Fields and Waves

Lesson 3.2

MAGNETOSTATICS

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Introduction to B & H Fields

- Introduced to Magnetostatic form of Maxwell's equations
- Calculate B and H fields from I and J in symmetric cases

Maxwell's Equations:

Ampere's Law

$$\nabla \times \vec{H} = \vec{j} \longrightarrow \oint \vec{H} \bullet d\vec{l} = \int \vec{j} \bullet d\vec{s} = I_{net}$$

Integral form

$$\nabla \bullet \vec{B} = 0 \longrightarrow \oint \vec{B} \bullet d\vec{s} = 0$$

For now, $\vec{B} = \mu_0 \cdot \vec{H}$ with μ_0 as a constant

Maxwell's equations

Magnetostatics

$$\nabla \times \vec{H} = \vec{j}$$

$$\nabla \bullet \vec{B} = 0$$

Electrostatics

$$\nabla \times \vec{E} = 0$$

$$\nabla \bullet \vec{D} = \rho$$

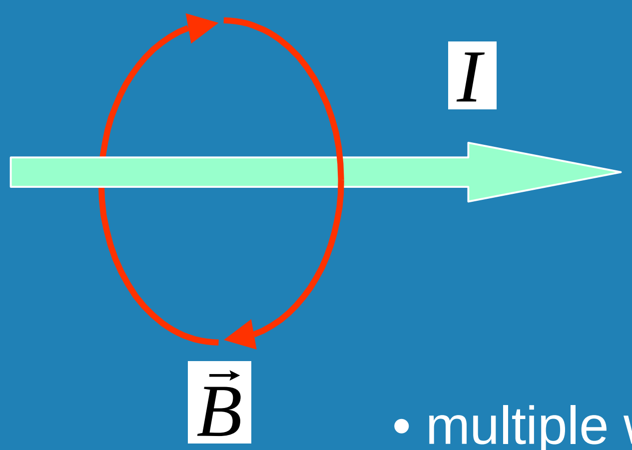
$\vec{B}, \vec{H}$ have curl (rotation) but no divergence (flux)

$\vec{D}, \vec{E}$ do not have curl (rotation) but have divergence (flux)

Do Problem 1

B-Fields

Direction of $\vec{B}$ → $\vec{B}$ wraps around I & $\vec{j}$



Use right-hand rule

- thumb along I & $\vec{j}$
- fingers are in $\vec{B}$
- multiple wires or segments - use superposition

Do problem 2 - add up B-field for different segments - see what cancels and what adds up - use symmetry

Ampere's Law - solve for B & H

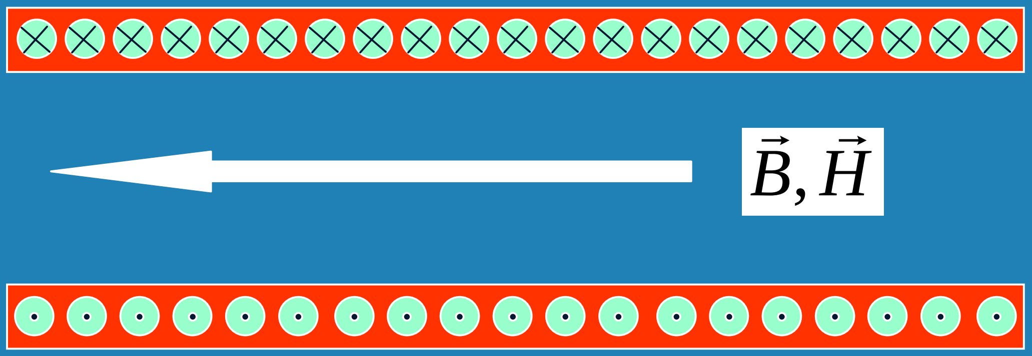
$$\oint \vec{H} \bullet d\vec{l} = \int \vec{j} \bullet d\vec{s} = I_{net}$$

or

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \cdot \int \vec{j} \bullet d\vec{s} = \mu_0 \cdot I_{net}$$

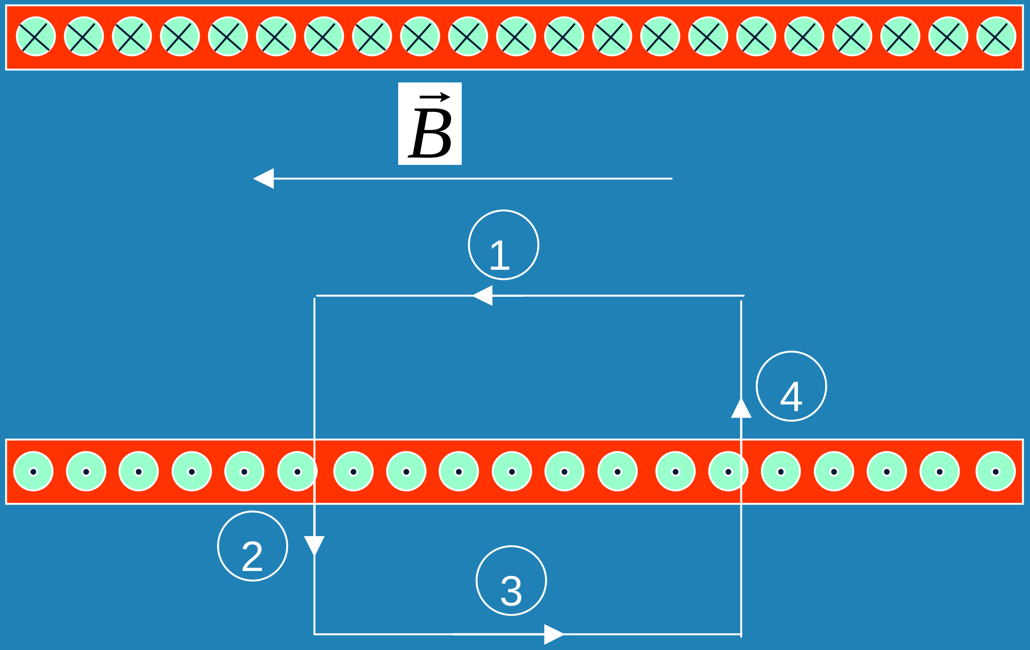
Approach similar to using Gauss' Law, use symmetry to get B-field out of integral

Example: Consider an infinite wire solenoid



sectional view

Ampere's Law - solve for B & H



Find $\vec{B}$

Solenoid has current I through **n** turns/length

STEP 1: Choose path for integral - $\oint \vec{B} \bullet d\vec{l}$

- Chosen paths are 1,2, 3 and 4 - they form a closed loop

Ampere's Law - Infinite Solenoid

STEP 2: Evaluate $\oint \vec{B} \bullet d\vec{l}$

- Segments 2 and 4 have $\vec{B} \perp d\vec{l}$

$$\therefore \oint \vec{B} \bullet d\vec{l} \Rightarrow 0$$

- Segment 3 has $\vec{B} \rightarrow 0$ (will show later)

- Segment 1 has $\oint \vec{B} \bullet d\vec{l} = \int_0^l B_z \cdot dz = B_z \cdot l$

arbitrary length

Ampere's Law - Infinite Solenoid

STEP 3: find I_{net}

- current passing through loop :

$$I_{net} = n \cdot l \cdot I$$

STEP 4: solve for $\vec{B}$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \cdot \int \vec{j} \bullet d\vec{s} = \mu_0 \cdot I_{net}$$

$$\Rightarrow \quad B_z \cdot l = \mu_0 \cdot n \cdot l \cdot I \quad \Rightarrow \quad \vec{B} = \mu_0 \cdot n \cdot I \cdot \hat{a}_z$$

Do problem 3a and 3b