}_z$$

cross-section

points in direction of current flow

In general,

$$I = \int \vec{j} \bullet d\vec{s}$$

Current & Current Density

$$I = \int \vec{j} \bullet d\vec{s}$$

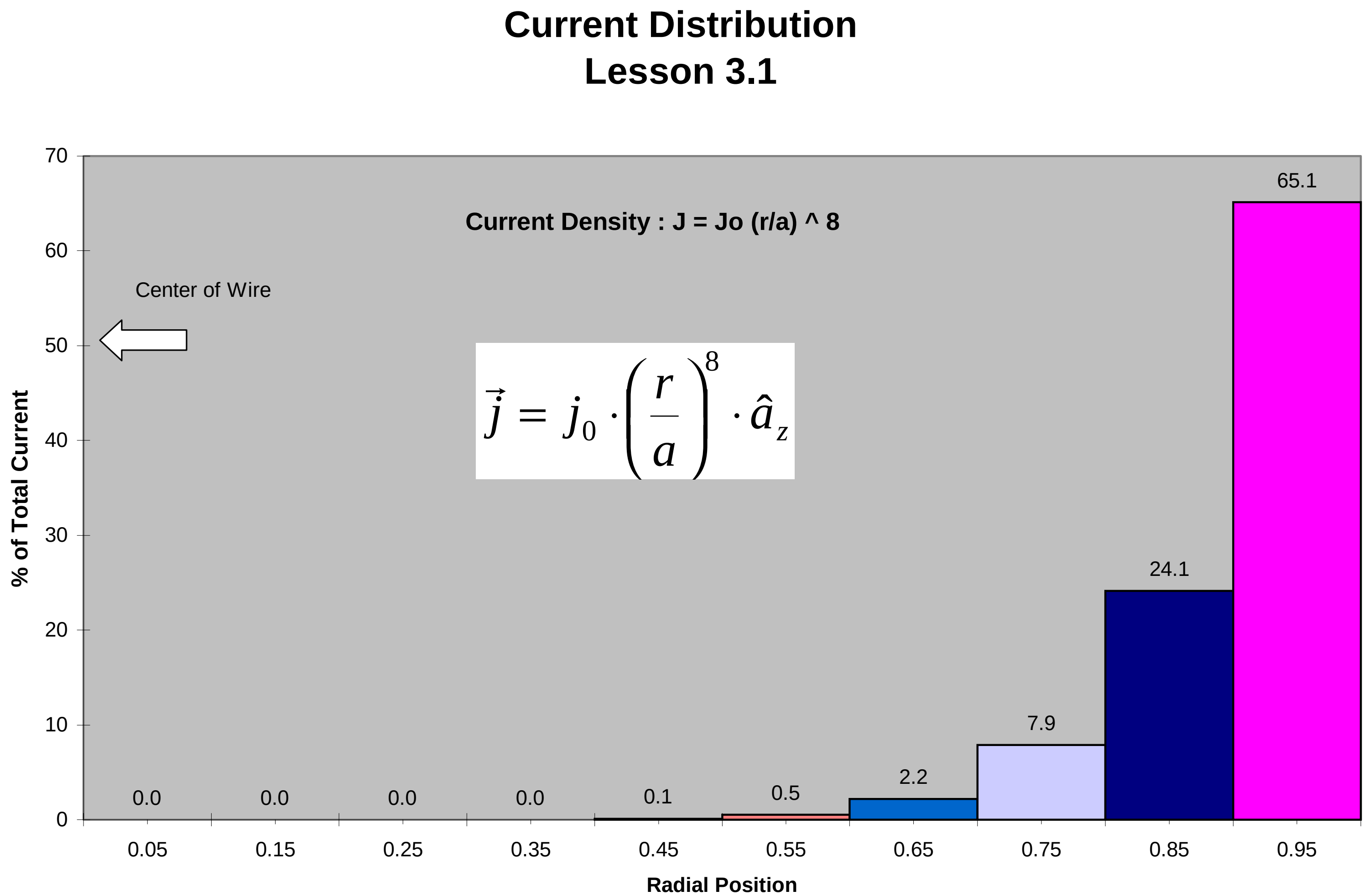
Example: wire with constant current density $\vec{j} = j_0 \cdot \hat{a}_z$

$$\Rightarrow \quad I = \int_0^{2\pi}\int_0^{a} j_0 \cdot \hat{a}_z \bullet r \cdot dr \cdot d\varphi \cdot \hat{a}_z = j_0 \cdot \pi \cdot a^2$$

$d\vec{s}$, in cylindrical geometry

Problem 1a - Find I in terms of J_0 and invert $\rightarrow$ $\vec{j} = j_0 \cdot \left(\frac{r}{a}\right)^8 \cdot \hat{a}_z$

Problem 1:Current & Current Density

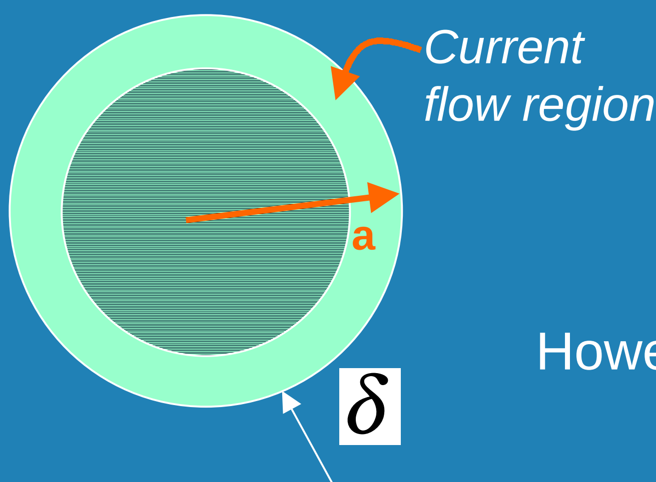


Surface Current Density

Surface Current Density J_s (in units of A/m)

- conductors often have current flowing in thin sheet (eg. High Freq.)

Example: current flow in a wire



$$j = \frac{I}{Area} \cong \frac{I}{2 \cdot \pi \cdot a \cdot \delta}$$

But, $\delta \xrightarrow{\lim it} 0 \Rightarrow j \rightarrow \infty$

However, define:

$$j_s = \frac{I}{Area/\delta} = \frac{I}{distance}$$

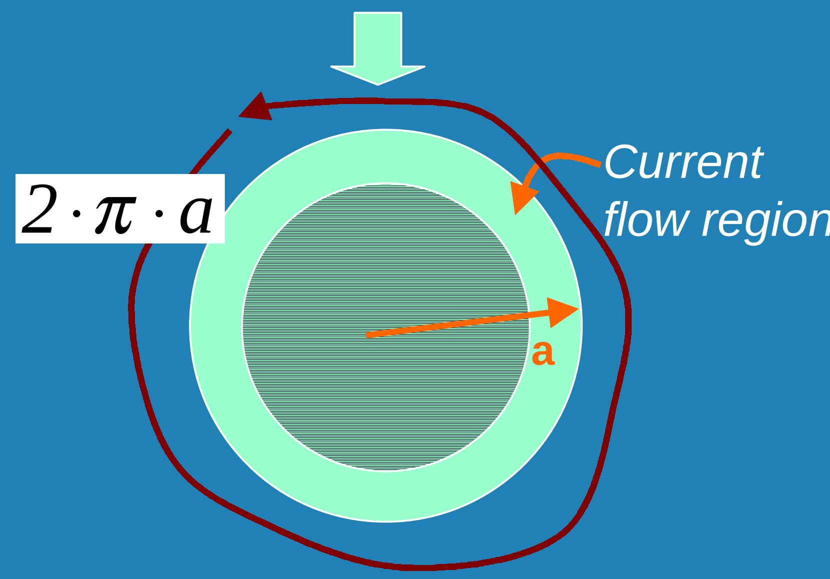
$$= \frac{I}{2 \cdot \pi \cdot a}$$

Surface Current Density

$$j_s = \frac{I}{Area/\delta} = \frac{I}{distance}$$

$$= \frac{I}{2 \cdot \pi \cdot a}$$

distance over which current is distributed



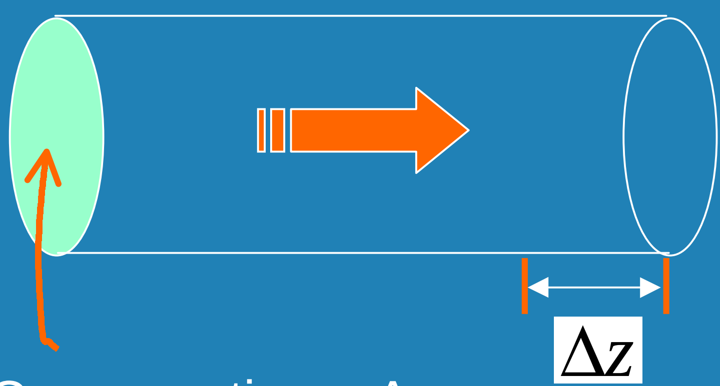
Do Problem 1b

“think of cross-section area and throw out δ”

Current Density

We now obtain an alternate expression for current density

Wire with current, I



Cross-section = A

$$I = \frac{\Delta Q}{\Delta t} \quad \Rightarrow \quad j = \frac{\Delta Q}{A \cdot \Delta t}$$

Assume all particles move at same *v*

- In Δt, all particles within Δz=v.Δt will pass through the right face

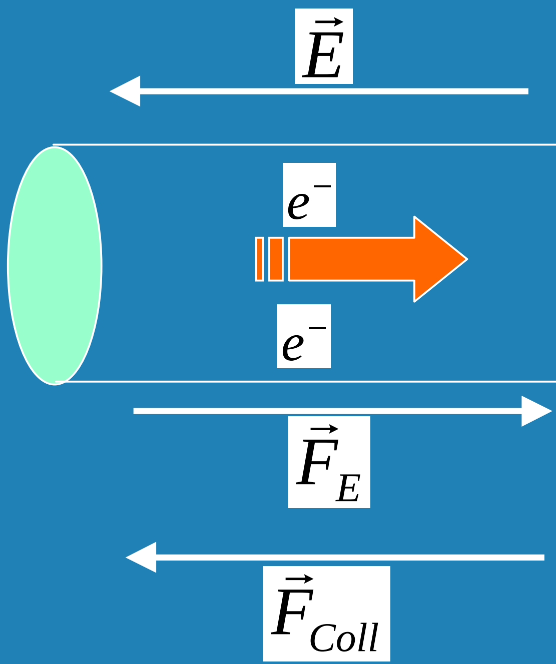
$$\therefore \Delta Q = \rho \cdot \Delta z \cdot A = \rho \cdot v \cdot A \cdot \Delta t$$

$$\Rightarrow j = \frac{\Delta Q}{A \cdot \Delta t} = \frac{\rho \cdot v \cdot A \cdot \Delta t}{A \cdot \Delta t} = \rho \cdot v \quad \Rightarrow \quad \vec{j} = \rho \cdot \vec{v}$$

Average velocity

Do problem 1c

Resistance



$\vec{E}$, in wire pushed electrons to the right

e^- s, collide with lattice ions

$\vec{F}_E$, force (from E-field) driving electrons

$\vec{F}_{Coll}$, balances $\vec{F}_E$ at some $\vec{v}_{average}$

Since $\vec{j} = \rho \cdot \vec{v}_{average}$ $\Rightarrow$ Balance occurs for some value of $\vec{j}$ for a particular $\vec{E}$

Ohm's Law

$$\vec{j} = \sigma \cdot \vec{E}$$

Conductivity - units of S/m or 1/ohm-m

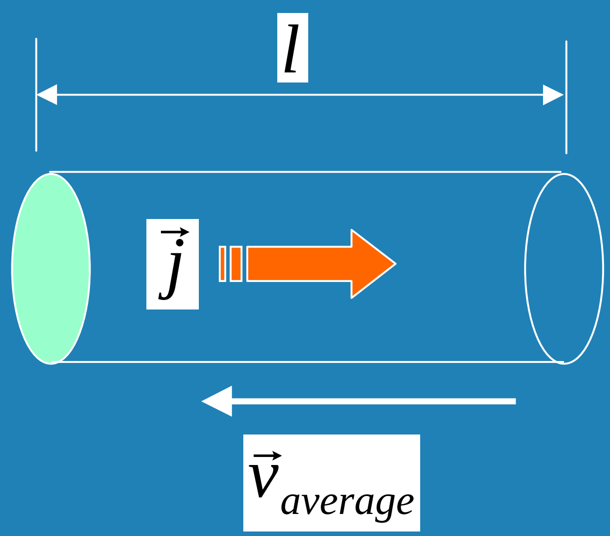
• varies from 10^{7} to 10^{-15}

Good conductor eg. Cu

Good insulator

$\vec{j} = \sigma \cdot \vec{E}$, is Fields and Waves version of Ohm's Law

Ohm's Law



$$I = j \cdot A = \sigma \cdot E \cdot A$$

$$V = -\int \vec{E} \bullet d\vec{l} = E \cdot l$$

$$\frac{V}{I} = \frac{E \cdot l}{\sigma \cdot E \cdot A} = \frac{l}{\sigma \cdot A} = R$$

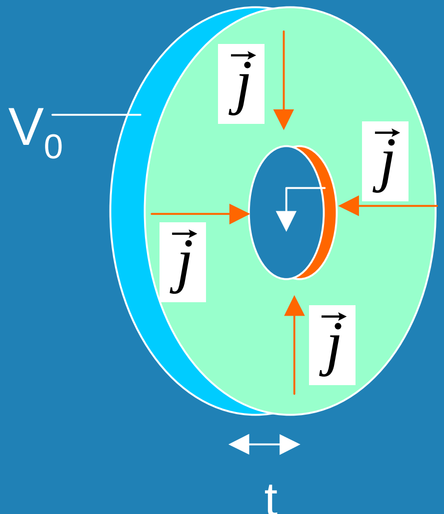
Do problem 2a and 2b…..note for 2b the effective area is an annulus

Ohm's Law

$$R = \frac{l}{\sigma \cdot A}$$ ⇒ Valid if only *j* and *A* are constant

What if they are not? Compute V and I separately and form V/I

Example: Disk with Radial Current



⇒ Look at wedge (sketch)

I is the same at the inner and outer part

Ohm’s Law - alternative calculation

I is the same at the inner and outer part of disk

$$j = \frac{I}{Area} = -\frac{I}{2\cdot\pi\cdot r\cdot t}\cdot\hat{a}_r$$

$$\vec{E} = \frac{\vec{j}}{\sigma} = -\frac{I}{2\cdot\pi\cdot r\cdot\sigma\cdot t}\cdot\hat{a}_r$$

Compute potential difference between inner and outer part of disk:

$$V_b - \underbrace{V_a}_{0} = -\int_a^b \vec{E}\bullet d\vec{l} = \frac{I}{2\cdot\pi\cdot\sigma\cdot t}\Big|_a^b \frac{dr}{r} = \frac{I}{2\cdot\pi\cdot\sigma\cdot t}\ln\left(\frac{b}{a}\right)$$

$$\therefore R = \frac{V}{I} = \frac{\ln(b/a)}{2\cdot\pi\cdot\sigma\cdot t}$$

Do problem 2c